$$\dot{y} = f(t, y)$$

Last time we saw the 4th order Runge-Kutta integrator (RK4):

RK4 ('ode45' based on this)

$$y_{k+1} = y_k + \frac{\Delta t}{6}\left(f_1 + 2f_2 + 2f_3 + f_4\right)$$

$$f_1 = f(t_k, y_k)$$

$$f_2 = f\left(t_k + \Delta t/2,\ y_k + (\Delta t/2) f_1\right)$$ [evaluate vector field after taking half Euler step using f_1]

$$f_3 = f\left(t_k + \Delta t/2,\ y_k + (\Delta t/2) f_2\right)$$ [evaluate VF using half Euler step w/ f_2]

$$f_4 = f\left(t_k + \Delta t,\ y_k + \Delta t\, f_3\right)$$ [take full Euler step w/ f_3].

- Very accurate ($\mathcal{O}(\Delta t^5)$) local accuracy per time step.
- Uses four evaluations of $f(t,y)$, which is typically expensive.

[More evaluations of $f(t,y)$ per time step than Forward Euler, but, many fewer Δt's required for same accuracy!]

Example: Lorentz Equation (1963, atmospheric convection model)

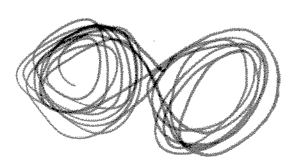
$$\left.\begin{aligned} \dot{x} &= \sigma(y - x) \\ \dot{y} &= x(\rho - z) - y \\ \dot{z} &= xy - \beta z \end{aligned}\right\}$$

parameters may lead to 'chaos'

$\sigma = 10$

$\beta = 8/3$ $\Longrightarrow$

$\rho = 28$



'Butterfly effect'