CHAPTER 6: POLYHEDRAL STRUCTURES

The Platonic Solids

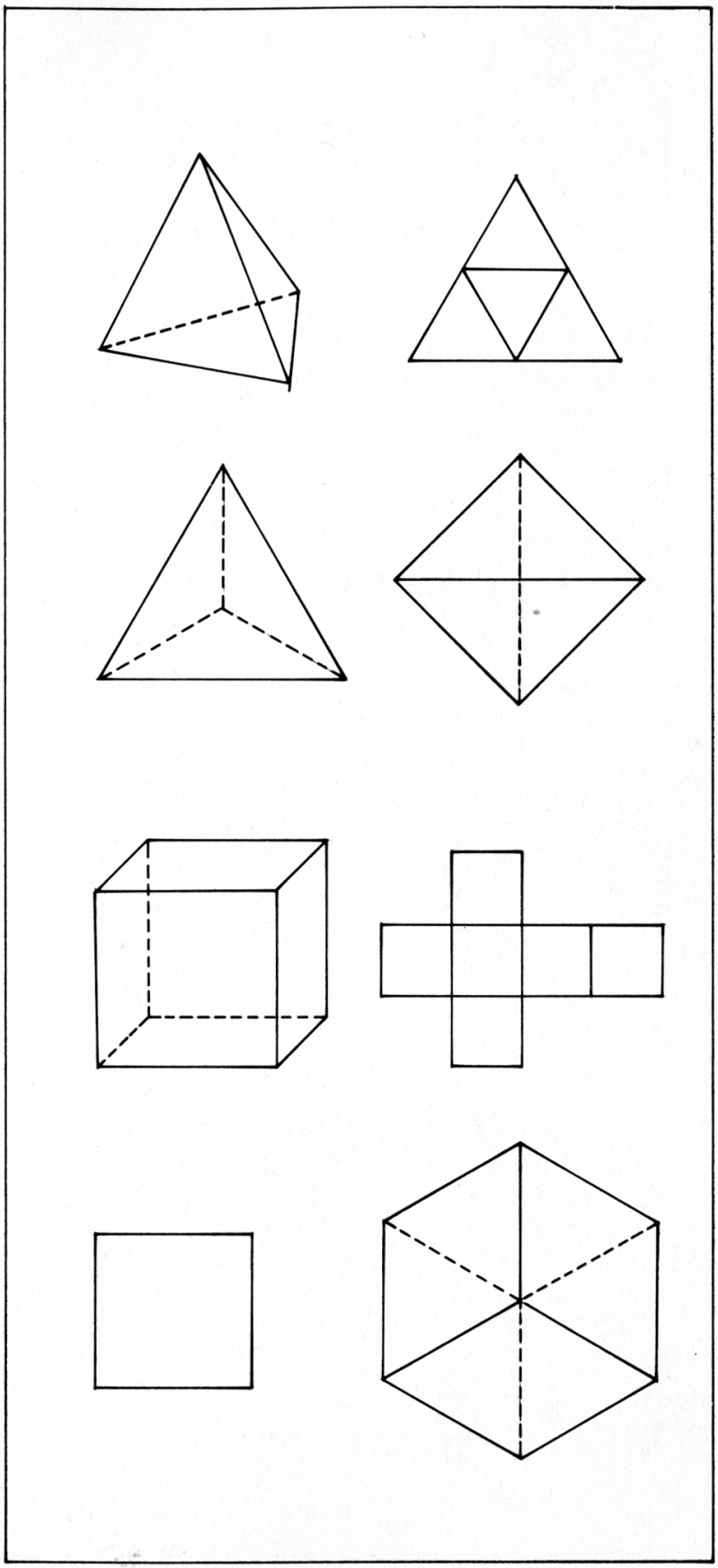
Polyhedra are fascinating shapes, which can be adopted as basic structures in three-dimensional design. Among them are five fundamental regular geometric solids that are of prime importance. As a group they are called Platonic solids, and include the tetrahedron (four faces), the cube (six faces), the octahedron (eight faces), the dodecahedron (twelve faces), and the icosahedron (twenty faces). Each is constructed of regular faces, all congruent, and their vertices are regular polyhedral angles.

The tetrahedron contains four faces, four vertices, and six edges. Each face is an equilateral triangle. (Fig. 204)

If it rests on one of its faces, the plane view is an equilateral triangle. If it rests on one of its edges, in a rather unstable way, then its plane view is, unexpectedly, a square. (Fig. 205)

The tetrahedron is the simplest among the Platonic solids, but it is the strongest structure that can be made by man.

The cube is the best known shape among the Platonic solids. We have mentioned it frequently right from the beginning of this book. It contains the three primary directions and is indispensable for the establishment of the three basic views (see Chapter 1).



