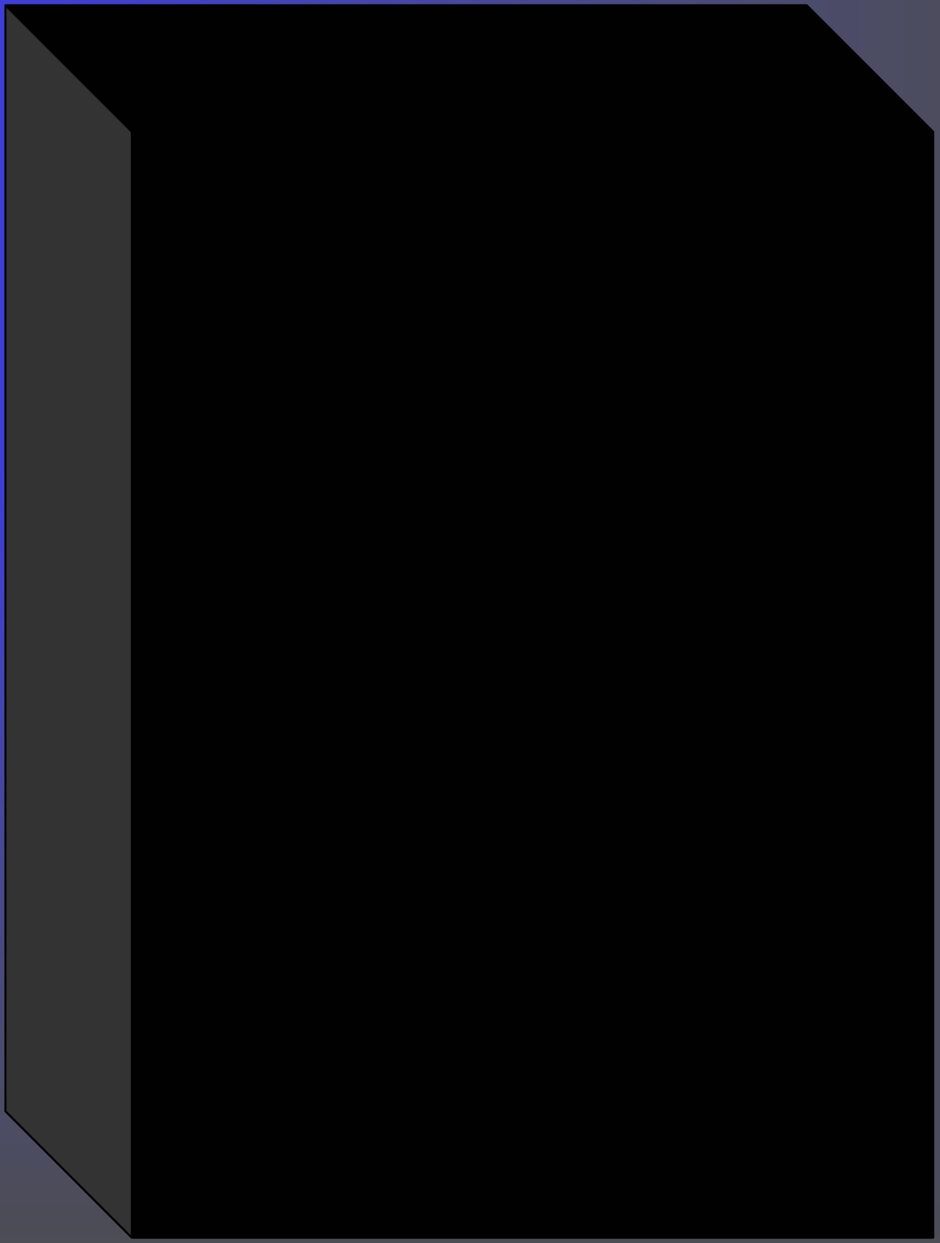
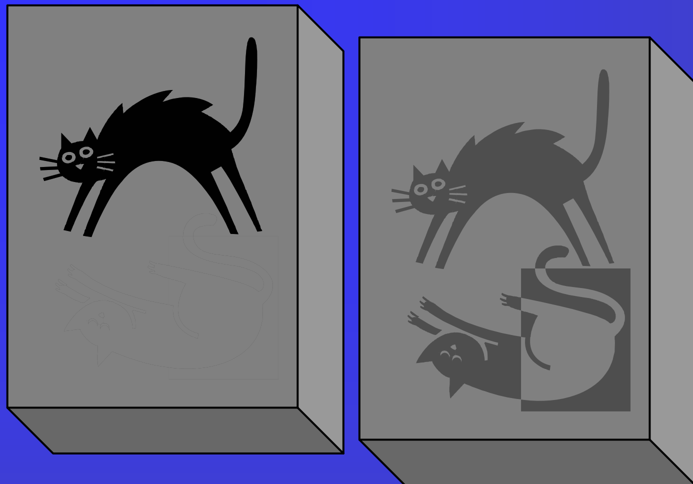
The Cat



