Modeling Criminal Careers as Departures From a Unimodal Population Age-Crime Curve: The Case of Marijuana Use

(2012)

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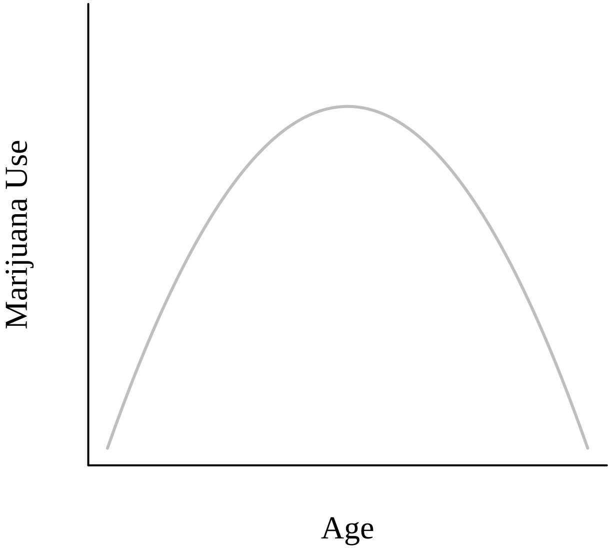
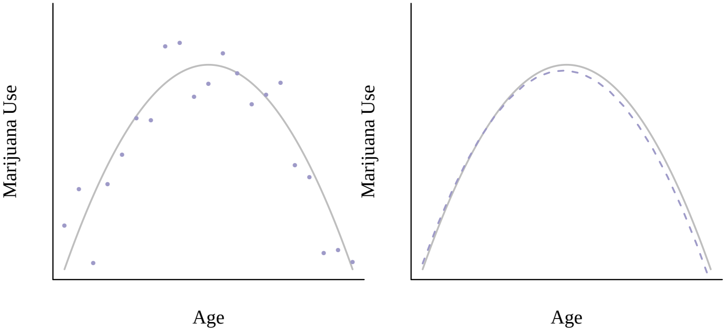
- Telesca et al. (2012) extends Telesca and Inoue (2008) by applying the methods of Telesca and Inoue (2008) to a study of individual age-crime curves in the context of marijuana use over the course of young adulthood

- Representative of criminal career trajectories
- Used to study:
  - Age of onset of offending
  - Timing of peak offending
  - Duration of criminal careers
  - "Types" of criminal careers
  - Other features of criminal careers
- When combined with individual characteristics, can also be used to describe relationship between individual characteristics and features of criminal careers

- Relatively new field of statistics
- The same/similar methods referred to as curve registration, warping regression models, structural averaging ...
- Useful when:
  - Data is observed as a time series for many individuals
  - Features of the time-outcome relationship (peaks, inflection points) are of interest
  - Substantial variation in individual curves

What would we need the data to look like for these questions to be easy to answer?


Marijuana Use
Age


Marijuana Use
Age
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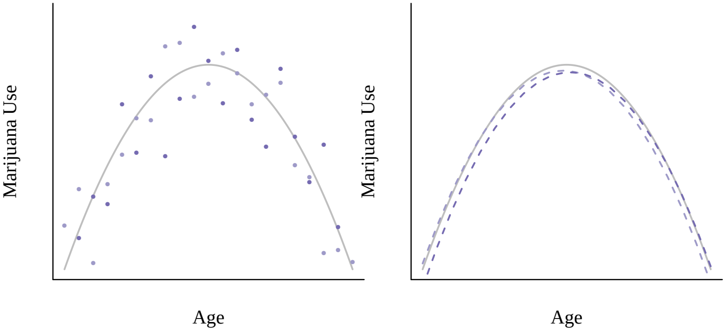
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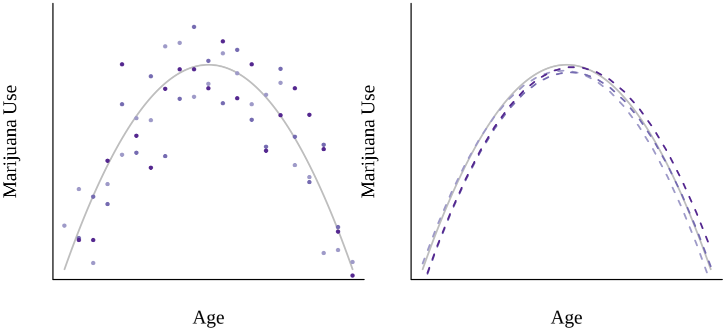
Marijuana Use
Age
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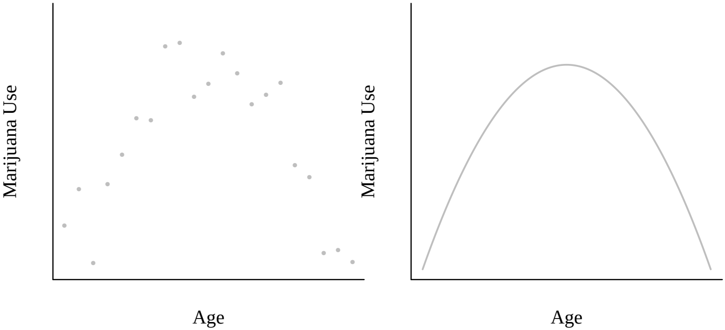
Age

