Statistical Inference in a Stochastic Epidemic SEIR Model with Control Intervention: Ebola as a Case Study

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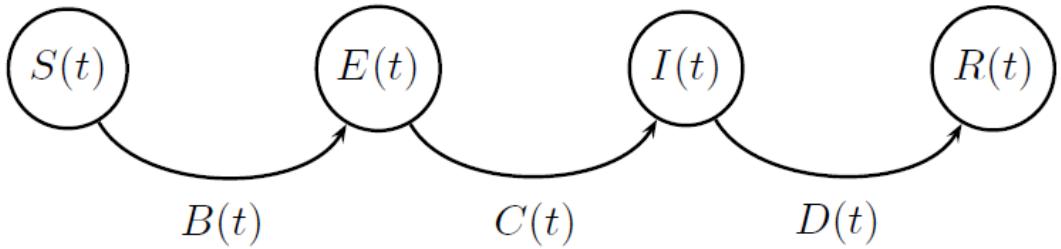
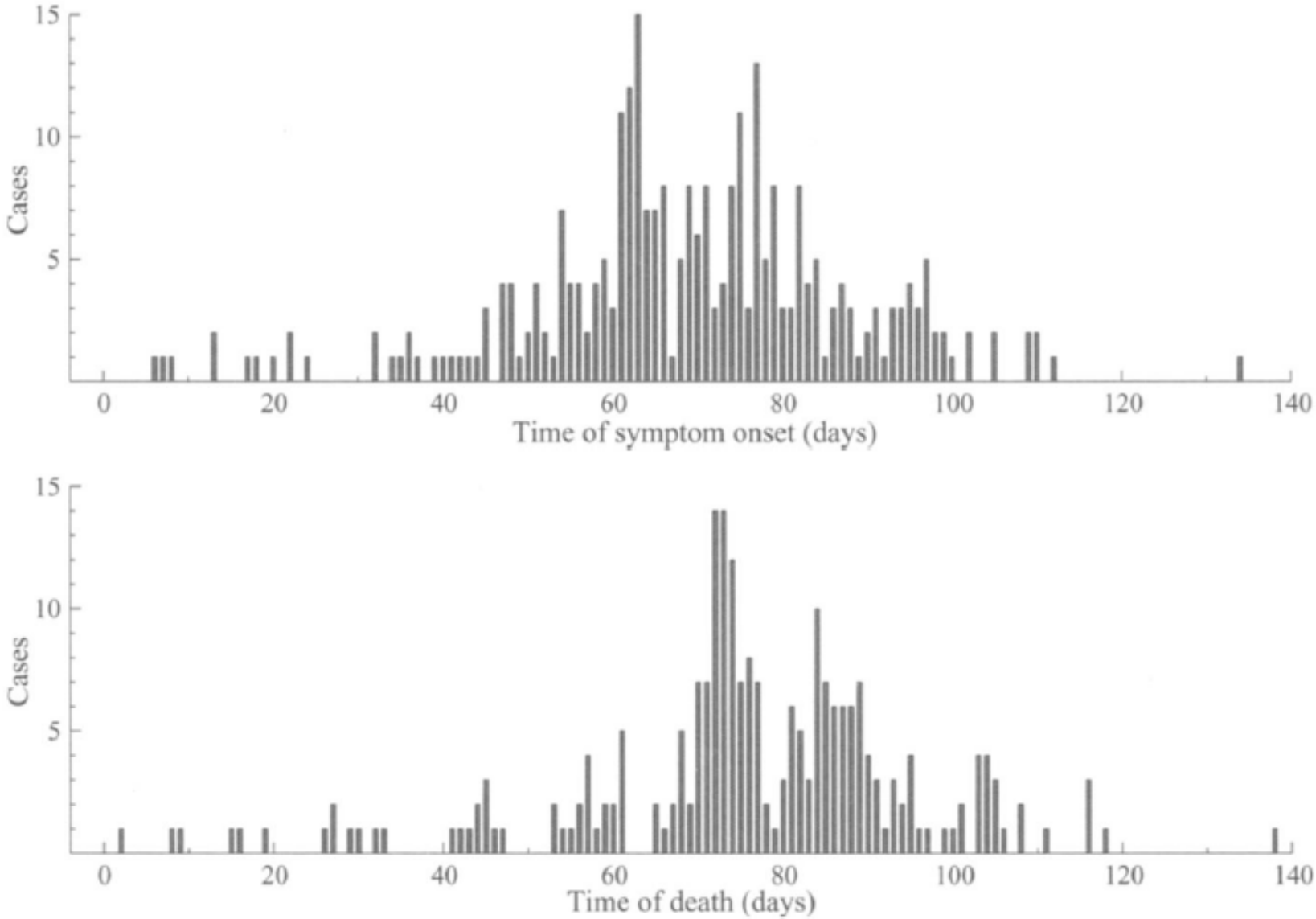
Ebola modeling

- Ebola outbreak in DRC in 1995 infected 316, had an 81% mortality rate.
- There's no cure, but measures to prevent the spread of the disease are very effective.
- Question: How does preventative intervention influence the course of the epidemic?
- Model the progression of the disease with a susceptible-exposed-infectious-recovered (SEIR) model.



