Biost/Stat 533
Spring 2008

Homework Assignment #1
(due in class Monday, April 7, 2007)

Reading: Seber & Lee (S&L): Read Chapter 1 and Appendix A, preview Chapter 2

Note: Throughout the course you are encouraged to work as many of the problems as you can in the relevant reading. Hints and partial solutions to many of the problems are contained in the back of the Seber & Lee text.

Homework:

1. For example 2 in lecture notes 1, what is the rank of the design matrix X? Assume $n > p$. You may have to describe cases. For cases where the rank is < p, describe what this means in "practical" terms – what does the scatterplot of the data look like?

2. For example 4 in lecture notes 1, what is the rank of the design matrix X? Assume $n > p$. You may have to describe cases. For cases where the rank is < p, describe what this means in "practical" terms – what does the scatterplot of the data look like?

3. For example 5 in lecture notes 1, what is the rank of the design matrix X? Assume $J \geq 2$.

4. For example 6 in lecture notes 1, what is the rank of the design matrix X? Assume $K \geq 2$.

5. Repeat the exercise on page 10 of lecture notes 1 for the dataset $\{(x, y) = (1, 0), (1, 2), (0, 0)\}$.

6. Let $\mathbf{P} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$ and $\mathbf{Q} = \mathbf{I} - \mathbf{P}$. (For the exercise to make sense, assume $\mathbf{X}'\mathbf{X}$ is non-singular.) Prove: (a) $\mathbf{P}$ and $\mathbf{Q}$ are symmetric and idempotent; (b) $\mathbf{QX} = \mathbf{0}$ and $\mathbf{X}'\mathbf{Q} = \mathbf{0}$.

7. Let $\mathbf{J}_n$ be the $n \times n$ matrix of 1's. Show $\frac{1}{n}\mathbf{J}_n$ is idempotent.

8. Let $\mathbf{A}$ be a symmetric $n \times n$ matrix, let λ_i be its eigenvalues, and let s be a non-zero integer. Prove trace$(\mathbf{A}^s) = \sum_{i=1}^{n} \lambda_i^s$.