What does the data actually look like?


Marijuana Use
Age
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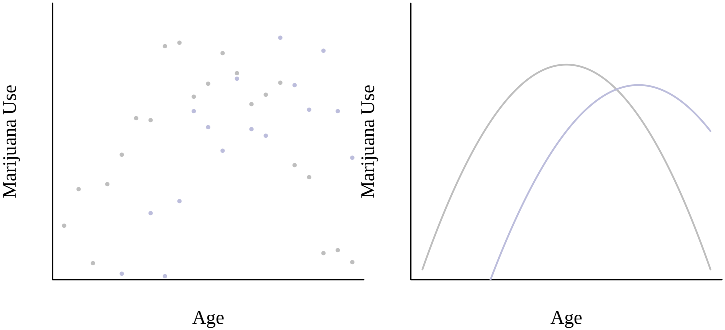
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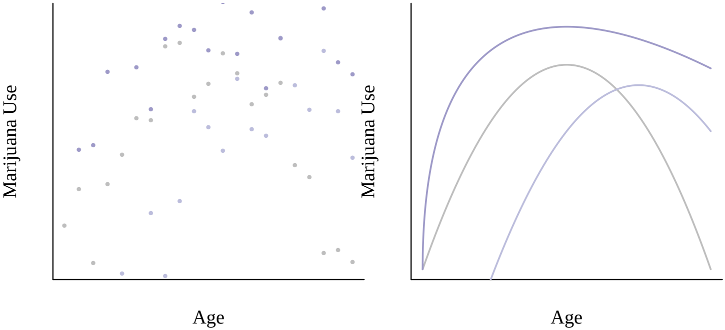
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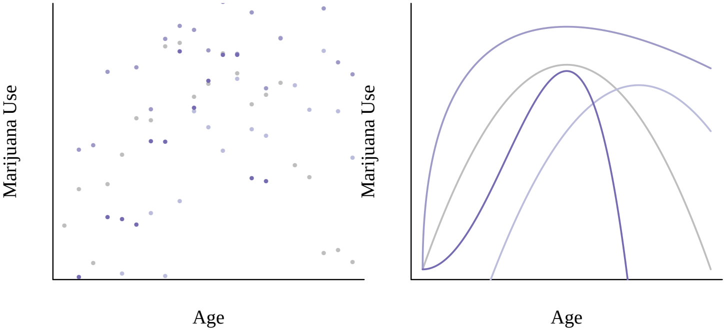
Marijuana Use
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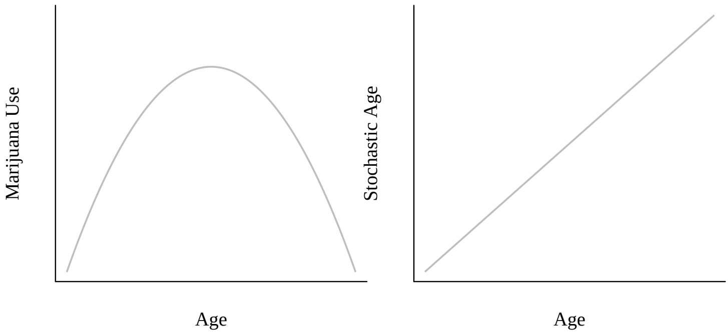
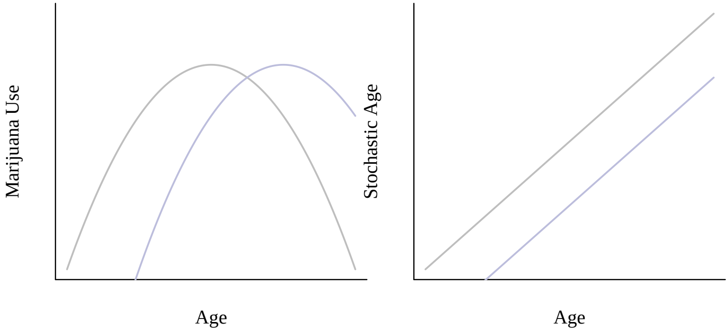
Age

