ATMOSPHERIC SCIENCES 340

WINTER QUARTER 2009

PROBLEM SET 5

DUE FRIDAY MARCH 13 AT 9:30 AM

(Problem 6 is on reverse)

1. Prove that the equilibrium saturation vapor pressure over a pure water droplet of radius, *r*, decreases with increasing temperature if:

$$r < \frac{2\sigma}{L\rho_l}$$

where σ is the surface energy of water, *L* is the latent heat of condensation, and ρ_l is the density of liquid water. (Hint: use Kelvin's equation, the Clausius-Clapeyron equation, and the ideal gas equation.) If $\sigma = 72 \times 10^{-3}$ N m^{-1}, L = 2.5 x 10^6 J kg^{-1}, and 1000 kg m^{-3}, how small does *r* have to be to meet this condition?

2. Calculate the equilibrium relative humidity (in %) over a droplet of radius 0.04 μm at a temperature of 15°C if the droplet has 2×10^{-17} gram NaCl dissolved in it. Use a surface energy of 7.3 x 10^{-2} N m^{-1}, and assume the density is approximately the same as that of pure liquid water (1000 kg m^{-3}). Atomic weights of H, O, Na, and Cl are 1, 16, 23 and 35, respectively. The gas constant for water is 461 J K^{-1} kg^{-1}.

3. Derive an expression for the height above cloud base, *h*, as a function of time, *t*, of a cloud droplet that is growing by condensation only in a cloud with a steady updraft velocity, *w*, and supersaturation, *S*. Assume that the droplet's terminal fall speed is given by $\nu_t = ar^2$, where *a* is a constant and *r* is the droplet radius.

4. How long would it take for a drop which has an initial radius of 50 μm to grow to a radius of 500 μm by the continuous collection of very small droplets that have a liquid water content of 0.5 g m^{-3}? Assume that the terminal fall speed of the drop is given by $\nu_t = ar^2$, where *a* is a constant equal to 3 x 10^{-3} cm s^{-1} μm^{-2} and *r* is the drop radius in μm. Also assume that the collected droplets are stationary and are collected with an efficiency of 0.5. The density of liquid water is 1000 kg m^{-3}.

5. Calculate the radius and the mass of an ice crystal after it has grown by deposition from the vapor phase for half an hour in a water-saturated environment at -5°C. Assume that the crystal is a thin circular disk with a constant thickness of 10 μm. The electrostatic capacity, *C*, of a circular disk

of radius, r, is given by $C = 8\pi\varepsilon_0$. Assume the density of ice is 917 kg m^{-3}. Hint: an estimate needs to be made from W&H Fig. 6.39.

6. A flat snowflake is falling through a region of smaller ice crystals and growing by collection. How long does it take the snowflake to grow from a radius of 1 mm to a radius of 1 cm? Assume the snowflake's volume is given by $V = \frac{r^3}{2}$; its density is 100 kg m^{-3}; its velocity is given by $v_1 = A\left(\frac{r}{r_0}\right)^{0.25}$, where $A = 1.0$ m s^{-1} and $r_0 = 2.0$ mm; the water content of the small ice particles is 0.3 g m^{-3}; they have zero fall velocity; and they are collected with an efficiency of 0.85.