Of Ice and Statisticians†: Interpreting Measurements of Arctic Sea Ice Thickness

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†with apologies to William Shakespeare and John Steinbeck

Considering a Career in Statistics?
You Might Be Interest to Know That ...

- according to a comprehensive ranking of 200 different jobs by JobsRated.com, the three best professions are
  1. mathematician
  2. actuary
  3. statistician!!!
- the three worst professions are

198. taxi driver
199. dairy farmer
200. lumberjack

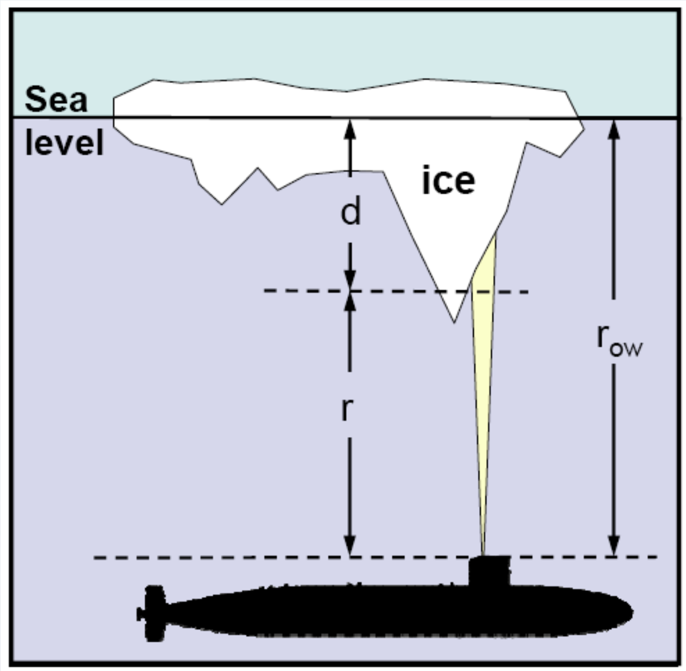
Why I Like Being a Statistician

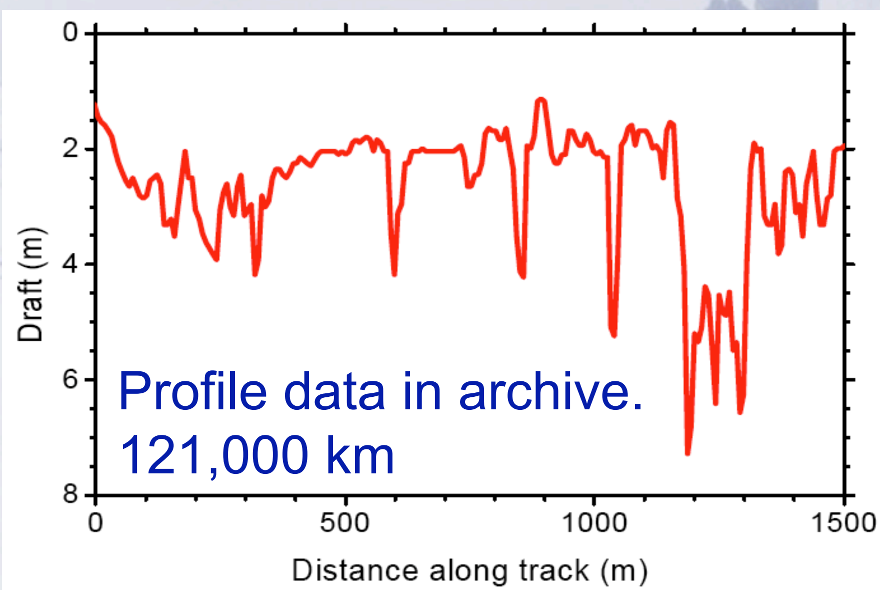
- in exchange for providing help with statistics, get to work with highly motivated folks passionate about their fields of expertise
- since statistics is used in a wide range of applications, have an opportunity to learn something about *many* different areas (don't have to get 'stuck' in any one particular area of interest)
- some problems I have had a chance to work on:
  - assessing performance of atomic clocks
  - deciphering Martian annual atmospheric pressure cycles
  - characterizing vertical shear and turbulence in the ocean
  - assessing effect of hormone therapy on menopausal transition
  - forecasting hazards to coastal communities due to tsunamis
  - interpreting thickness of Arctic sea ice (today's topic)

