Biost/Stat 533
Spring 2008

Homework Assignment #8
(due in class Wednesday, May 28, 2008)

Homework:

1. (S&L 4b.3) Let $Y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, where ϵ_i are *iid* $N(0, \sigma^2)$. Assume $\bar{x} = 0$. Derive an F-statisitc for testing $H : \beta_0 = \beta_1$. (Side question (do not turn in): I didn't say "without loss of generality $\bar{x} = 0$" as I often do. Why not?)

2. (S&L 9MISC.1) Suppose the postulated reression model is

$$E(Y) = \beta_0 + \beta_1 x$$

when, in fact, the true model is

$$E(Y) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3$$

(a) If we have observations at $x = -3, -2, -1, 0, 1, 2, 3$ and fit the postulated model, what bias will be introduced to those estimates?
(b) Answer the same question if the true and postulated models are reversed.

The previous two questions are to give you practice using the material covered in class but aren't especially interesting. Here is a more interesting question (also more work!)

3. Consider a randomized clinical trial for the effect of a treatment on some positive continuous trait (blood pressure, cholesterol level, body weight...). z is the pre-treatment value of the trait and y is the post-treatment value. x denotes assignment to the active treatment ($x = 1$) or placebo ($x = 0$).

Suppose that in truth the effect of treatment is linear on the *relative change* in the trait. The true model is

$$(y_i - z_i)/z_i = \alpha_0 + \alpha_1 x_i + \epsilon_i$$

(a) Write the true model in matrix notation (assume subjects are randomized equally to treatment and placebo).

Suppose that the data are modeled using *absolute change*:

$$y_i - z_i = \beta_0 + \beta_1 x_i + \beta_2 z_i + \epsilon_i$$

(b) Write the model for absolute change in matrix notation and then show the least squares estimator of β_1 has expectation $E(\beta_1) \approx \bar{z}\alpha_1$.
(c) Would testing the hypothesis $H : \beta_1 = 0$ be a valid test for a treatment effect? Explain.
(d) Suppose one tested $H : \beta_1 = 0$ with a Wald test, i.e., one used the test statistic $T = \hat{\beta}_1/\sqrt{\hat{\text{var}}(\hat{\beta}_1)}$. Would the test be conservative? Anit-conservative? Explain.

4. Let the true and fitted models be reversed from question 4. Find $E[\hat{\alpha}_1]$