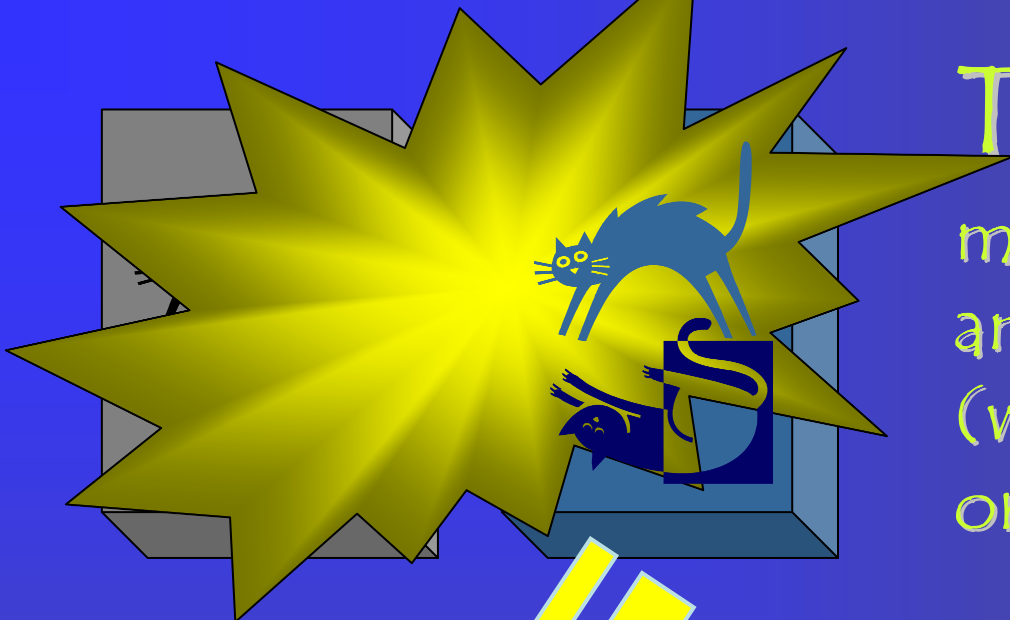


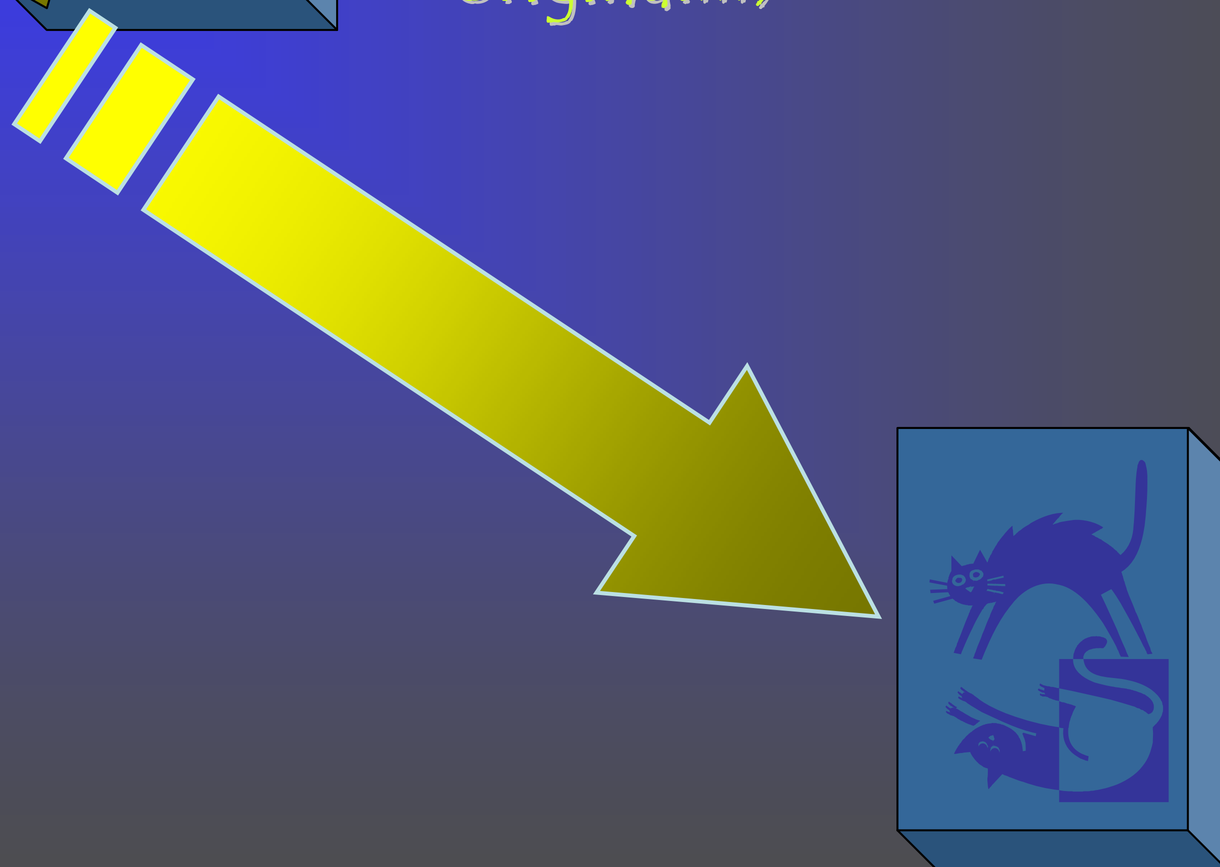
$|\Psi\rangle = \alpha|$  $\rangle + \beta|$  $\rangle$

Entanglement of cats

Teleportation:

making an exact replica of an arbitrary quantum state (while destroying the original...)