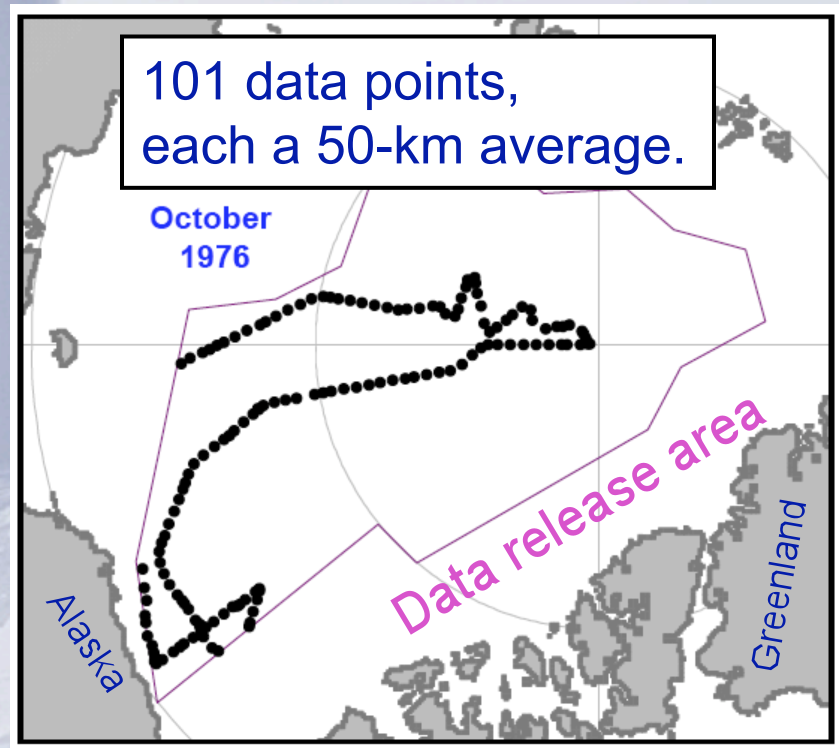
Some Background

- joint effort with Drew Rothrock (APL), Tilmann Gneiting (Department of Statistics), Mark Wensnahan (APL) and Alan Thorndike (Department of Physics, University of Puget Sound)
- scientific question of interest: has the average thickness of Arctic sea ice declined significantly over the past 30 years?
- thickness can be deduced from measurements of draft (submerged portion of sea ice)
- draft measured using upward-looking sonars on submarines
- our effort differs from previous ones by use of
  - a new statistical model for draft measurements
  - newly archived data for submarine cruises from 1975 to 2001 (almost doubling the amount of available data)

Ice Draft from Upward-Looking Sonar





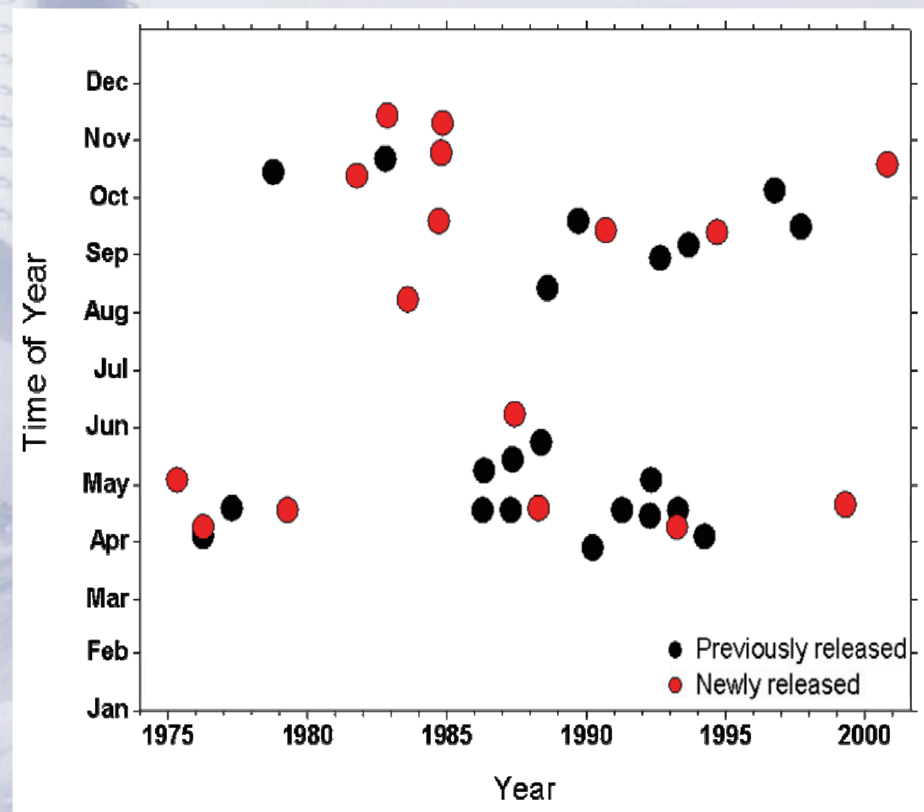


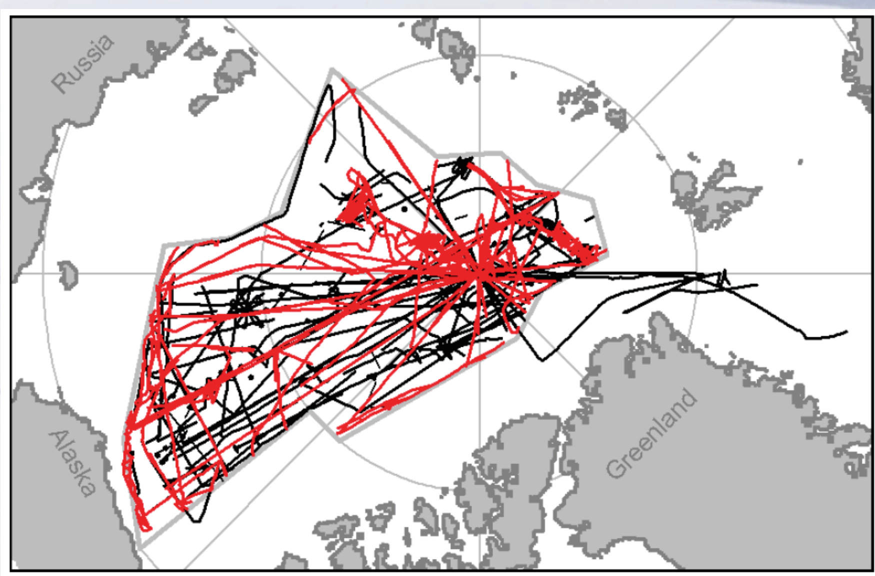
(Wensnahan et al., *EOS*, Jan., 2007)

Submarine Cruises

- NSIDC has draft data from 121,000 km of cruise tracks
- New data (2006) in red added 81%.

