Cox Regression
vs.
Logistic Regression

	Cox Regression	Logistic Regression
Outcome	T = time to event continuous, positive	Y = indicator of event binary (0/1): Yes/No (usually individuals followed for the same time)

	Cox Regression	Logistic Regression
What we model	(log) Hazard rate $h(t) = \lim_{\triangle \to 0} \frac{P(t \leq T < t+\triangle \mid T \geq t)}{\triangle}$	(log) Odds $\text{odds} = \frac{P(Y=1)}{1-P(Y=1)}$
Units	time^{-1}	unitless

Model	
Cox Regression	$\log(h(t\|X)) = \log(h_o(t)) + \beta_1 X_1 + \beta_2 X_2 + ..$
Logistic Regression	$\log(odds(X)) = \text{logit}\ (\pi(X)) = \beta_o + \beta_1 X_1 + \beta_2 X_2 + ..$
	where $\pi(X) = P(Y{=}1\|X)$

	Cox Regression	Logistic Regression
Interpretation in terms of	Hazard ratios (e^{β})	Odds ratios (e^{β})
	between two groups (after controlling for other covariates)	

	Cox Regression	Logistic Regression
Type of model	Semiparametric Form of baseline hazard $(h_o(t))$ not specified Estimated only hazard ratios between reference and other groups	Fully parametric Form of (log) odds fully specified through β's

	Cox Regression	Logistic Regression
Assumptions	Independent observations Censoring independent of time to event Proportional hazard (rates) = hazard ratio between two groups constant over time	Independent observations

	Cox Regression	Logistic Regression
Restrictions	May be invalid if proportional hazard assumption is violated	Cannot be used when the outcome is censored

Cox Regression - Example

Addicts Data

- Outcome = time to event (dropout from the maintenance treatment program) in days
- Status = censored(0) / observed(1)
- Clinic = A/B $\longrightarrow$ clinicB = indicator of clinic B
- Dose = dose of methadone (50-110 mg) $\longrightarrow$ Dose_{50} = Dose - 50

Let's consider models:

(1) $\log(h(t|X)) = \log(h_o(t)) + \beta_1 \text{ClinicB}$

(2) $\log(h(t|X)) = \log(h_o(t)) + \beta_2 \text{Dose}$

(3) $\log(h(t|X)) = \log(h_o(t)) + \beta_1 \text{ClinicB} + \beta_2 \text{Dose}$

(4) $\log(h(t|X)) = \log(h_o(t)) + \beta_1 \text{ClinicB} + \beta_2 \text{Dose}_{50} + \beta_3 \text{ClinicB} * \text{Dose}_{50}$

Interpretation of parameters?

(1) $\log(h(t|X)) = \log(h_o(t)) + \beta_1 \text{ClinicB}$

- β_1 - Log hazard ratio of dropout from the maintenance treatment program between patients in clinic B and A.

(2) $\log(h(t|X)) = \log(h_o(t)) + \beta_2 \text{Dose}$

- β_2 - Log hazard ratio of dropout from the maintenance treatment program between two individuals whose dosage differs by 1 mg.

3) $\log(h(t|X)) = \log(h_o(t)) + \beta_1 \text{ClinicB} + \beta_2 \text{Dose}$

- β_1 - Log hazard ratio of dropout from the maintenance treatment program between patients in clinics B and A who take the same dosage of methadone.
- β_2 - Log hazard ratio of dropout from the maintenance treatment program between two individuals who are at the same clinic and whose dosage differs by 1 mg.

(4) $\log(h(t|X)) = \log(h_o(t)) + \beta_1 \text{ClinicB} + \beta_2 \text{Dose}_{50} + \beta_3 \text{ClinicB} * \text{Dose}_{50}$

- β_1 - Log hazard ratio of dropout from the maintenance treatment program between patients in clinics B and A who take 50-mg dose of methadone.
- β_2 - Log hazard ratio of dropout from the maintenance treatment program between two individuals who are at clinic A and whose dosage differs by 1 mg.
- β_3 - Difference in log hazard ratio of dropout from the maintenance treatment program corresponding to a 1 mg difference in the methadone dosage between clinic B and A.
- $\beta_2 + \beta_3$ - Log hazard ratio of dropout from the maintenance treatment program between two individuals whose dosage differs by 1 mg and who are at clinic B.