Biost/Stat 533
Spring 2008

Homework Assignment #7
(due in class Monday, May 19, 2008)

Reading: Seber & Lee (S&L): Finish 4.3, 4.4, 4.6, Begin Chapter 9

Homework:

1. (a) Argue that if $\mathbf{A}$ has rank q then $\mathbf{A}(\mathbf{X}'\mathbf{X})^{-}\mathbf{A}'$ is invertible.

(b) Show the result on page 10 of Lecture 13 that if $\mathbf{A}$ has rank q

$$E[RSS_H - RSS] = \sigma^2 q + (\mathbf{A}\boldsymbol{\beta})'[\mathbf{A}(\mathbf{X}'\mathbf{X})^{-}\mathbf{A}']^{-1}(\mathbf{A}\boldsymbol{\beta})$$

. (*Hint*: Use theorem 13.2.1 and identify a quadratic form for the random vector $\mathbf{A}\hat{\boldsymbol{\beta}}$.)

2. (Seber & Lee 4b.4) Let

$$\begin{aligned} Y_1 &= \theta_1 + \theta_2 + \epsilon_1 \\ Y_2 &= 2\theta_2 + \epsilon_2 \\ Y_3 &= -\theta_1 + \theta_2 + \epsilon_3 \end{aligned}$$

where the ϵ_i are independent $N(0, \sigma^2)$. Derive and F-statistic for testing the hypothesis $H : \theta_1 = 2\theta_2$. Note: there is a typo in the hint given in the book.

3. (Seber & Lee 4MISC.2) Given the two regression lines

$$Y_{ki} = \beta_k + \epsilon_{ki} \quad (k = 1, 2; i = 1, \ldots, n)$$

show that the F-statistic for testing $H : \beta_1 = \beta_2$ can be put in the form

$$F = \frac{(\hat{\beta}_1 - \hat{\beta}_2)^2}{2S^2(\sum_i x_i^2)^{-1}}.$$

Obtain RSS and RSS_H and verify that $RSS_H - RSS = \frac{1}{2}\sum_i x_i^2(\hat{\beta}_1 - \hat{\beta}_2)^2$.

4. (Seber & Lee 4MISC.4) A series of $n+1$ observations $Y_i (i = 1, 2, \ldots, n+1)$ are taken from a normal distribution with unknown variance σ^2. After the first n observations it is suspected that there is a sudden change in the mean of the distribution. Derive a test statistic for testing the hypothesis that the $(n+1)^{th}$ observation has the same population mean as the previous observations.

5. Prove the result on page 6 of Lecture 14 that $\frac{1}{\sigma^2}(RSS_H - RSS)$ has a non-central chi-squared distribution with non-centrality parameter $\lambda = \frac{1}{2\sigma^2}\boldsymbol{\mu}'(\mathbf{P}_\Omega - \mathbf{P}_\omega)\boldsymbol{\mu}$.