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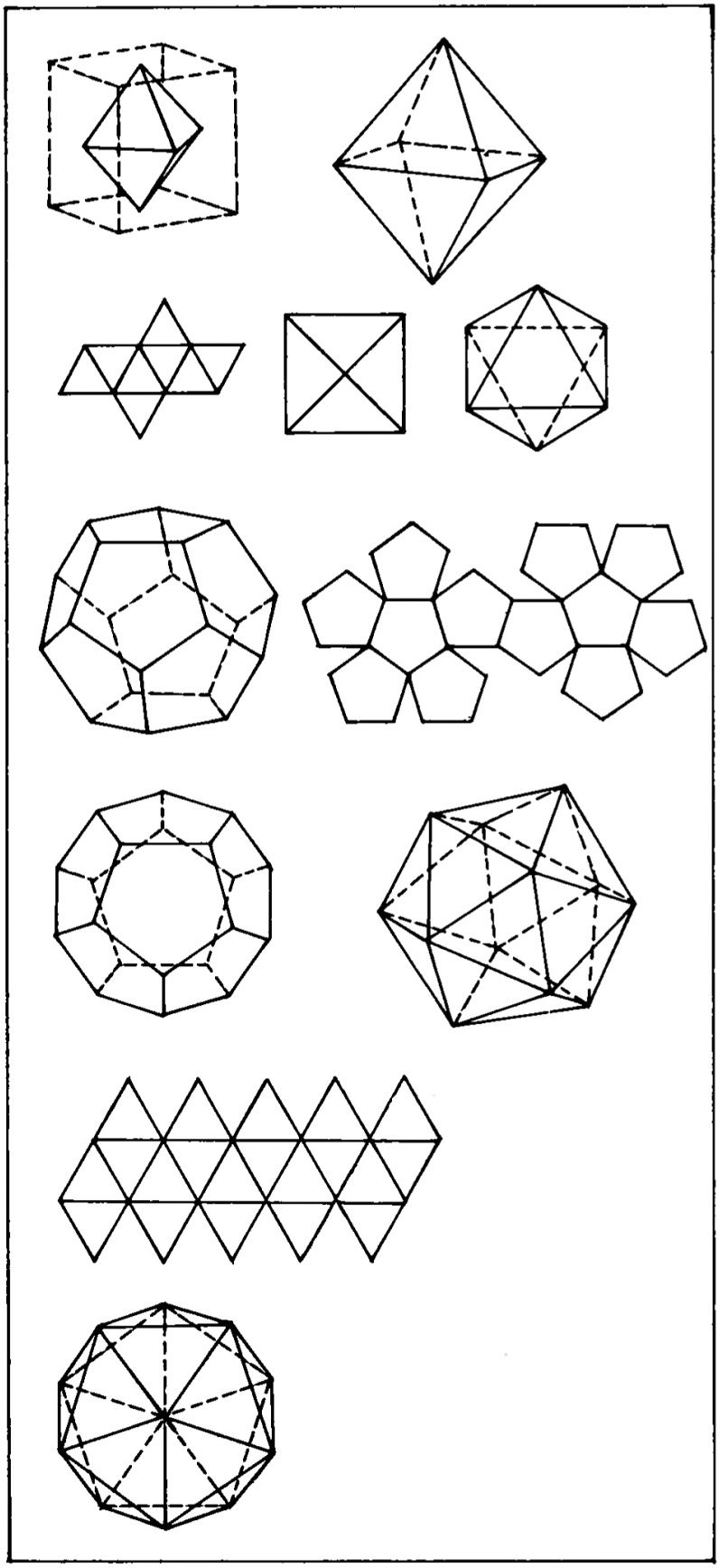


There are six faces, eight vertices, and twelve edges in a cube. Each face is a square. All angles are right angles. (Fig. 206)

If it rests on one of its faces, the plane view is a square. If it rests on one of its vertices, then its plane view is a regular hexagon (six sides). (Fig. 207)

The octahedron is the dual of the cube. This means that to form an octahedron, each vertex of the cube is replaced by a face of the octahedron, and each face of the cube by a vertex of the octahedron. (Fig. 208)

An octahedron has eight faces, six vertices, and twelve edges. Each face is an equilateral triangle. (Fig. 209)

If it rests on one of its vertices, the plane view is a square. If it rests on one of its faces, the plane view is a hexagon (six sides). (Fig. 210)

The dodecahedron is composed of regular pentagons (five sides). It has twelve faces, twenty vertices, and thirty edges. (Fig. 211)

If it rests on one of its faces, the plane view is a regular decagon (ten sides). (Fig. 212)

The icosahedron is the dual of the dodecahedron. It has twenty faces, twelve vertices, and thirty edges. (Fig. 213)

Each face is an equilateral triangle, just as in the tetrahedron and the octahedron. (Fig. 214)

If it rests on one of its vertices, the plane view is also a regular decagon (ten sides). (Fig. 215)

The Archimedean Solids

Besides the five Platonic solids, which are completely regular polyhedra, there are quite a number of semi-regular polyhedra called the Archimedean solids. These semi-regular polyhedra are also constructed of regular polygons. The difference between the Platonic and the Archimedean solids is that each Platonic solid is built of only one type of regular polygon, whereas each Archimedean solid is built of more than one type of regular polygon.