Homework #6

• Problems 1 (30 points) and 3 (40 points) from Chapter 2 of “Exploring Black Holes” (handouts).

Due Wednesday, November 28.

Physics 311
General Relativity

Lecture 17:

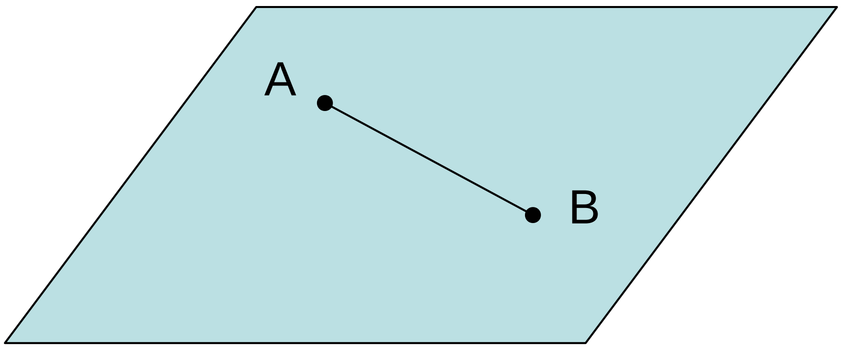
Geodesics, tidal accelerations and *gravitational waves*.