Month vs Year

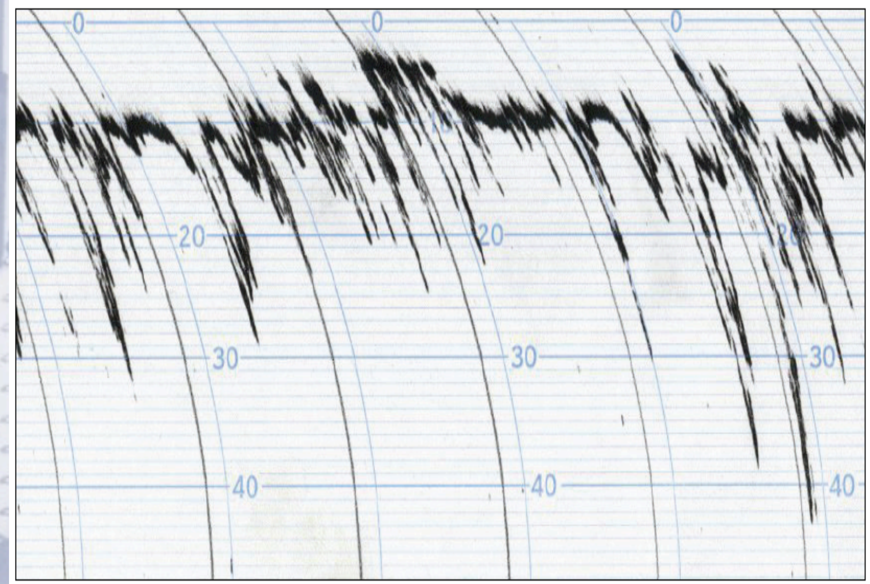


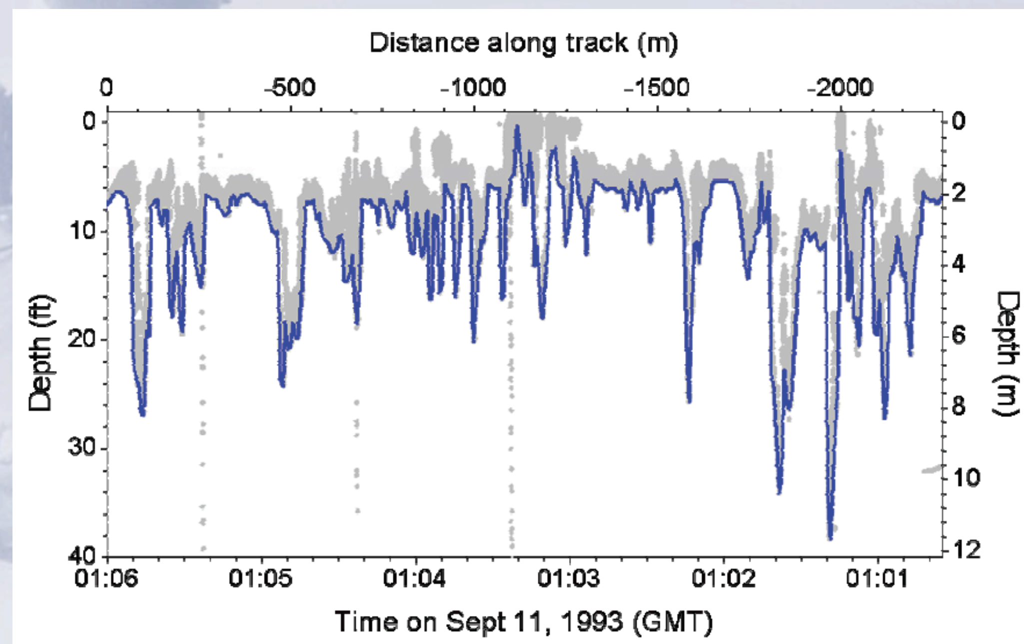


Data are archived at the National Snow and Ice Data Center
[*Google*: NSIDC]

Paper Charts (Analog)

are scanned and
a digital trace extracted.





(Wensnahan & Rothrock, *GRL*, 2005; Wensnahan et al., *EOS*, 2007)

Model for Sea Ice Thickness Data

- let $\overline{H}_{\mathbf{x},t}$ represent average of 1 km measurements taken at location $\mathbf{x}$ and time t ($\mathbf{x} = [0, 0]$ = Pole & $1975 \leq t \leq 2001$)
- let τ represent time relative to start of year
- assume so-called 'multiple regression' model:

$$\overline{H}_{\mathbf{x},t} = C + I(t) + A(\tau) + S(\mathbf{x}) + \epsilon_{\mathbf{x},t},$$

  where

  - C is the overall mean ice thickness
  - $I(t)$ is the interannual variation (from one year to the next)
  - $A(\tau)$ is the variation within a year (annual cycle)
  - $S(\mathbf{x})$ is the spatial field
  - $\epsilon_{\mathbf{x},t}$ is an error term

Understanding the Statistical Properties of $\overline{H}_{\mathbf{x},t}$: I

- at a given time t and location $\mathbf{x}$, we form $\overline{H}_{\mathbf{x},t}$ by averaging together $L = 1000$ basic measurements $H_{\mathbf{x},t,l}$, $l = 1, 2, \ldots, L$
- each basic measurement $H_{\mathbf{x},t,l}$ comes from the upward-looking sonar and is the ice thickness averaged over a 1 meter patch
- assume each $H_{\mathbf{x},t,l}$ comes from a population with an unknown mean μ and unknown variance σ^2 (i.e., a standard deviation of σ), where $\mu = C + I(t) + A(\tau) + S(\mathbf{x})$
- since basic measurements $H_{\mathbf{x},t,l}$ and data $\overline{H}_{\mathbf{x},t}$ are related by

$$\overline{H}_{\mathbf{x},t} = \frac{1}{L}\sum_{l=1}^{L} H_{\mathbf{x},t,l},$$

  $\overline{H}_{\mathbf{x},t}$ is a sample mean that estimates the unknown μ

Understanding the Statistical Properties of $\overline{H}_{\mathbf{x},t}$: II

- can estimate unknown variance σ^2 using the sample variance:

$$\hat{\sigma}^2 = \frac{1}{L-1}\sum_{l=1}^{L}\left(H_{\mathbf{x},t,l} - \overline{H}_{\mathbf{x},t}\right)^2$$

- σ^2 is the variance associated with each individual $H_{\mathbf{x},t,l}$
- if the $H_{\mathbf{x},t,l}$ came from a random sample, theory says variance associated with the sample mean $\overline{H}_{\mathbf{x},t}$ is σ^2/L; i.e., the variability in $\overline{H}_{\mathbf{x},t}$ would decrease at a rate given by L^{-1}
- alas, standard statistical theory is problematic because $H_{\mathbf{x},t,l}$ cannot reasonably be regarded as coming from a random sample
- Q: what can go wrong if you don't have a random sample?

