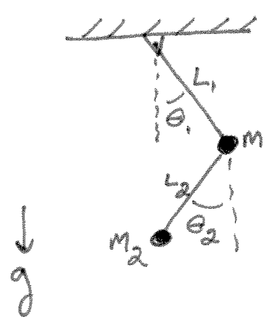
A few words on 'chaos':

- sensitive dependence on initial conditions.
  * We saw this with the small cube of particles in the Lorenz vector field
  * any small errors will be amplified (exponentially) in time

- Example: Double Pendulum Demo.



  * Very simple physical system that exhibits chaos
  * four dimensional phase space: trying to fit into

$$\frac{d}{dt}\theta_1 = \dots$$
$$\frac{d}{dt}\dot{\theta}_1 = \dots$$
$$\frac{d}{dt}\theta_2 = \dots$$
$$\frac{d}{dt}\dot{\theta}_2 = \dots$$

- Chaos may be very challenging for numerical integration
  * most integrators minimize local error at every time step

    However, even very small (10^{-16}) errors will grow rapidly in a chaotic system.

  * Alternative: instead of minimizing local errors, try to preserve conserved quantities:
    - conserve energy ⟹ 'symplectic' integrator
    - make Lagrange's equations as close to satisfied as possible ⟹ 'variational' integrator

Best strategy known today.

Interestingly, RK78 is approximately symplectic...

Example: predicting motion of the planets

* one of the oldest problems in physics/mathematics
* motivation for Poincare to discover chaos in first place.

aside. → (also why Gauss discovered FFT in 1805).
(even two years before Fourier' work)
(150 years before Cooley & Tukey).

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  Development Ephemeris
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Example: Double Gyre ... model of ocean basin mixing.

* show demo Keynote
* show code.