STAT/BIOSTAT 571: Coursework 4

To be handed in on Friday 6th February, 2009.

1. For the data of question 2 of coursework 1 we will carry out a Bayesian analysis using WinBUGS. Assume independent priors with an improper flat prior on $\mu$, an improper prior $\pi(\sigma_e^2) \propto \sigma_e^{-2}$ on $\sigma_e^2$, and the Ga$(0.05,0.01)$ prior on $\sigma_0^{-2}$.

   (a) Report posterior medians and 90% credible intervals for $\mu$, $\sigma_0^2$, $\sigma_e^2$ and compare (where possible) with the answers you obtained in the frequentist (REML) analysis.

   (b) Carry out inference for the posterior distribution of $\sigma_0^2/(\sigma_0^2 + \sigma_e^2)$, and hence answer the original question of interest concerning how much of batch-to-batch variability explains the total variation in yield.

2. In this question we will analyze the first group of rats in the diet-growth data of coursework 2, question 3, using a Bayesian analysis. Specifically, suppose $Y_{ij}$ is the body weight of rat $i$ at time $t_j$, and consider the three-stage model:

   **Stage 1:**
   
   $Y_{ij} = \beta_0 + b_i + \beta_1 t_j + \epsilon_{ij}$
   
   with $\epsilon_{ij} \sim iid N(0, \tau^{-1})$, $i = 1, \ldots, m$, $j = 1, \ldots, n$.

   **Stage 2:** $b_i \sim iid N(0, \tau_0^{-1})$, with $b_i$ independent of the $\epsilon_{ij}$, $i = 1, \ldots, m$, $j = 1, \ldots, n$.

   **Stage 3:** Independent hyperpriors with:

   $\pi(\beta) \propto 1$, $\pi(\tau) \propto \tau^{-1}$, $\pi(\tau_0) \sim Ga(0.1, 0.5)$

   where $\beta = (\beta_0, \beta_1)$.

   (a) Find the form of the conditional distributions that are required for constructing a Gibbs sampling algorithm to explore the posterior $p(\beta, \tau, b_1, \ldots, b_m, \tau_0 \mid y)$:

   • $p(\beta \mid \tau, b_1, \ldots, b_m, \tau_0, y)$
   • $p(\tau \mid \beta, b_1, \ldots, b_m, \tau_0, y)$
   • $p(\tau_0 \mid \beta, \tau, b_1, \ldots, b_m, y)$
   • $p(b_i \mid \beta, \tau, b_j, j \neq i, \tau_0, y)$, $i = 1, \ldots, m$.

   (b) Implement this algorithm for the data of the 8 rats in the control group. Provide traceplots of selected parameters to provide evidence of convergence of the Markov chain. Report two sets of summaries (5%,50%,95% quantiles) from two chains run from different starting values.

   (c) Check your answers by coding up and running the above model in WinBUGS.