Altogether there are thirteen Archimedean solids, but only the simpler and more interesting ones are introduced here.

The cuboctahedron is one which contains fourteen faces, twelve vertices, and twenty-four edges. (Fig. 216)

Among the fourteen faces, eight are equilateral triangles and six are squares. (Fig. 217)

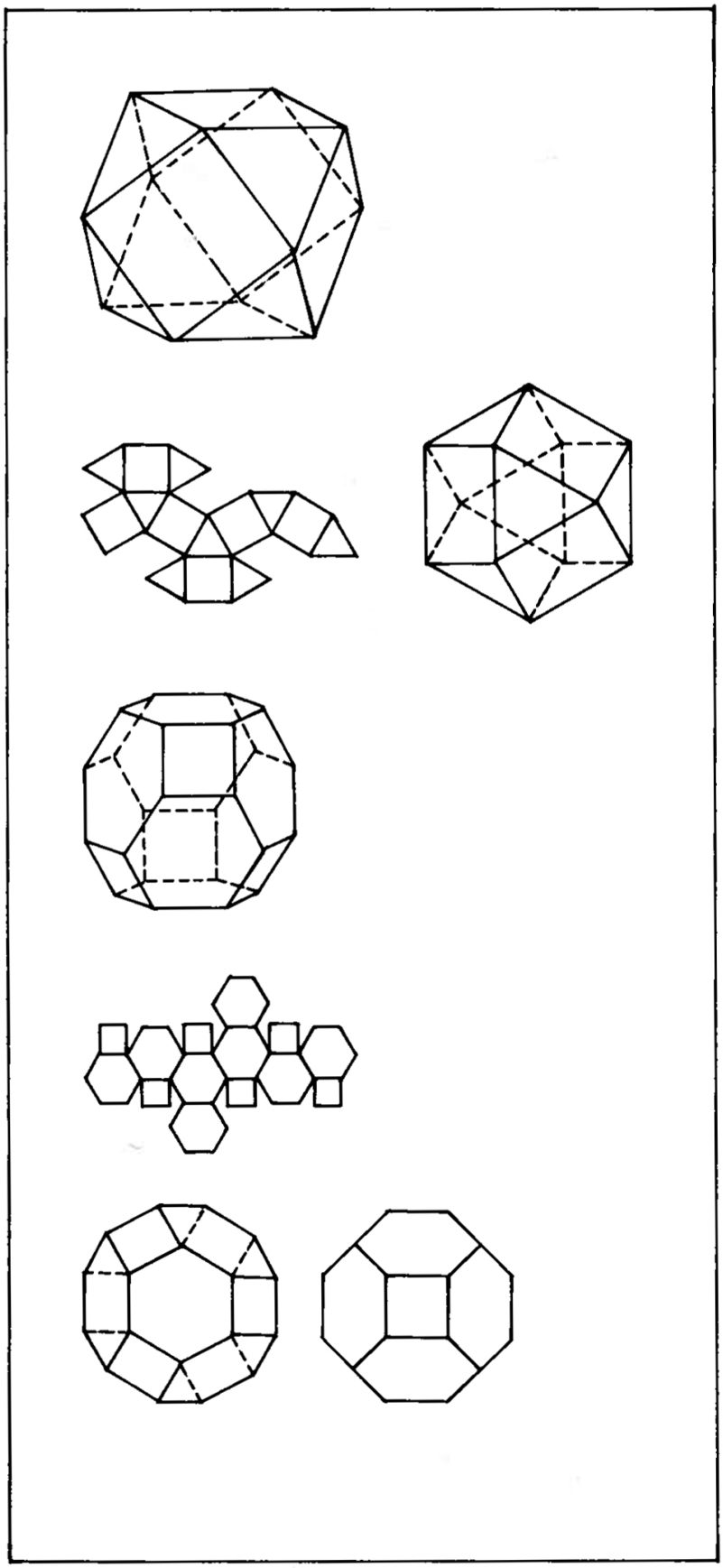
If it rests on one of the triangular faces, the plane view is a hexagon (six sides). (Fig. 218)

The truncated octahedron is one which contains fourteen faces, twenty-four vertices, and thirty-six edges. (Fig. 219)

It is obtained by chopping away the six vertices of an octahedron, and replacing them by six square faces.

