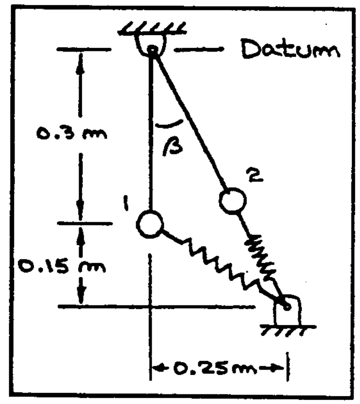
ME 469 Test #2

110 min

(1) The driver of a 3000 ~~lb.~~ kg car moving at 40 ~~mph~~ Km/h applies an increasing force on the brake pedal. The magnitude of the resulting force exerted on the car by the road is $f = 250 + 6s$ ~~lb~~ ,N where s is its horizontal position relative to its position when the brakes were applied. Assuming that the car's tires do not slip, determine the distance required for the car to stop (a) by using Newton's second law; (b) by using the principle of work and energy.

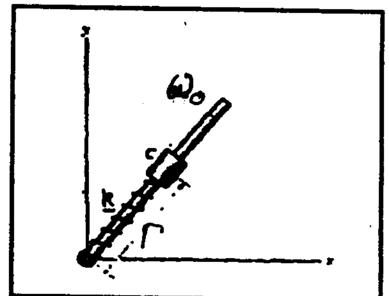
(2) The mass $m = 1\text{-}kg$, the spring constant $k = 200\ N/m$, and the unstretched length of the spring is $0.1\text{-}m$. When the system is released from rest in the position shown, the spring contracts, pulling the mass to the right. Use conservation of energy to determine the magnitude of the velocity of the mass when the string and spring are parallel.



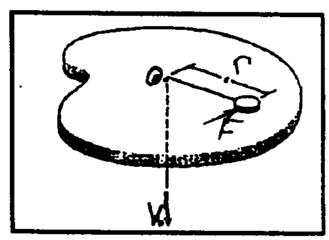
(3) The bar rotates *in the horizontal plane* about a smooth pin at the origin. The *2-kg* sleeve C slides on the smooth bar, and the mass of the bar is negligible in comparison to that of the sleeve. The spring constant $k = 40\ N/m$, and the unstretched length of the spring is $0.8\text{-}m$. At $t = 0$, the angular velocity of the bar is $\omega_0 = 6 rad/s$, $r = 0.2m$, and the radial velocity of the sleeve is $v_r = 0$..

a) What is the angular velocity of the bar when the spring is unstretched?

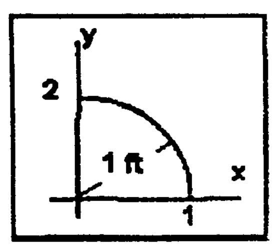
b) What is the total velocity of the sleeve when the spring is unstretched?



(4) In Example 5.7, determine the disk's velocity as a function of time if the force is $F = Ct$, where C is a constant.
Example 5.7: A disk of mass m slides on a smooth horizontal table under the action of a constant transverse force F. The string is drawn through a hole in the table a O at a constant velocity v_o. At $t = 0$, $r = r_o$ and the transverse velocity of the disk is zero. What is the disk's velocity as a function of time?



(5) The potential energy associated with a force $\vec{F}$ acting on an object is $V = -r\sin\theta + r^2\cos^2\theta\ \ ft\text{-}lb$, where r is in feet. (a) Determine $\vec{F}$. (b) If the object moves from point 1 to point 2 along the circular path, how much work is done by $\vec{F}$?



$$\nabla = \frac{\partial}{\partial r}\bar{e}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\bar{e}_\theta$$

20pts each

① a)

$$ma = -f$$

$$m\frac{dv}{dt} = -f$$

$$m\frac{dv}{ds}\frac{ds}{dt} = -f$$

$$mv\frac{dv}{ds} = -(250 + 6s)$$

↓ integrate

$$v^2 = -\frac{2}{m}(250s + 3s^2) + C$$

$$s = 0 \qquad v(0) = 40\left(\frac{1000}{3600}\right)\frac{m}{s} = 11.1\ \frac{m}{s}$$

$$C = v(0)^2 = 123.5\left(\frac{m}{s}\right)^2$$

$$0 = -\frac{2}{m}(250s + 3s^2) + V_0^2$$

$$0 = s^2 + \frac{250}{3}s - \frac{mV_0^2}{6}$$

$$\boxed{s_{1,2} = 210.3\ m}$$

b)