Proper Use of Standard Statistical Theory: I

- in late March of every year, the Acme Beer Corporation (ABC) sends the Ninety-Nine Company (NNC) three packages of beer, with 33 bottles in each package, for a total of 99 bottles of beer (considered to be a random sample from a population of beers)
- NNC has 99 employees, identified by $l = 1, 2, \ldots, 99$
- at 5PM on 1 April of each year, employee #1 opens the three packages and takes out the 99 beers, pouring the contents into 99 glasses labelled by $l = 1, 2, \ldots, 99$
- employee $\#l$ takes glass l, but, prior to drinking the beer, pours it through a beer analyzing machine (BAM), which measures % hop content H_l (with no error and no loss of beer)
- employee #99 sends data H_1, H_2, …, H_{99} back to ABC

Proper Use of Standard Statistical Theory: II

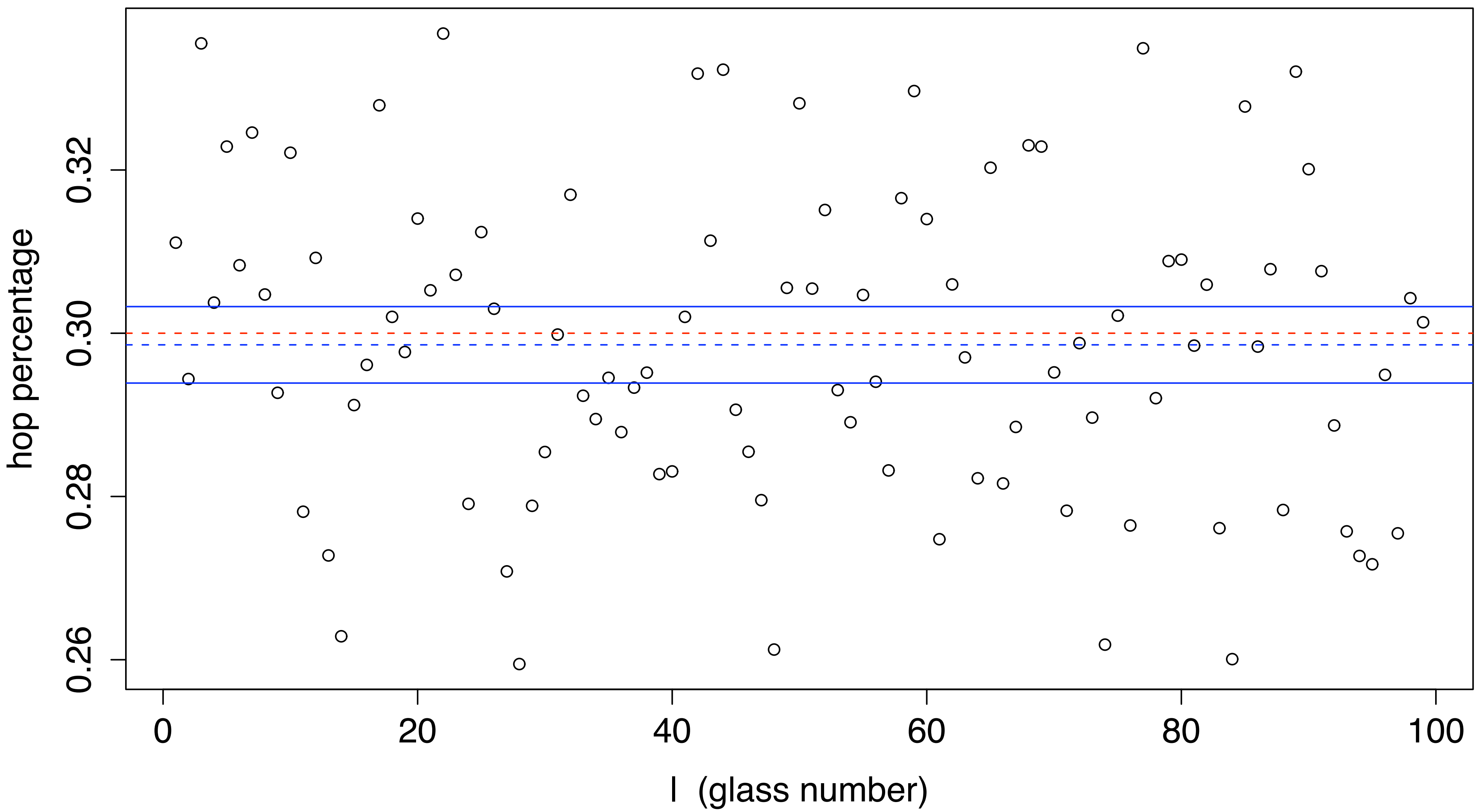
- based upon $H_1, H_2, \ldots, H_{99}$, an ABC statistician† computes the sample mean $\overline{H}$, the sample variance $\hat{\sigma}^2$ and then a 95% confidence interval for the true unknown % hop content:

$$[\overline{H} - 1.96\sqrt{(\hat{\sigma}^2/99)}, \overline{H} + 1.96\sqrt{(\hat{\sigma}^2/99)}]$$

- if the target % hop content falls in this interval, upper management is happy and gives a bonus to everyone at ABC
- following plot shows data H_l for 2007 (circles), their sample mean (blue dashed line), the 95% confidence interval (blue solid lines) and the target % hop content (red dashed line)
- bonuses granted – hurray!

†quite happy, by the way, with his/her job!