How does the functional data analysis framework allow us to characterize data of this kind?


Marijuana Use
Age
Stochastic Age
Age


Marijuana Use
Age
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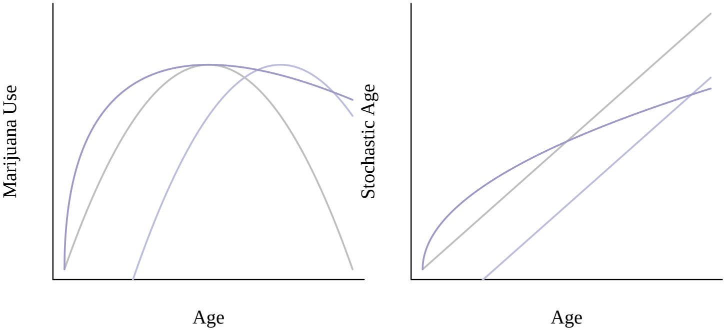
Age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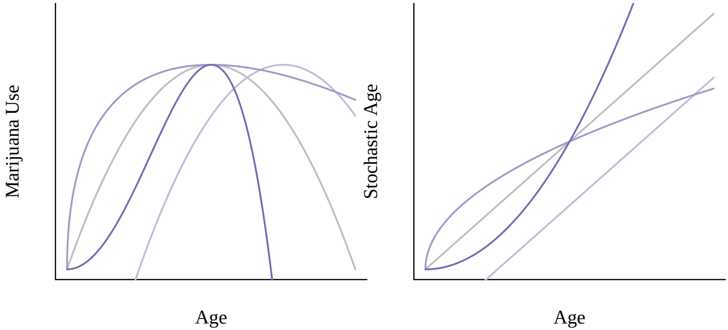
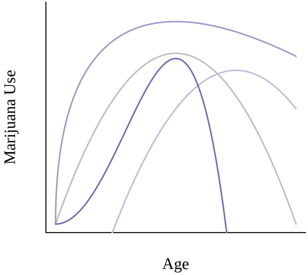

Marijuana Use
Age
Stochastic Age
Age

- Formulation used by Telesca et al. (2012) introduced in Ramsay and Li (1998)
- Introduces model where
    - All individuals share a stochastic time-response function
    - Variation in observed time-response curves is generated by individual, strictly monotonic functions mapping time to stochastic time, called "warping functions" and noise
        - Flexibility in modeling warping functions obtained by using B-spline basis functions
        - Smoothness of estimated warping functions obtained by penalization of relative curvature

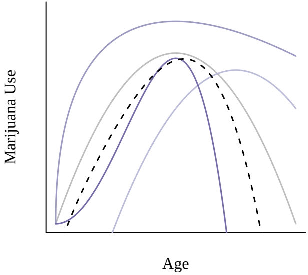
- Telesca and Inoue (2008) developed a Bayesian framework for functional data analysis based on Ramsay and Li (1998)
    - Allows information to be shared across curves
    - Introduces a temporal misalignment window
    - Incoporates smoothing penalty directly into the model
    - Gives a formal account of amplitude and phase variability
    - Allows for exact inference

- Telesca etal. (2012) is an extension of Telesca and Inoue (2008)
  - Extends the curve registration models, developed for continuous data, to count data
  - Allows scale and time-transformations to depend on individual covariates



- Model mean trajectory as a degree two polynomial in time/age
- Incorporated individual variation using random effects and/or latent classes

- **Random Effects**: Raudenbush and Chan (1993)
- **Latent Classes**: Nagin and Land (1993); Roeder et al. (1999)
- **Random Effects & Latent Classes**: Muthén and Shedden (1999)