$$U = T_2 - T_1$$

$$U = -T_1$$

$$\int_0^s f\,ds = -\frac{1}{2}mV_0^2$$

$$-\int_0^s (250 + 6s)\,ds = \frac{1}{2}mv_0^2$$

$$-(250s + 3s^2) = \frac{1}{2}mv_0^2$$

$$0 = s^2 + \frac{250}{3}s - \frac{mv_0^2}{6}$$

same as above

(2) $s_1 = \sqrt{(0.15)^2 + (0.25)^2} - 0.1 = 0.192\,m$

$s_2 = \sqrt{(0.3+0.15)^2 + (0.25)^2} - 0.3 - 0.1 = 0.115$

$\beta = \arctan \frac{0.25}{0.45} = 29.1^\circ$

$V_1 + T_1 = V_2 + T_2$

$\frac{1}{2} k s_1^2 - mg(0.3m) + \cancel{\frac{1}{2} m v_1^2}^{\,0} = \frac{1}{2} k s_2^2 - mg(0.3m \cos\beta) + \frac{1}{2} m v_2^2$

$v_2^2 = \frac{k}{m}(s_1^2 - s_2^2) - 2g\,0.3m(1-\cos\beta)$

$\boxed{v_2 = 1.99\,m/s}$

(3) a) $H = m r^2 \omega = m r_0^2 \omega_0$

$\omega = \frac{r_0^2}{r^2} \omega_0$

$= \frac{0.2^2}{0.8^2}\, 6\ rad/s$

$\boxed{\omega = 0.375\ rad/s}$

b) $T_0 + V_0 = T_1$

$\frac{1}{2} m r_0^2 \omega_0^2 + \frac{1}{2} k (r - r_0)^2 = \frac{1}{2} m r^2 \omega^2 + \frac{1}{2} m \dot{r}^2$

$v_r^2 = \dot{r}^2 = r_0^2 \omega_0^2 - r^2 \omega^2 + \frac{k}{m}(r - r_0)^2$

$= \left(r_0^2 - \frac{r_0^4}{r^2}\right)\omega_0^2 + \frac{k}{m}(r - r_0)^2$

$= \left(\left(0.2^2 - \frac{0.2^4}{0.8^2}\right)6^2 + \frac{40}{2}(0.8-0.2)^2\right)\left(\frac{m}{s}\right)^2$

$v_r = 2.92\ m/s$ $\qquad v_\theta = \omega \cdot r = 0.3\ m/s$

(4) $r = r_0 - V_0 t$

$$\bar{M} = \bar{r} \times \Sigma \bar{F} = r\bar{e}_r \times (-T\bar{e}_r + F\bar{e}_\theta)$$
$$= (r_0 - V_0 t) C t \, \bar{e}_z$$

$$\bar{H} = \bar{r} \times m\bar{v}$$
$$= r\bar{e}_r \times m (v_r \bar{e}_r + v_\theta \bar{e}_\theta)$$
$$= m V_\theta (r_0 - V_0 t) \bar{e}_z$$

$$\int_{t_1}^{t_2} \bar{M}\, dt = \bar{H}_2 - \bar{H}_1$$

$$\int_{t_1}^{t_2} (r_0 - V_0 t) C t \, \bar{e}_z \, dt = m V_\theta (r_0 - V_0 t) \bar{e}_z$$

$$V_\theta = \frac{C t^2 \left(\frac{r_0}{2} - \frac{V_0 t}{3} \right)}{m (r_0 - V_0 t)}$$

$$\boxed{\bar{V} = -V_0 \bar{e}_r + \frac{C t^2 \left(\frac{r_0}{2} - \frac{V_0 t}{3} \right)}{m (r_0 - V_0 t)} \bar{e}_\theta}$$

(5) a) $\bar{F} = -\nabla V = -\left(\frac{\partial}{\partial r} \bar{e}_r + \frac{1}{r} \frac{\partial}{\partial \theta} \bar{e}_\theta \right) (-r \sin\theta + r^2 \cos^2\theta)$

$$\bar{F} = (\sin\theta - 2r\cos^2\theta)\bar{e}_r + (\cos\theta + 2r \sin\theta \cos\theta)\bar{e}_\theta$$

b) $U_{1,2} = -(V_2 - V_1)$ @ $r = 1, \theta = 0$

$V_1 = 1$ ft-lb

$r = 1, \theta = \frac{\pi}{2}$

$V_2 = -1$ ft-lb

$\boxed{U_{1,2} = 2 \text{ ft-lb}}$