Today’s lecture:

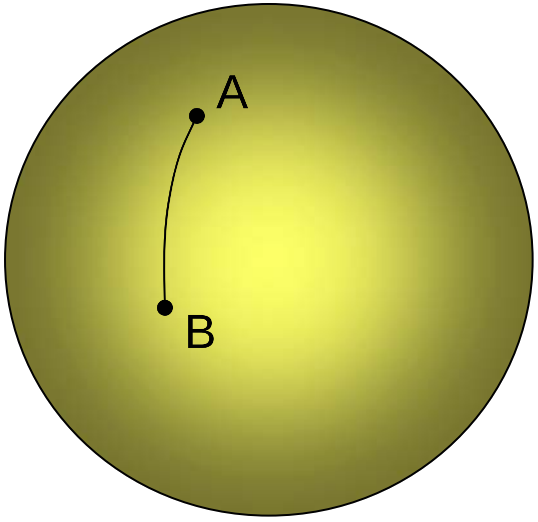
- Geodesics: motion on the metric
- Gravitational red shift, geodesics of Schwarzschild metric
- Tidal accelerations and space curvature
- Gravitational waves – time-dependent solution of Einstein field equation
- LIGO, LISA and such

Straight line – always the shortest distance?

- Term “**geodesics**” is a generalization of the notion of “straight line”, when applied to a curved space.
- Straight line is the shortest distance between two points – right?
- Sometimes it is, sometime it isn’t!



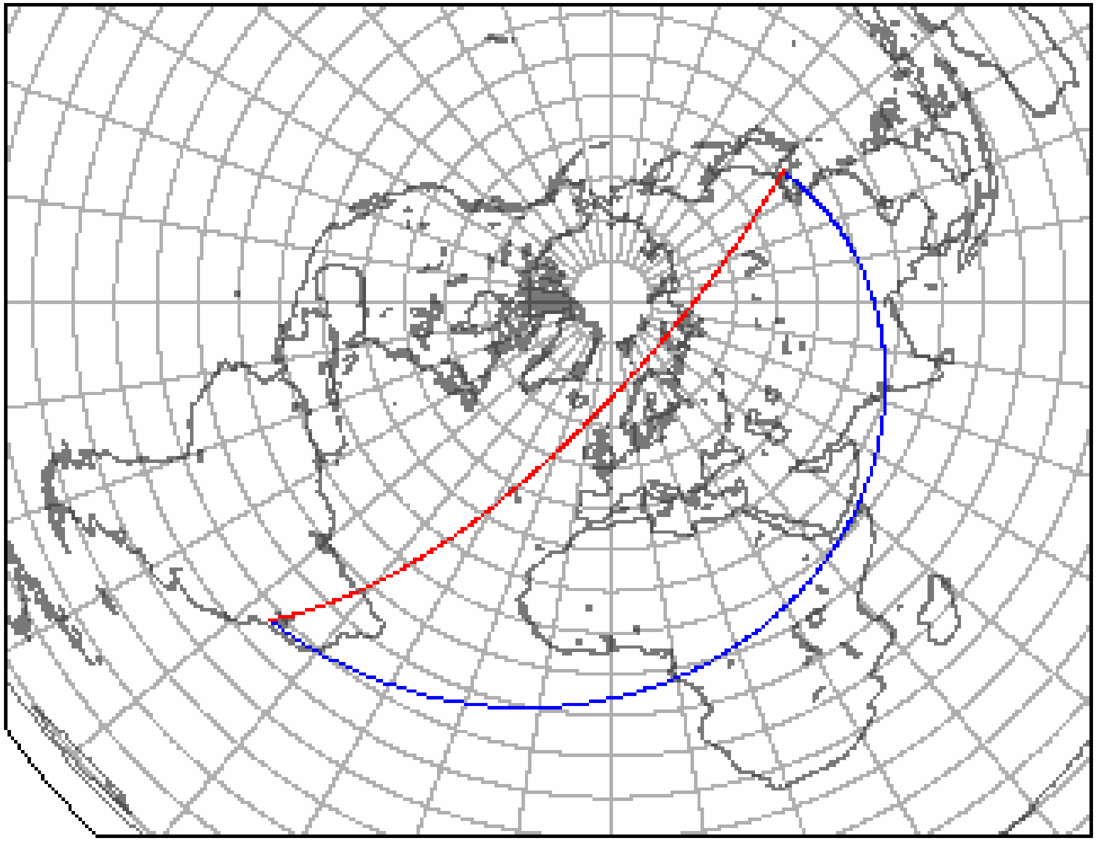
Shortest distance on a plane is a straight line