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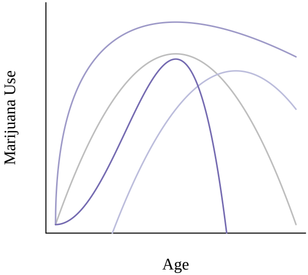

$$Y_{ij}|\lambda_i\left(t_j, \boldsymbol{X}_i\right) \sim \mathcal{P}\left(\lambda_i\left(t_j, \boldsymbol{X}_i\right)\right)$$

$$\begin{aligned}\lambda_i\left(t_j, \boldsymbol{X}_i\right) &= \underbrace{a_i\left(\boldsymbol{X}_i\right)}_{\text{Amplitude}} \underbrace{S\left(t_j, \boldsymbol{\beta}\right)}_{\text{Shape}} \circ \underbrace{\mu_i\left(t_j, \boldsymbol{\phi}_i; \boldsymbol{X}_i\right)}_{\text{Phase Shift}} \\ &= a_i\left(\boldsymbol{X}_i\right) S\left(\mu_i\left(t_j, \boldsymbol{\phi}_i; \boldsymbol{X}_i\right), \boldsymbol{\beta}\right)\end{aligned}$$

$a_i\left(\boldsymbol{X}_i\right)$

- $\mathbb{E}\left[a_i|\boldsymbol{X}_i\right] = \exp\left\{\boldsymbol{X}_i'\boldsymbol{b}_a\right\}$
- Gamma conditional prior distribution of $a_i|\boldsymbol{X}_i, \boldsymbol{b}_a$
    - Conjugate prior distribution for Poisson rate parameter
    - Naturally accommodates over dispersion

$S(t_j, \boldsymbol{\beta})$

- $S(t_j, \boldsymbol{\beta}) = \mathcal{S}_B(t_j)' \boldsymbol{\beta}$
  - $\mathcal{S}_B(t_j)$ set of K B-spline basis functions of order four evaluated at time t
- Positivity and unimodality of $S(t_j, \boldsymbol{\beta})$ imposed by restrictions on $\boldsymbol{\beta}$
- Second order shrinkage prior on a reparametrization of $\boldsymbol{\beta}$ shrinks $S(t_j, \boldsymbol{\beta})$ towards a piecewise linear trajectory

$\mu_i\left(t_j, \boldsymbol{\phi}_i; \boldsymbol{X}_i\right)$

- $\mu_i\left(t_j, \boldsymbol{\phi}_i; \boldsymbol{X}_i\right) = \mathcal{S}_\mu\left(t_j\right)' \boldsymbol{\phi}_i\left(\boldsymbol{X}_i\right)$
  - $\mathcal{S}_\mu\left(t_j\right)$ set of Q B-spline basis functions of order four evaluated at time t
- Monotonicity and boundedness of $\mu_i\left(t_j, \boldsymbol{\phi}_i; \boldsymbol{X}_i\right)$ imposed by restrictions on $\boldsymbol{\phi}_i$
- First order random walk shrinkage prior on a reparametrization of $\boldsymbol{\phi}_i$ shrinks $\boldsymbol{\phi}_i$ towards the B-spline coefficients that yield the identity function, $\mu_i\left(t_j, \boldsymbol{\phi}_i; \boldsymbol{X}_i\right) = t_j$
- $\mathbb{E}\left[\phi_{iq} | \boldsymbol{X}_i, \boldsymbol{b}_\phi\right] = \Gamma_q + \boldsymbol{X}_i' \boldsymbol{b}_\phi$

- Use Markov Chain Monte Carlo to fit model by combination of Gibbs sampling and Metropolis-Hastings sampling algorithms
- Use posterior predictive loss to choose number of interior knots

Model allows us to explicitly test:

- Relationship between level of offending and individual characteristics
- Unimodality of age-crime curve
- Relationship between phase shift and individual characteristics

- Better familiarity with relevant concepts in foundational papers:
  - Ramsay and Li (1998)
  - Lang and Brezger (2004)
  - Telesca and Inoue (2008)
- Work through derivations of:
  - Posterior predictive loss for Poisson case
  - Conditional posterior distributions of parameters
  - MCMC algorithm used
- Begin working through code

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