Results for 2007



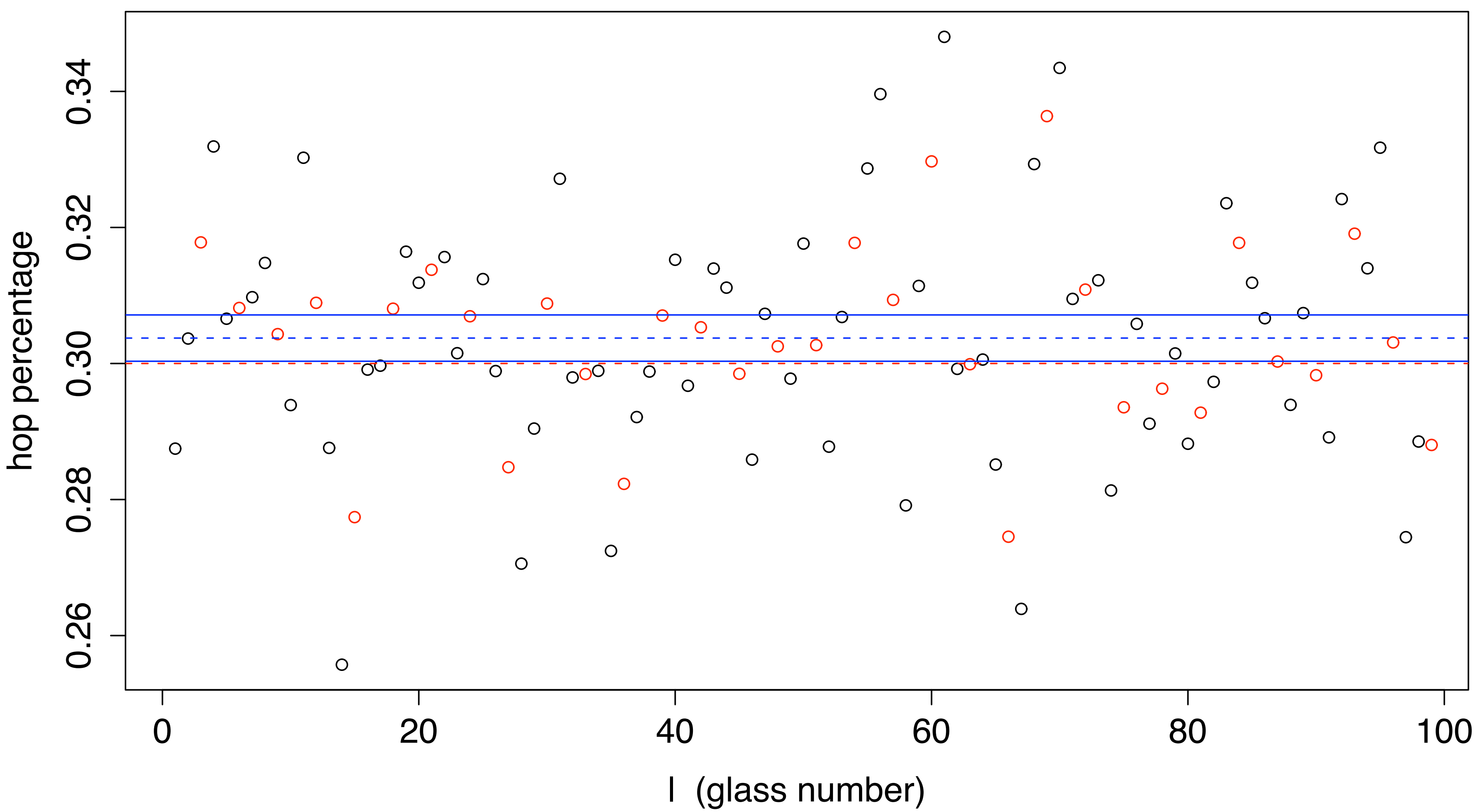
Misuse of Standard Statistical Theory: I

- in 2008, disaster struck: while unloading the beer, NNC employee #13 dropped one package, destroying 33 bottles of beer
- to save the day, employee #1 came up with following scheme
  - pour 66 bottles of beer into glasses 1,2,4,5,7,8,...,94,95,97,98, leaving glasses 3, 6, 9, ..., 96, 99 empty to start with
  - pour one third of glasses 2 & 4 into glass 3
  - pour one third of glasses 5 & 7 into glass 6
  - pour one third of glasses 8 & 10 into glass 9
  - ...
  - pour one third of glasses 95 & 97 into glass 96
  - pour one third of glasses 98 & 1 into glass 99
- now have 99 glasses of beer (each two thirds filled), so proceed as usual (NNC employees were drinking too much anyway!)