Shortest distance on a sphere is an arc

Airlines know that!

• Airliners take the shortest path between airports – which at first sight doesn't seem like the shortest! The name "geodesics" is taken from *geodesy* – the science of measuring the size and shape of Earth.



How to find the geodesics?

• The strict definition of a geodesics is a *locally* shortest path between two points on a metric.

• Being the shortest path, geodesics thus describes the motion of free particles. Thus, geodesics is the *world line* of a free particle in a given metric.

• What was the world line of a free particle in Special Relativity?

• Straight line! We can thus make a conclusion that the ***geodesics of Minkowski metric is a straight line***.

• Formally, geodesics between two points can be found by writing down the equation for the length of a curve, then minimizing the length of the curve using standard techniques of calculus and differential equations.

• That, in practice, is how you find geodesics for some funny metrics you may encounter...

Curved spacetime

• The geodesics is pretty boring in Special Relativity. In fact, we didn't even need the term there. In General Relativity, geodesics becomes very important.

• Recall the bending of light effect. Light always takes the shortest path, thus, light rays trace a geodesics in (the 4-dimensional) spacetime. What we observe in the 3-dimensional space is light deflection from apparently straight line. Light rays trace out the space part of the geodesics!

• There is a way to also "trace out" the time part of the geodesics. It comes from another effect of curved spacetime – the so-called ***gravitational red shift***.

Gravitational red shift

- Let's recall Schwarzschild metric:

$$ds^2 = [1-(2m/r)]dt^2 - [1-(2m/r)]^{-1}dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\phi^2$$

- This metric applies to spacetime around a spherically-symmetric mass, as around a planet, a star or a black hole.

- Three important features of Schwarzschild metric that we have not yet discussed:

  1) $c = G = 1$, which implies that mass is measured in meters!