Among the fourteen faces, eight are regular hexagons and six are squares. (Fig. 220)

If it rests on one of the hexagonal faces, the plane view is a dodecagon (twelve sides) with



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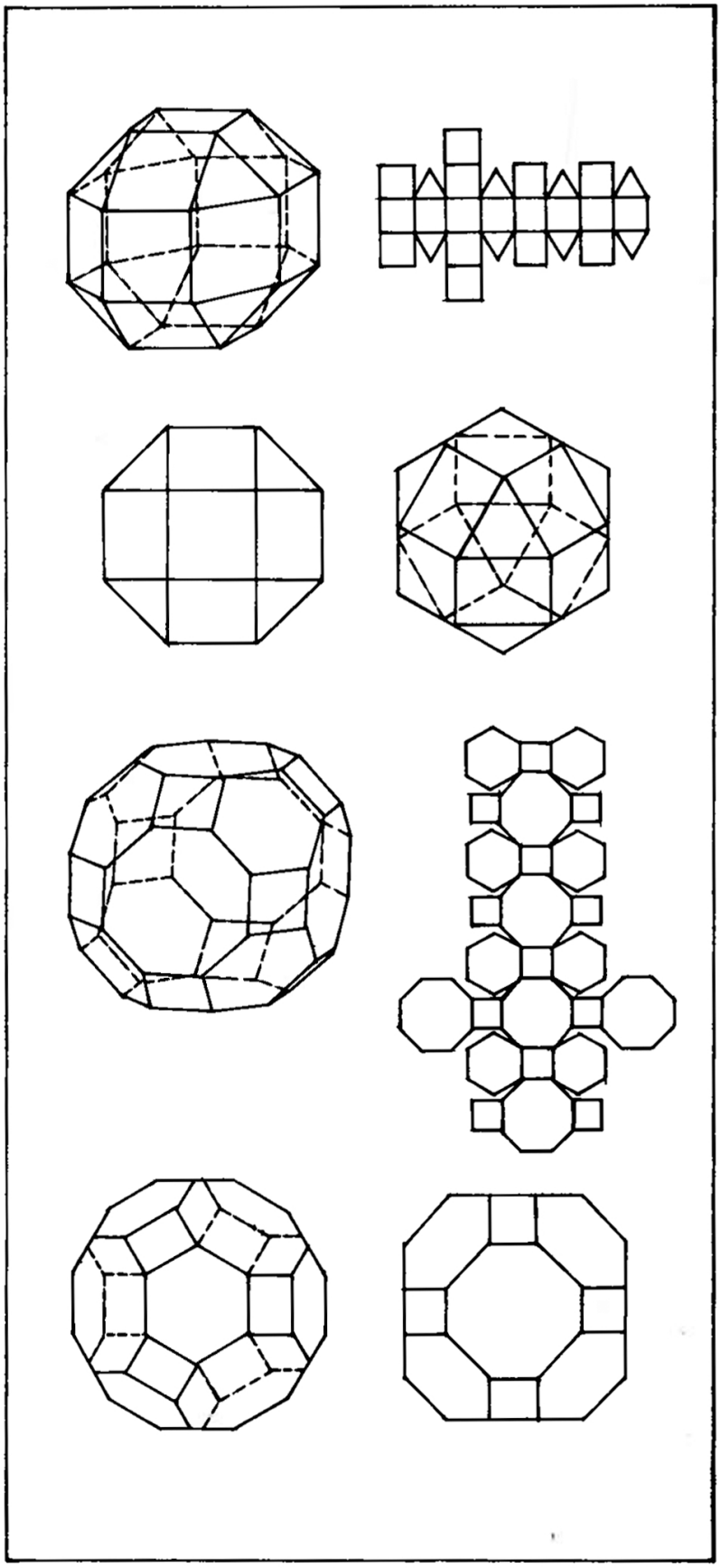

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unequal adjacent sides. If it rests on one of the square faces, the plane view is an octagon (eight sides) with unequal adjacent sides. (Fig. 221)

The rhombicuboctahedron, or small rhombicuboctahedron, to distinguish it from the great one, which is described next, is a solid which contains twenty-six faces, twenty-four vertices, and forty-eight edges. (Fig. 222)

Among the twenty-six faces, eight are equilateral triangles and eighteen are squares. (Fig. 223)

If it rests on one of the square faces, the plane view is a regular octagon (eight sides). If it rests on one of the triangular faces, the plane view is a regular hexagon (six sides). (Fig. 224)

The great rhombicuboctahedron (or truncated cuboctahedron) contains twenty-six faces, forty-eight vertices, and seventy-two edges. (Fig. 225)

Among the twenty-six faces, twelve are squares, eight are regular hexagons (six sides), and six are regular octagons (eight sides). (Fig. 226)

If it rests on one of the hexagonal faces, the plane view is a regular dodecagon (twelve sides). If it rests on one of the octagonal faces, the plane view is an octagon (eight sides) with unequal adjacent sides. (Fig. 227)

Interesting designs can be developed from any of the polyhedra. All provide the fundamental

structure for face treatment, edge treatment, and vertex treatment.

