Consider the model

$$Y_i = \beta_0 + \beta_x x_i + \varepsilon_i, \quad i = 1, \ldots, n;$$

We know

$$\hat{\beta}_x = \frac{\hat{\sigma}_{xy}}{\hat{\sigma}_x^2}$$

where $\hat{\sigma}_x^2$ is the sample variance of the x's and $\hat{\sigma}_{xy}$ is the sample covariance of x and Y.

Suppose the variable we are interested in, x, can only be measured or observed with error. That is, we cannot observe x but we observe $w = x + u$, where u is a random variable with mean 0 and variance σ_u^2. Assume the u's are uncorrelated with each other and with the ε's.

Since w is an unbiased estimate of x, we proceed by regressing Y on w. That is, we fit the model

$$Y_i = \beta_0 + \beta_w w_i + \varepsilon_i, \quad i = 1, \ldots, n;$$

$$\begin{aligned} \beta_w &= \frac{\sigma_{wy}}{\sigma_w^2} = \frac{\sigma_{wy}}{\sigma_x^2 + \sigma_u^2} = \frac{\sigma_{wy}}{\sigma_x^2 + \sigma_u^2} \times \frac{\sigma_x^2}{\sigma_x^2} \\ &= \frac{\sigma_{wy}}{\sigma_x^2} \times \frac{\sigma_x^2}{\sigma_x^2 + \sigma_u^2} \end{aligned}$$

Now $\text{cov}(w, y) = \text{cov}(x + u, y) = \text{cov}(x, y) + \text{cov}(u, y) = \text{cov}(x, y)$, so

$$\begin{aligned} \beta_w &= \frac{\sigma_{wy}}{\sigma_x^2} \times \frac{\sigma_x^2}{\sigma_x^2 + \sigma_u^2} = \frac{\sigma_{xy}}{\sigma_x^2} \times \frac{\sigma_x^2}{\sigma_x^2 + \sigma_u^2} \\ &= \beta_x \frac{\sigma_x^2}{\sigma_x^2 + \sigma_u^2} \end{aligned}$$

Thus $E[\hat{\beta}_w] \neq \beta_x$ but rather $E[\hat{\beta}_w] = \beta_x \lambda$ where $\lambda = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_u^2}$ is the "attenuation factor."

As a "rule of thumb, " errors in regression covariates cause attenuation in parameter estimates. That is, estimates are biased towards 0.

Because of this, some mistakenly believe it is ok to ignore measurement error since biases in regression parameters are in the conservative direction. However, this rule of thumb is not always true.

BIOST 578 (Special Topics): Measurement Error (Has been taught by K. Rice, D. Yanez Spring '08)

- Regression Calibration
- Simulation Extrapolation
- Likelihood Methods
- Bayesian Methods