The real data



What would complete data look like?

	Time of exposure	Time of contagion	Time of removal
Person 1	1.32	6.45	14.90
Person 2	Not exposed	—	—
Person 3	Not exposed	—	—
⋮	⋮	⋮	⋮
Person N	145.01	152.77	155.81

Complete data would be:

- linked
- continuous
- complete (i.e. have $B(t)$, $C(t)$, and $D(t)$ with no missing records.)

The plan for making inferences

- Define a model that gives a likelihood function for unlinked, discrete-time data given a set of parameters.
- Choose prior distributions for parameters.
- Alternate sampling from distributions for (missing data|parameters) and (parameters|data).
- Make inferences based on posterior distribution of parameters.

A model for the complete data

Assume a population of size N with one initial infection. At the individual level:

- Time until $S \to E$ transitions is exponentially distributed with rate $\beta(t)I(t)/N$, where

$$\beta(t) = \begin{cases} \beta & \text{if } t < t_* \\ \beta e^{-q(t-t_*)} & \text{if } t \geq t_*. \end{cases}$$

- Time until $E \to I$ transitions is exponentially distributed with rate ϱ.
- Time until $I \to R$ transitions is exponentially distributed with rate γ.

Parameters

β: base transmission rate per infected individual (before intervention)

q: decay in rate of transmission after intervention

$1/\varrho$: mean incubation period

$1/\gamma$: mean infectious period

Put Gamma priors on β, q, ϱ, and γ.

We don't need linked or continuous data

We don't need linked data:

- Exponential distribution is memoryless.
- Sufficient statistics look like dwell times summed over all individuals.

We don't need continuous data. On a discrete time scale, we can use

$$B(t) \sim \text{Bin}\left(S(t), \left(1 - \exp\left[-\frac{\beta(t)}{N} I(t)\right]\right)\right)$$
$$C(t) \sim \text{Bin}\left(E(t), \left(1 - e^{-\varrho}\right)\right)$$
$$D(t) \sim \text{Bin}\left(I(t), \left(1 - e^{-\gamma}\right)\right).$$

We can impute $B(t)$, $C(t)$, and $D(t)$

How to sample from $B|C, D, \Theta$:

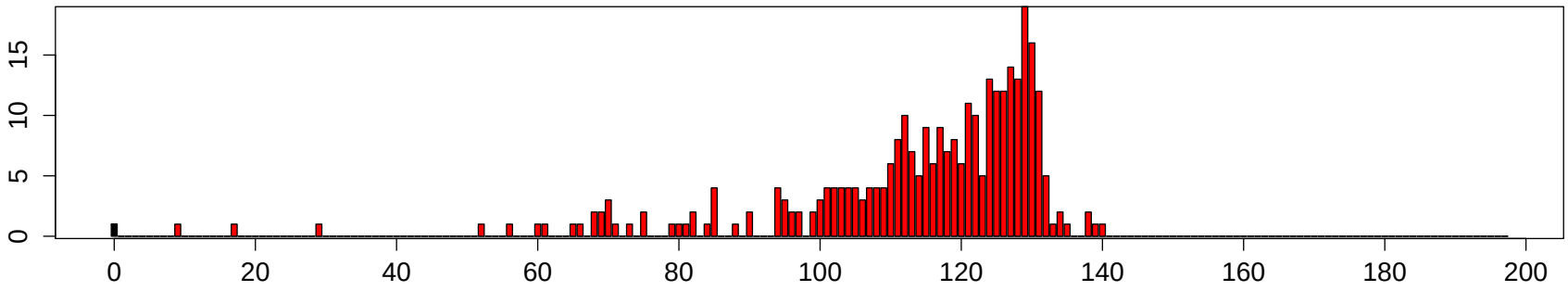
1. Start with some B that is not impossible. (e.g. B must be such that no new infections can occur if there are no infectious.)
2. Propose a configuration B' by randomly picking days t_+ and t_-. Take $B'(t_+) = B(t_+) + 1$ and $B'(t_-) = B(t_-) - 1$.
3. Accept the move to the proposed B' with probability