Face Treatment

If the polyhedron has been constructed so that it is hollow inside, the simplest face treatment is to make negative shapes on some or all of the faces, revealing the empty space inside. (Fig. 228)

Each entire flat face of the polyhedron can be replaced by an inverted or projected pyramidal shape, constructed of joined or interlocking planes. In this way the external appearance of the polyhedron may be transformed into a stellated polyhedral shape. (Fig. 229)

Separately constructed shapes can be attached to the faces of the polyhedron. (Fig. 230)

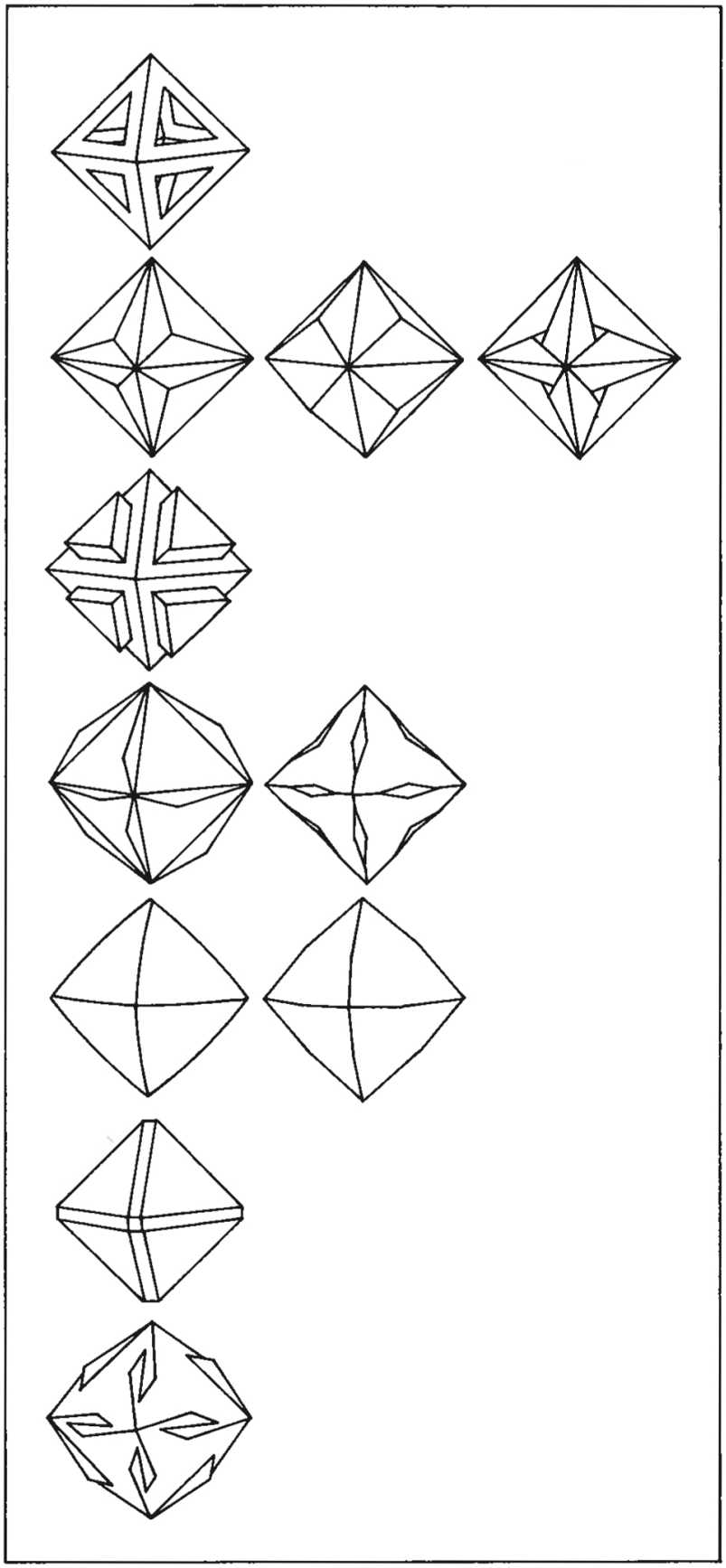
Edge Treatment

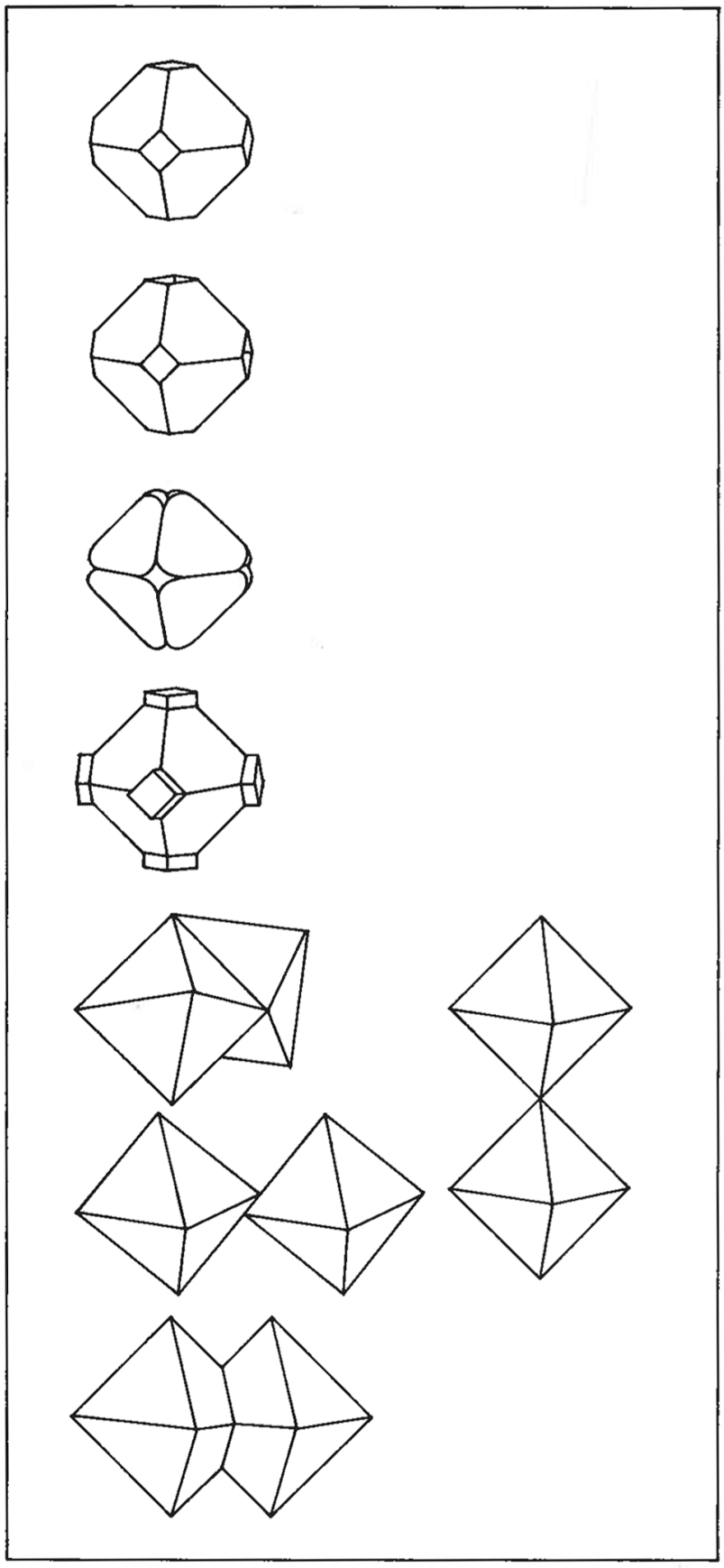
Along the edges of a polyhedron, shapes can be added or subtracted. When they are subtracted, faces are also affected because we cannot remove anything from an edge without removing a part of the adjoining faces. (Fig. 231)

Straight edges of the polyhedron can become curvilinear or bent. This will cause the flat faces to bulge or cave in, in accordance with the new edge shapes. (Fig. 232)

Each single-line edge can be replaced by double- or multi-line edge, and this will lead to the creation of new faces. (Fig. 233)

Interlocking of the face planes along the edges can take place in varied ways. (Fig. 234)





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Vertex Treatment

