Biost/Stat 533
Spring 2008

Homework Assignment #5
(due in class before the midterm, Monday, May 5, 2008)

Reading: Seber & Lee (S&L): Continue reading in chapter 3

Homework:

1. Argue directly from the definition of estimability and the facts we know about estimable quantities that if the design matrix $\mathbf{X}$ has full rank, every $\mathbf{a}'\boldsymbol{\beta}$ is estimable and in particular every individual parameter β_i is estimable.

2. Suppose $\text{rank}(\mathbf{X}) < p$. Show $\boldsymbol{\beta}$ is not estimable. That is, show there is no matrix $\mathbf{C}$ such that $\mathbf{CY}$ is an unbiased estimate of $\boldsymbol{\beta}$. (Equivalently, show that if $\boldsymbol{\beta}$ is estimable then $\mathbf{X}$ has full rank.)

3. Consider the following model:

$$\begin{aligned} Y_1 &= \tau_1 + \tau_2 + \tau_3 + \epsilon_1 \\ Y_2 &= \tau_1 \quad\quad + \tau_3 + \epsilon_2 \\ Y_3 &= \quad\quad \tau_2 \quad\quad + \epsilon_3 \end{aligned}$$

(a) Write out the model in matrix form. What is the rank of the design matrix?

(c) Which (if any) of the individual parameters are estimable?

(d) Is $\tau_1 - 2\tau_2 + \tau_3$ estimable? Explain how you know.

(e) Find the BLUE of $\tau_1 - 2\tau_2 + \tau_3$. In addition, find another estimate of $\tau_1 - 2\tau_2 + \tau_3$ that is not the BLUE.