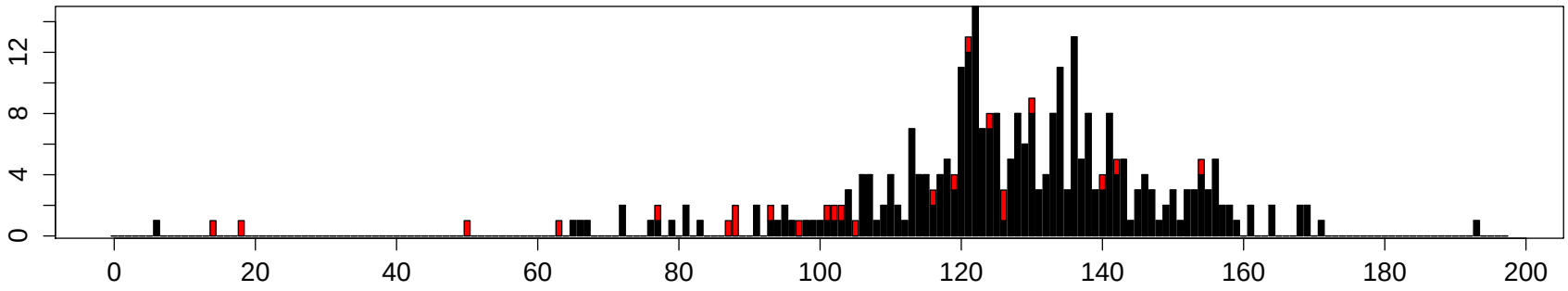
$$\min\left(\frac{\pi(B'|C, D, \Theta)}{\pi(B|C, D, \Theta)}\frac{p(B' \to B)}{p(B \to B')}, 1\right)$$

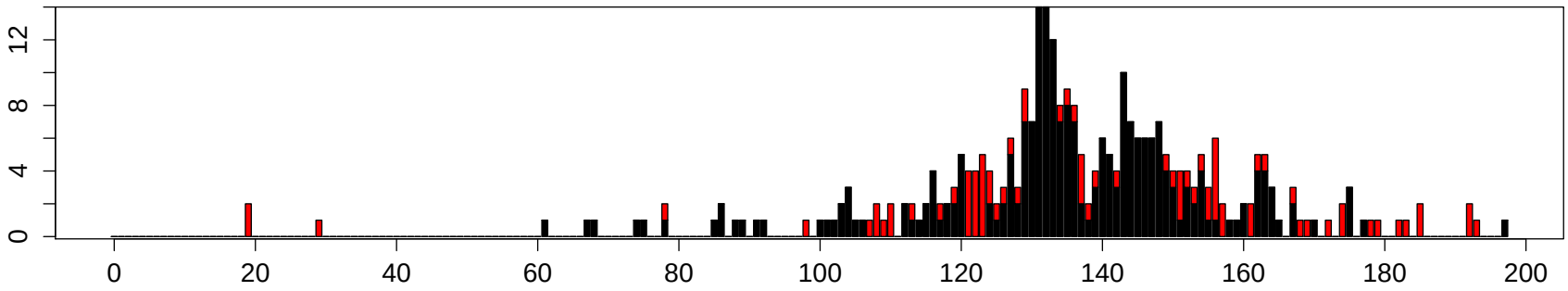
4. Repeat steps 2 and 3.

The same procedure will work on the missing observations from $C(t)$ and $D(t)$.