Vertex treatment normally affects all the faces which join one another at the single point of the vertex. One way to treat vertices is by truncation, which means that the vertices are cut off and new faces are formed on the cut areas. Truncation usually leads to creation of a new polyhedral shape. We have already described the truncated octahedron among the Archimedean solids. The polyhedron illustrated here, however, is not an Archimedean solid because the triangular faces have not been transformed into regular hexagons after truncation. (Fig. 236)

If the polyhedron is hollowed, truncation reveals a hole at each vertex. Such holes may be specially treated so that the borders are not just simple straight lines. (Fig. 237)

Additional shapes can be formed on the vertices. (Fig. 238)

Joining of Polyhedral Shapes

For a more complicated structure, two or more polyhedral shapes of the same or different designs can be joined together by face contact, edge contact, or vertex contact. (Fig. 239)

For greater structural strength or for design reasons, vertices can be truncated during vertex contact, edges flattened during edge contact, or the volume of one polyhedral shape made to penetrate the volume of another. (Fig. 240)

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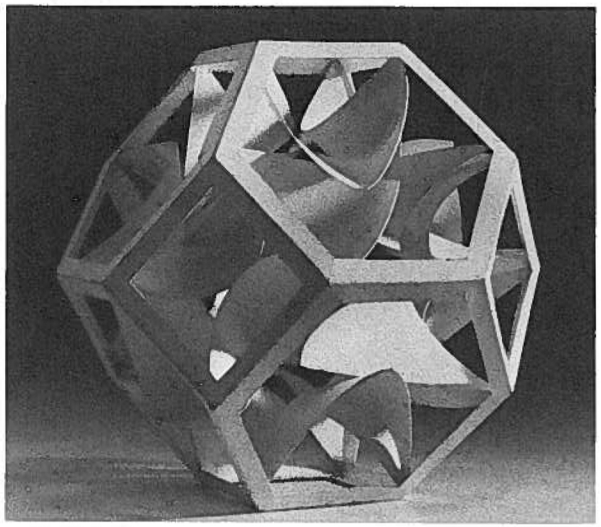

