“Tensor Regression with Applications in Neuroimaging Data Analysis”
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Scientific Motivation

- Mental health disorders are difficult to diagnose and treat
- Physiology of the brain is not well understood
- Neuroimaging can elucidate the brain’s physiology
- Several types of neuroimaging, e.g. PET, MRI, fMRI

Brain Areas Associated with ADHD from fMRI
Image Source: MIT Tech Review
BOLD & fMRI

Blood Oxygen Level Dependent Effect (BOLD)
Image Source: FMRIB Oxford

MRI Scanning Machine
Image Source: Discover Magazine
fMRI Data

- **Slice:** Section of the brain in 1-D
- **Voxel:** 3-D pixel, $\approx 3\text{mm}^3$
- **Measure BOLD signal at each voxel at each time point**
- **Final data is a 4-D array**

Image Source: Victor J. Montemayor
Statistical Motivation

▶ Want to use fMRI images as a covariate
▶ Naive approach: use image as vector covariate
  ▶ Lots of data $\implies$ lots of parameters ($\approx 16$ million)
  ▶ Ignores spatial and temporal correlation
▶ This paper tries to solve this problem
▶ Current Methods
  ▶ Voxel-Based Methods
  ▶ Functional Data Methods
  ▶ Two-Stage Reduction Methods
Current Methods

- **Voxel Based Methods**
  - Analysis of each voxel as response variable
  - Assumes voxels independent–ignores spatial correlation

- **Functional Data Methods**
  - Collapses data into one parameter function
  - Commonly used for 2-D data, extension to 3-D data is complex

- **Two-Stage Reduction Methods**
  - Reduce the dimension of the data, possibly more than once, then model the reduced data
  - Theoretical properties are intractable and reduction maybe unrelated to response
Rank-R Generalized Linear Tensor Regression

Extend the GLM to tensor covariates by assuming you can decompose the tensor parameter into a simple form and estimate

GLM framework (McCullagh & Nelder)

- Outcome $Y \sim$ exponential family
- Vector Covariate: $Z$
- Link function: $g(\mu) = \alpha + \gamma^T Z$
Rank-R Generalized Linear Tensor Regression

Extend the GLM to tensor covariates by assuming you can decompose the tensor parameter into a simple form and estimate

Rank-R Generalized Linear Tensor Regression

- Outcome \( Y \sim \) exponential family
- Vector covariate: \( Z \), Tensor covariate: \( X \)
- Matrix Covariate (\( D = 2 \))
  - Link function: \( g(\mu) = \alpha + \gamma^T Z + \beta_1^T X \beta_2 \)
- Tensor Covariate (\( D > 2 \))
  - Assume \( B = [[B_1, \ldots, B_D]] = \sum_{r=1}^{R} \beta_1^{(r)} \odot \ldots \odot \beta_D^{(r)} \)
  - Link function: \( g(\mu) = \alpha + \gamma^T Z + \langle (B_D \odot \ldots \odot B_1) 1_R, \text{vec}(X) \rangle \)
Rank-R Generalized Linear Tensor Regression

- Maximum Likelihood Estimation
- Estimation Algorithm
  1. Estimate the vector covariate parameters, assuming $B = 0$
  2. Estimate each tensor covariate parameter $B_d$, using current estimates for $B_k, k < d$ and previous estimates for $B_k, k > d$
  3. Iterate until the likelihood converges
- Authors show identiability, consistency and asymptotic normality
Summary

- Analysis of complex neuroimages is important for understanding brain physiology
- fMRI data is complex: 4-D array with spatial and temporal correlation
- Current analysis methods ignore one or more of these features
- Tensor regression, an extension of GLM to array covariates allows efficient analysis without ignoring these features
- Next Steps: literature review of current methods, simulations & proofs