Example imputation

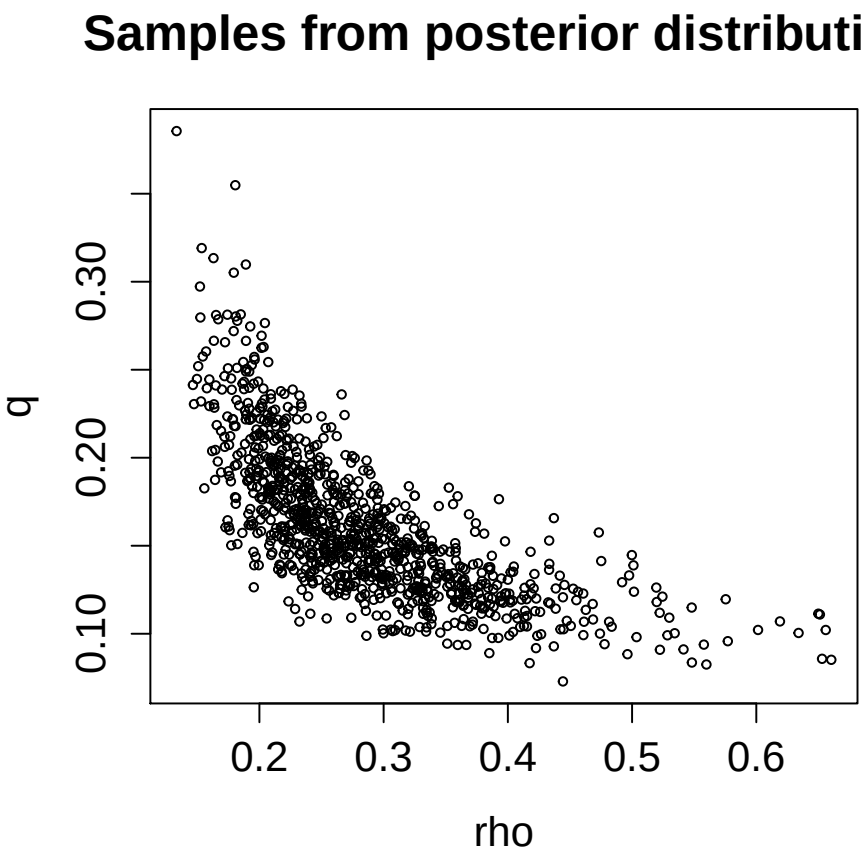


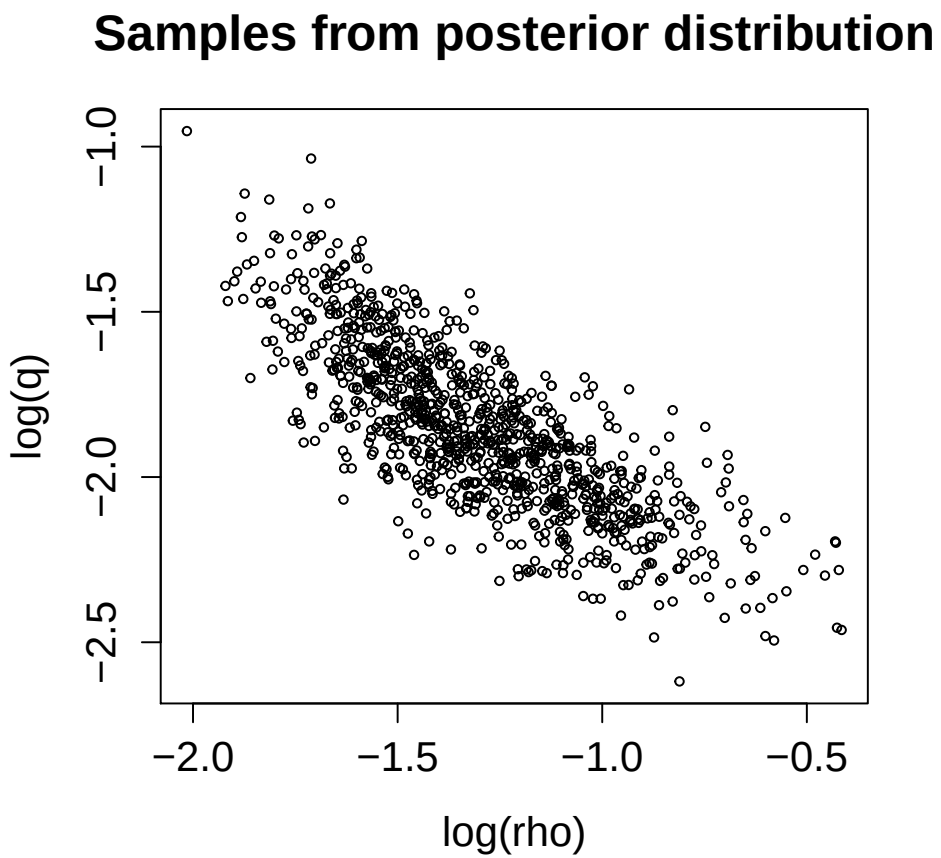




Challenge: Markov chain convergence

Imputation of B induces a negative correlation between posterior draws of ϱ (rate of $E \to I$ transition) and q (rate of transmission decay).





Summary

Scientific contribution: How does intervention impact the spread of Ebola?

Statistical contribution: How can we make inference in an SEIR model when the $S \rightarrow E$ transitions are entirely unobserved?

Problem (for me): Markov chain convergence.