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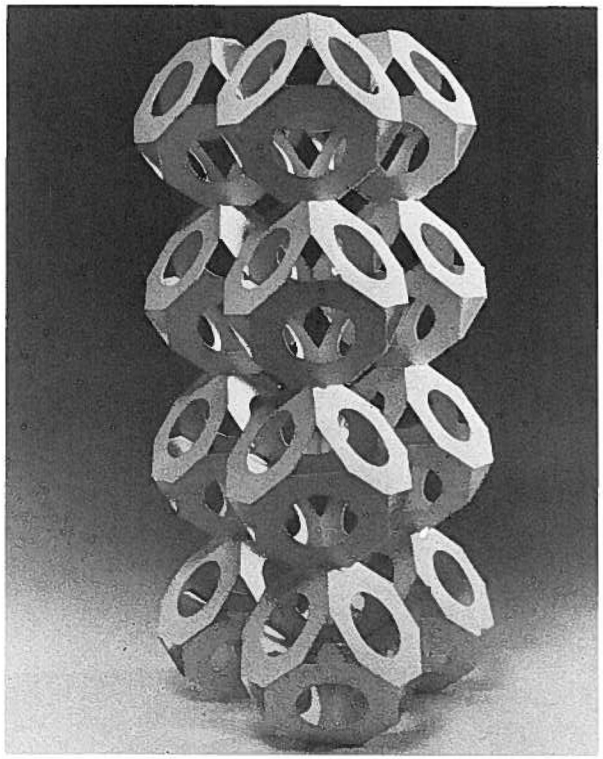

Figure 249—most parts of the faces of the truncated octahedron have been cut away. The main activity of the design takes place inside the polyhedral framework.

Figure 250—twelve truncated cubes have been used to compose this design. Each face of the cubes contains a negative circular shape which resonates visually with the triangular holes formed at the truncated vertices.

Figure 251—here the faces of the great rhombicuboctahedron have been treated with shapes projecting both inward and outward.

Figure 252—a dodecahedron has been used as the basic structure for this design. On each of the pentagonal faces, a pentagonal pyramid is built, but all the faces are stripped to the edges. The vertex of the pyramid, instead of projecting all the way out, is pushed inward. The result is a complicated design composed completely of linear elements.

Figure 253—this design is composed of two truncated octahedra, each of which shows a play of negative shapes and concave and convex forms.

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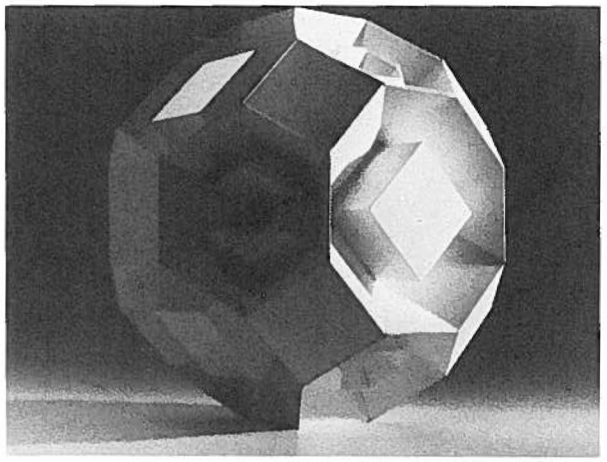

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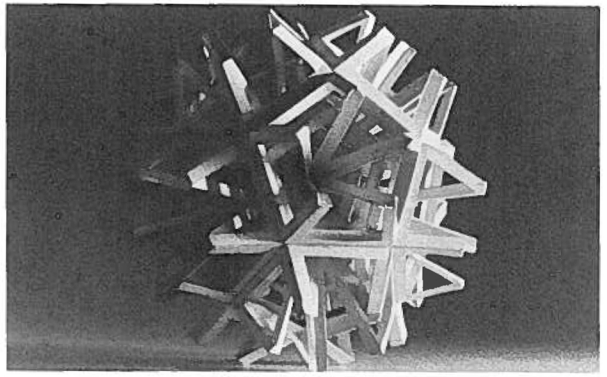

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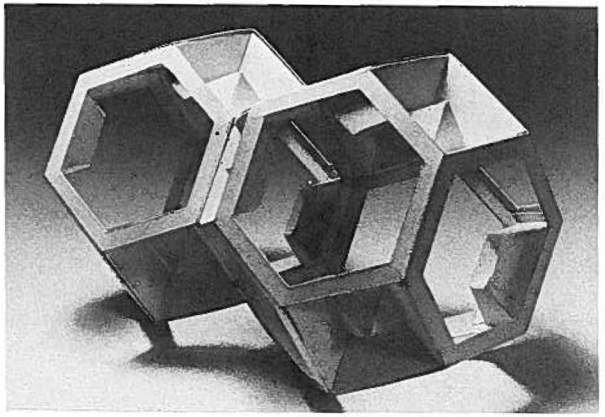