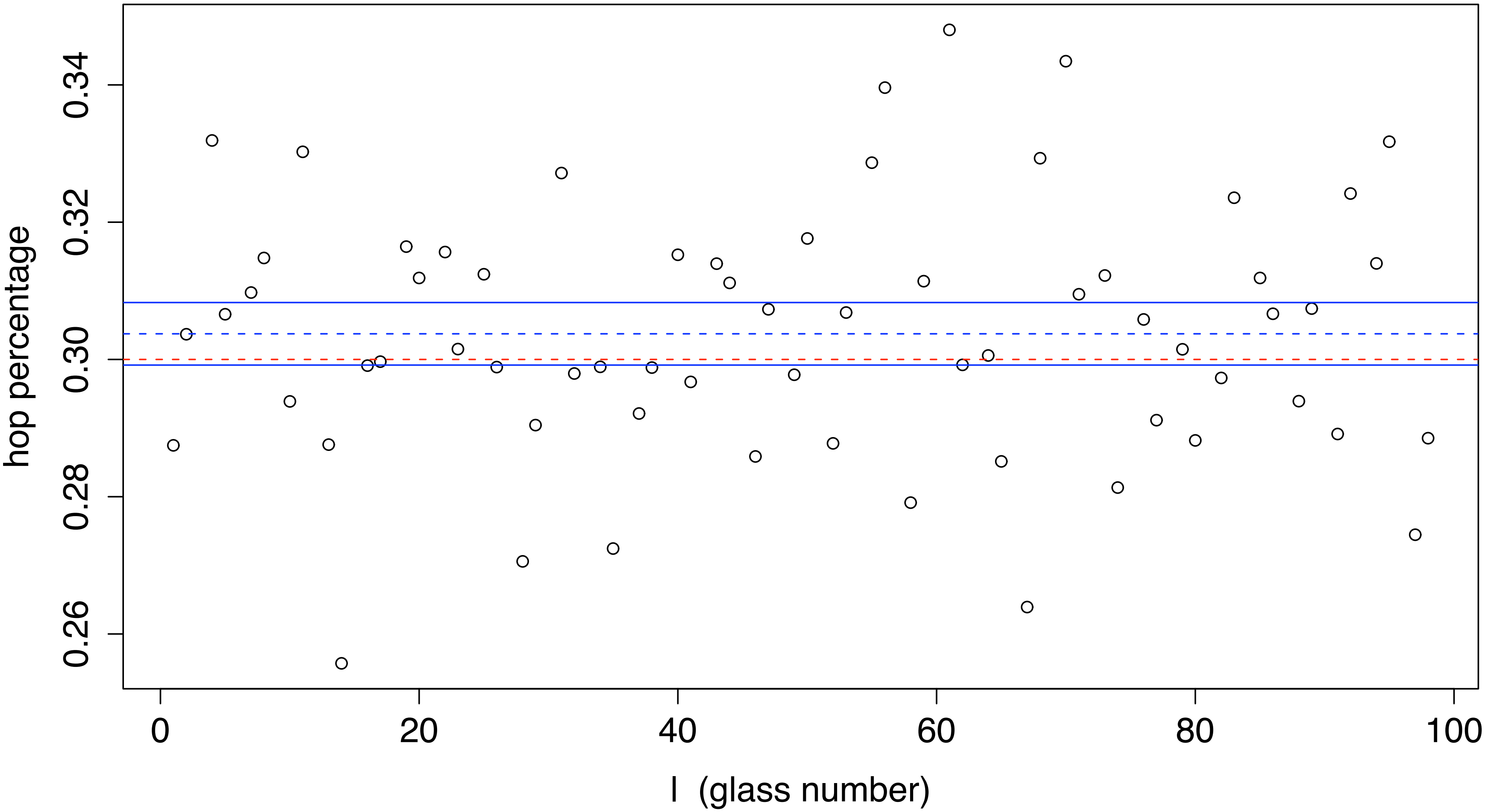
Misuse of Standard Statistical Theory: II

- employee #1's scheme introduces *correlation* into the data: data for glass 3 is an exact average of those for glasses 2 & 4 etc.
- we no longer have a random sample of size 99, so treating data as such leads to potential problems, as illustrated by the following two plots

Incorrect Results for 2008



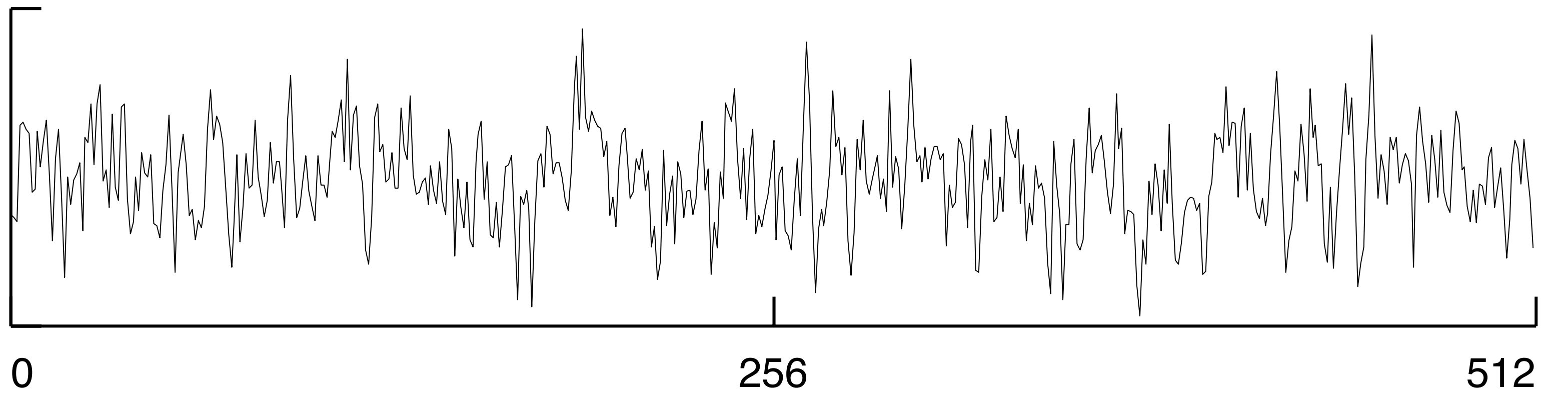
Correct Results for 2008



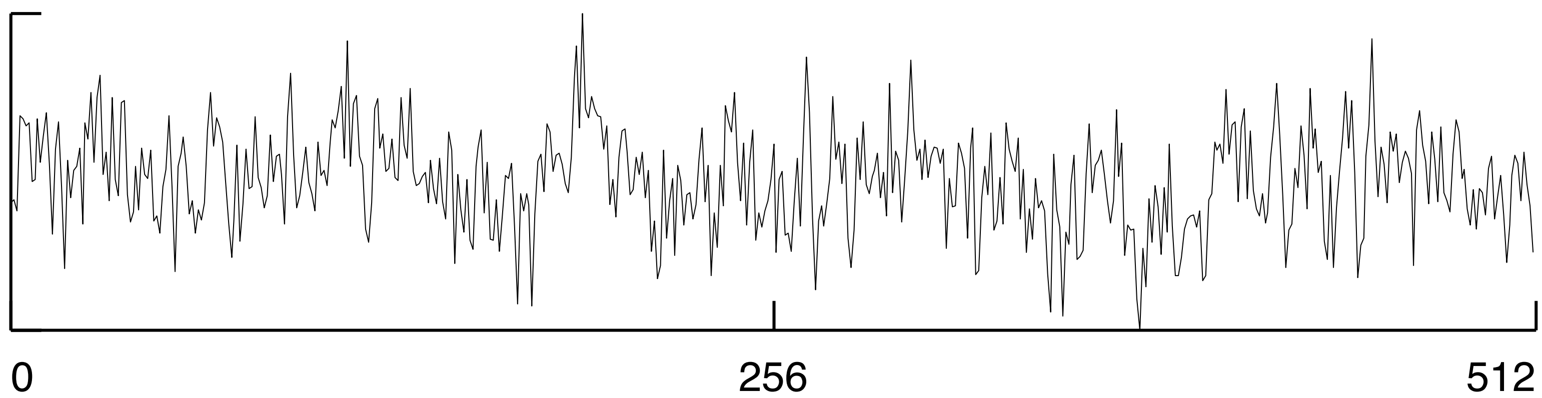
Taking Correlated Measurements into Account: I