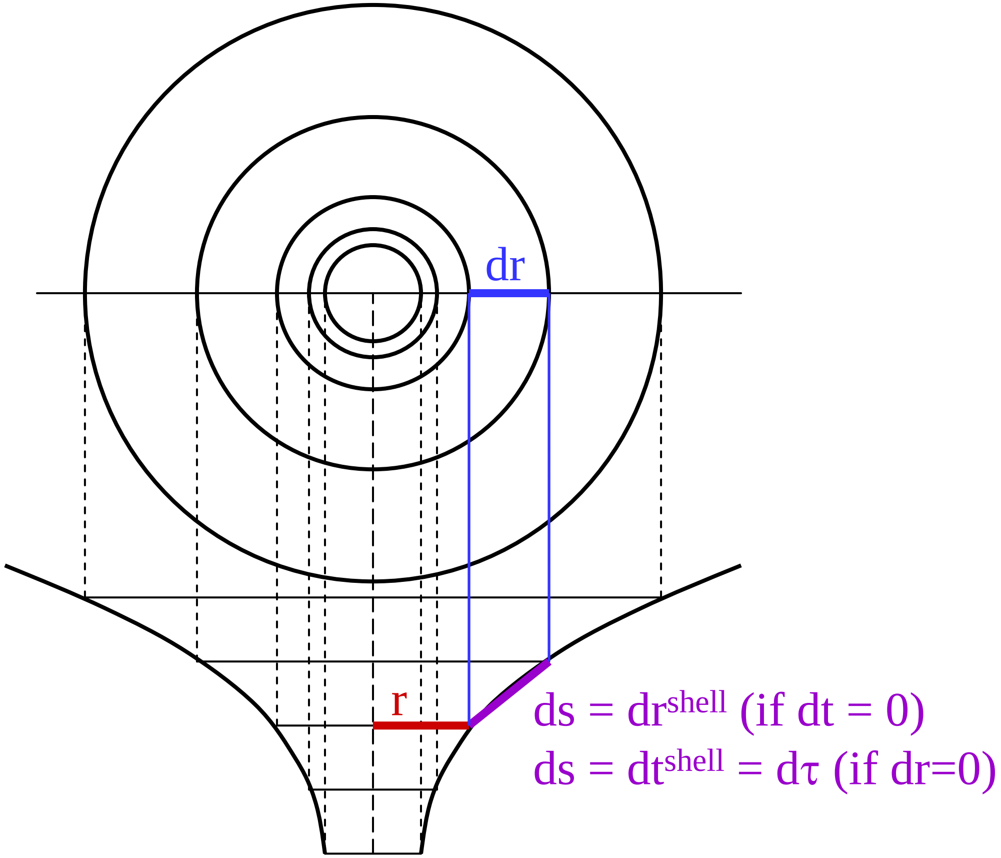
  2) Direct measurement of radius r in the curved space is impossible. Instead, we define $r = C/2\pi$, where C is the circumference of the great circle around the center of attraction.

  3) To similarly avoid the effects of the curvature of *time* near the heavy mass, we measure time with *faraway* clocks.

Schwarzschild coordinates

- In Schwarzschild geometry, there's r – the radial coordinate, defined as circumference/2π (a.k.a. "reduced circumference"), and there's the r^{shell} – the *local* radial coordinate.

- Same story for time: the Schwarzschild time t is measured by a faraway clock; the shell time t^{shell} (or $d\tau$) is measured locally.



More on Schwarzschild coordinates

- Schwarzschild metric:

$$ds^2 = [1-(2m/r)]dt^2 - [1-(2m/r)]^{-1}dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\phi^2$$

- For an observer located near the mass giving rise to the metric (the "shell observer"), we define local radial and temporal displacements as:

$$dr^{shell} = [1-(2m/r)]^{-1/2}dr$$

$$dt^{shell} = [1-(2m/r)]^{1/2}dt$$

These are equivalents of the "proper length" and the "proper time" of the Special Relativity!

Gravitational red shift - 2

• Let's fix the spatial position, so that $dr = d\theta = d\phi = 0$ and look at events that are only separated in time, not in space. (To lift the suspense: the events we are interested in are arrivals of the crests of an electromagnetic wave at the place where we are observing them).

