

Homework 3: Vorticity-Streamfunction Equations

DUE: Friday, November 4, 2016 (actually at 3am on 11/5)

The time evolution of the vorticity $\omega(x, y, t)$ and streamfunction $\psi(x, y, t)$ are given by the governing equations:

$$\omega_t + [\psi, \omega] = \nu \nabla^2 \omega \quad (1)$$

where $[\psi, \omega] = \psi_x \omega_y - \psi_y \omega_x$, $\nabla^2 = \partial_x^2 + \partial_y^2$, and the streamfunction satisfies

$$\nabla^2 \psi = \omega \quad (2)$$

Boundary Conditions: Assume periodic boundary conditions for both vorticity and streamfunction.

(a) Using the `spdiags` command, generate the three matrices $\mathbf{A} = \partial_x^2 + \partial_y^2$, $\mathbf{B} = \partial_x$, and $\mathbf{C} = \partial_y$ which take these derivatives in two dimensions.

ANSWERS: With $x, y \in [-10, 10]$, $n = 8$, write out the matrix solutions of your numerical evolution as A1.dat, A2.dat and A3.dat for \mathbf{A} , \mathbf{B} and \mathbf{C} respectively.

NOTE: You can't write out sparse matrices to ASCII files, so be sure to first make the matrices full, i.e. you can let `$\mathbf{A}=\text{full}(\mathbf{A})$` in MATLAB and it will turn a sparse matrix into a full matrix.