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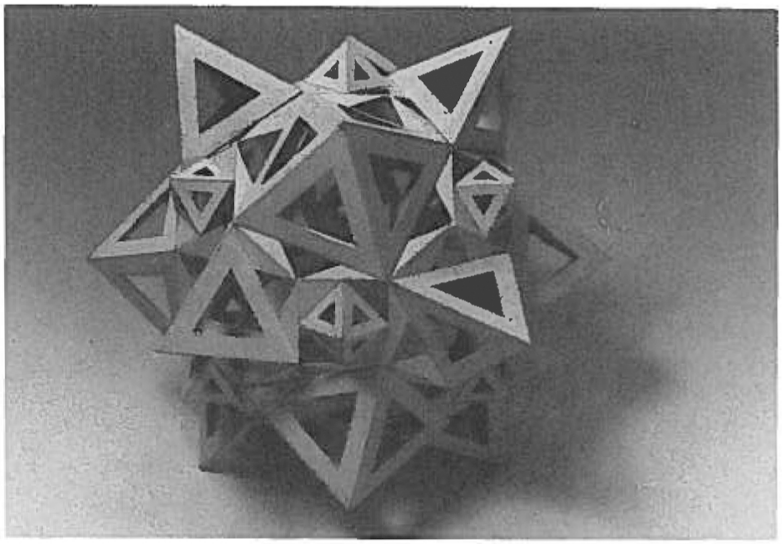

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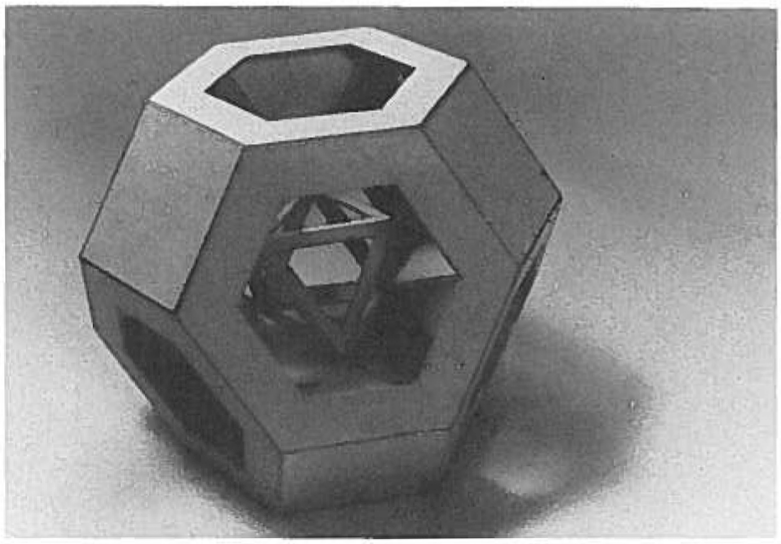

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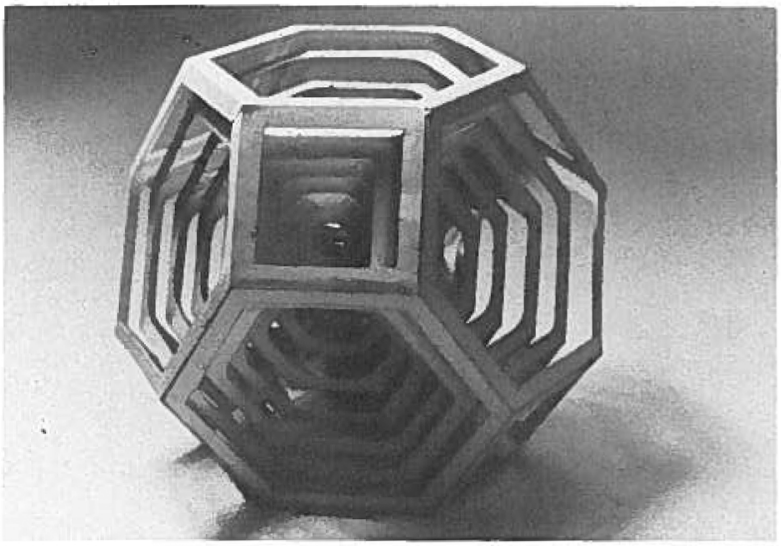

Figure 244—the structure for this complicated design is the great rhombicuboctahedron, which consists of octagonal, hexagonal, and square faces. Negative shapes are cut on all the faces and tetrahedral and semi-octahedral shapes are added.

Figure 245—a negative hexagonal shape is made on each of the hexagonal faces of a truncated octahedron, through which one can see the interesting interior polyhedral shape. It is a linear octahedron set among inwardly pointing square and hexagonal pyramids built on the underside of the faces.

Figure 246—the structure of this design is also the truncated octahedron. All the faces have been stripped to the edges, revealing six layers of the same shape in size gradation contained inside.