- assessing the variability in a sample mean $\overline{H}$ when dealing with correlated data requires an adjustment to the rule that the variance of $\overline{H}$ is given by σ^2/L
- depending on the exact nature of the correlation, appropriate adjustment can take different forms
- for sea ice data, considered two models for correlation
  - short-range correlation: measurements that are close in distance to one another are correlated, but correlation disappears rapidly with increasing distance
  - long-range correlation: similar to short-range case, but now correlation does *not* disappear rapidly with increasing distance
- as next 2 plots show, difference between these models is subtle

Simulated Data with Short-Range Correlation



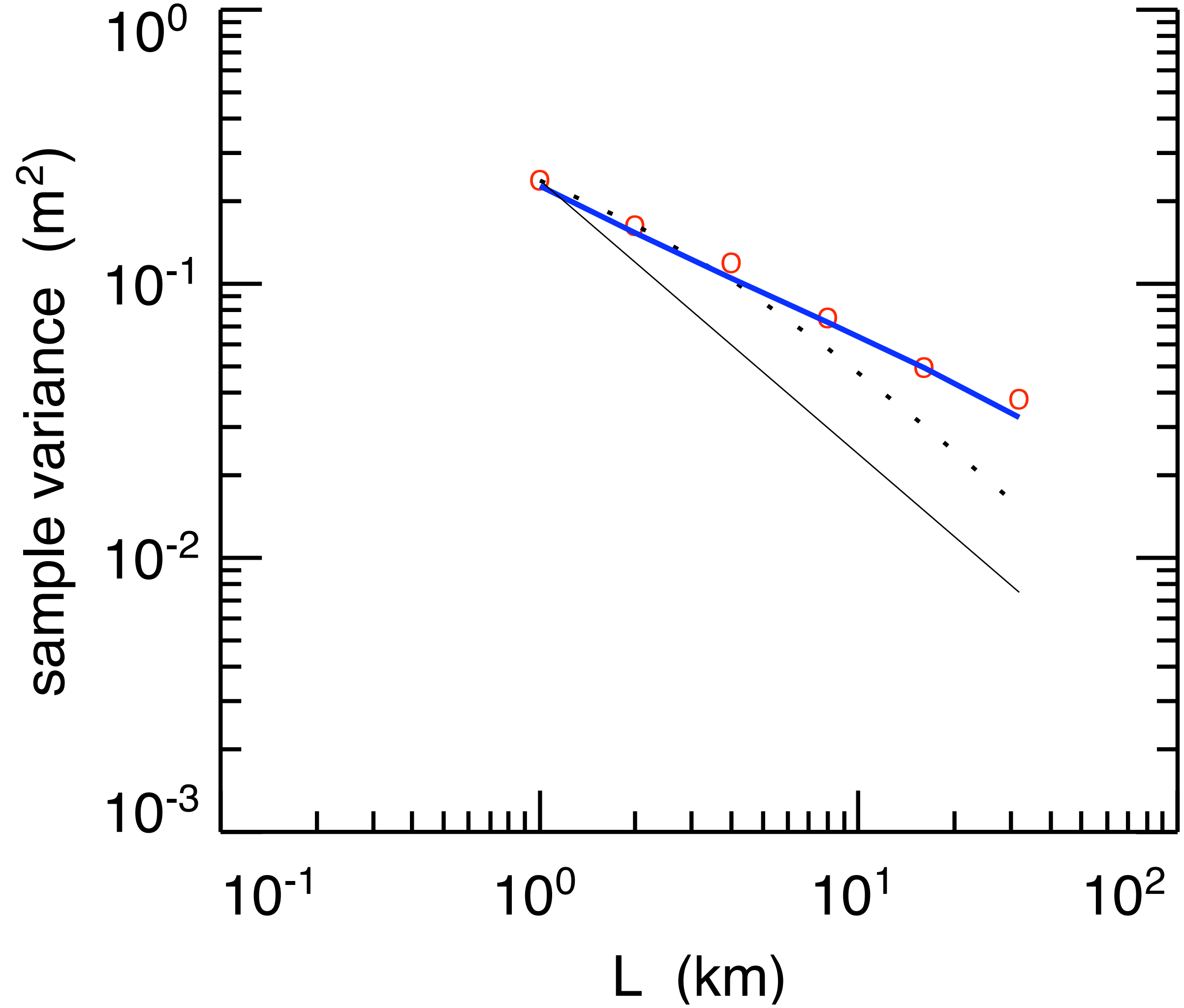
Simulated Data with Long-Range Correlation



Taking Correlated Measurements into Account: II

- for short-range correlation (assuming L is not too small), correction to σ^2/L takes the form σ^2/L', where typically L' (the 'effective number' of data points) is less than L (note that rate of decay is still L^{-1}, the same as for a random sample)
- for long-range correlation, correction to σ^2/L takes the form to σ^2/L^{α}, where $0 < \alpha < 1$; i.e., the variance of $\overline{H}$ decreases at a *slower rate* than if we have either a random sample or short-range correlation
- as following plot shows, empirical evidence suggests that sea ice data have long-range dependence

Sample (Circles) and Theoretical Variances versus L



blue: long-range

dotted: short-range

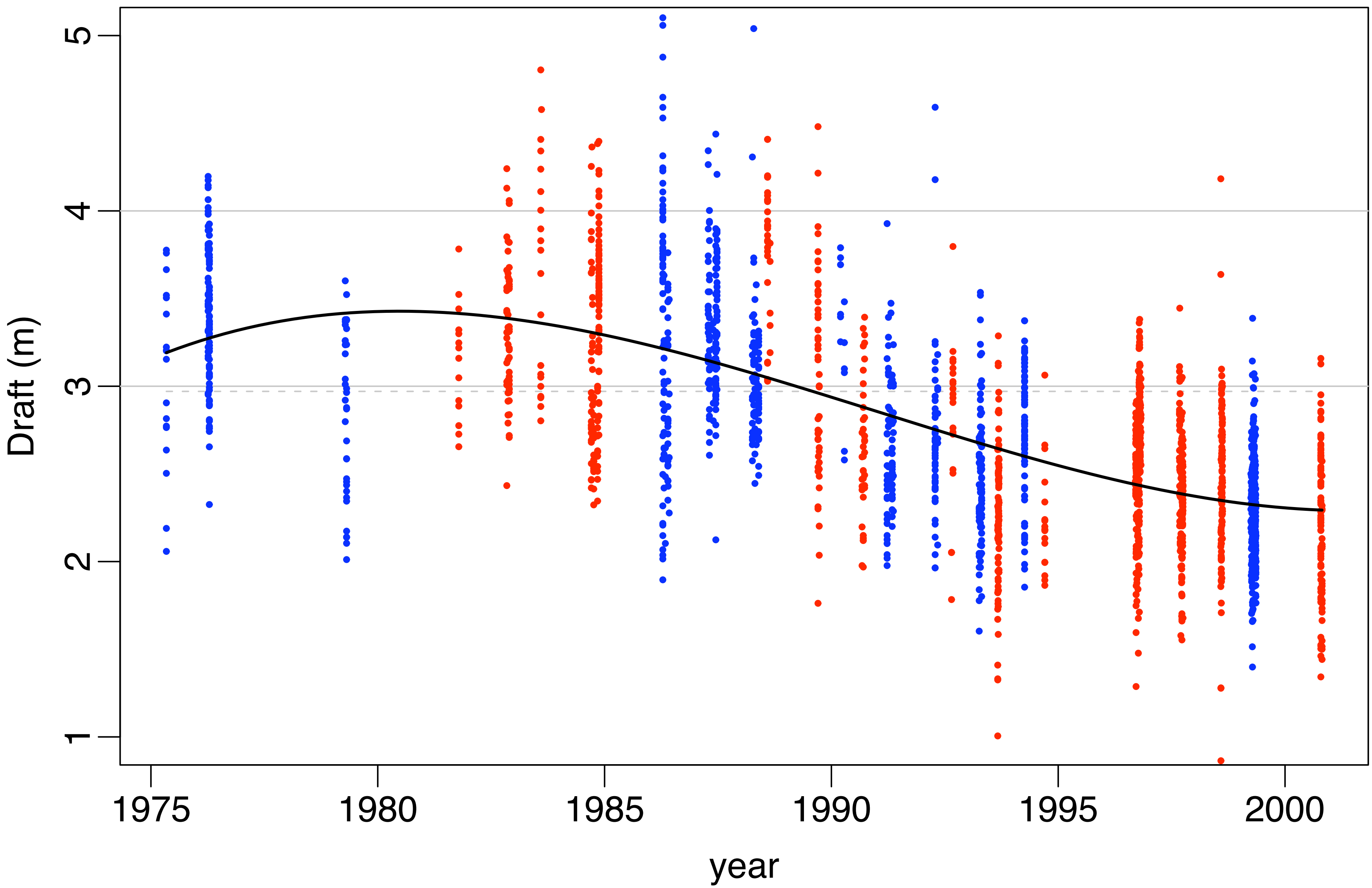
solid: standard theory

Results from Multiple Regression Model: I

- returning now to our multiple regression model

$$\overline{H}_{\mathbf{x},t} = C + I(t) + A(\tau) + S(\mathbf{x}) + \epsilon_{\mathbf{x},t},$$

  can use what we have learned about how the $\overline{H}_{\mathbf{x},t}$ variables are correlated to specify the statistical properties of $\epsilon_{\mathbf{x},t}$
- fitting model to the data gives us estimates of its components
- following plot shows estimated interannual variation $I(t)$, along with residuals (estimates of $\epsilon_{\mathbf{x},t}$) about fit (blue for January to June data, red for rest of year)



Draft (m)
1
2
3
4
5
1975
1980
1985
1990
1995
2000
year

Results from Multiple Regression Model: II

- change from 1981 to 2000 is -1.13 m
- steepest decline (-0.08 m/yr) occurred in 1991
- no recovery by 2000
- much fuller data set strengthens previous results (Rothrock *et al.*, 1999, and Tucker *et al.*, 2001)
- multiple regression model explains 79% of variance in data (standard deviation is 0.98 m)
- unexplained variance has standard deviation of 0.46 m
- estimated standard deviation of measurement errors is 0.25 m

Concluding Comments

- statistical analysis is central to properly interpreting data in almost all areas of science and technology
- statisticians can make a huge impact on issues of central interest to society
- demand for well-qualified statisticians remains high – there is much work to be done, and hopefully you will decide to join in!

References

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2. D. A. Rothrock, D. B. Percival and M. Wensnahan (2008), 'The Decline in Arctic Sea-ice Thickness: Separating the Spatial, Annual, and Interannual Variability in a Quarter Century of Submarine Data,' *Journal of Geophysical Research – Oceans*, **113**, C05003, doi:10.1029/2007JC004252.

3. D. A. Rothrock and M. Wensnahan (2007), 'The Accuracy of Sea-Ice Drafts Measured from U. S. Navy Submarines,' *Journal of Atmospheric and Oceanic Technology*, **24**(11), pp. 1936–1949.

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5. W. B. Tucker, J. W. Weatherly, D. T. Eppler, L. D. Farmer, and D. L. Bentley (2001), Evidence for Rapid Thinning of Sea Ice in the Western Arctic Ocean at the End of the 1980s,' *Geophysical Research Letters*, **28**(14), pp. 2851–2854.