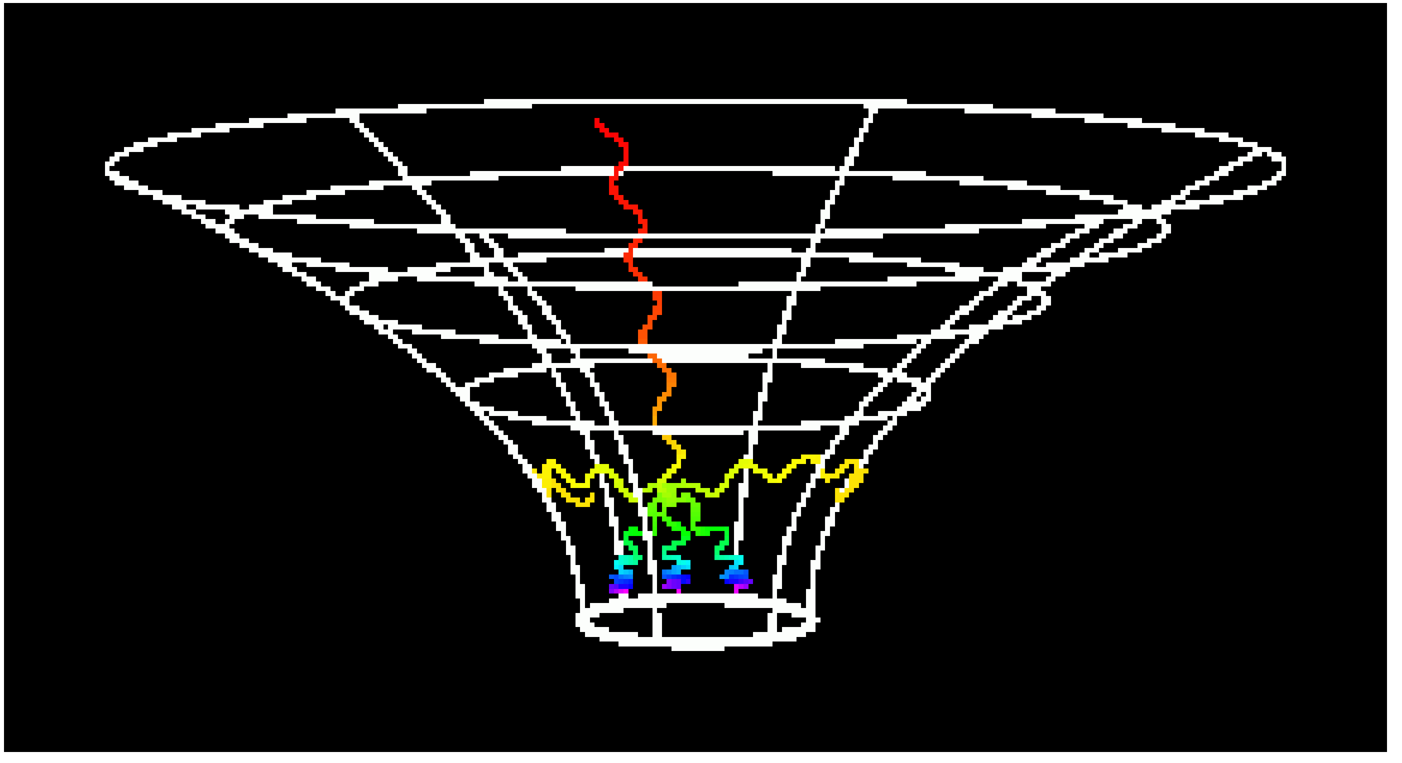
• Then the metric is just the proper time:

$$ds^2 = (dt^{shell})^2 = [1-(2m/r)]dt^2 \text{ or } dt^{shell} = [1-(2m/r)]^{1/2}dt$$

• Remember that time dt is measured at infinity, while the proper time dt^{shell} is measured locally, near the mass, the black hole or what have you.

• The quantity $2m/r$ is less than or equal to 1 outside of the black hole, while $r = 2m$ defines the famous *event horizon*. This means that the period of wave crests will appear <u>longer</u> for a remote observer.

• This lengthening of the period is known as the gravitational red shift (experimentally verified!).



Geodesics of Schwarzschild metric

- So, what is the expression for the geodesics of Schwarzschild metric?
- Well, it is not as simple as the Minkowski geodesics! In flat spacetime, straight line worked for all kinds of event separations – timelike or lightlike (spacelike geodesics is nonsense – why?).
- In Schwarzshild spacetime geodesics will be different for different types of event separation, and for different types of motion. They can be calculated using the recipe described a few slides back.
- The *radial* geodesics are:

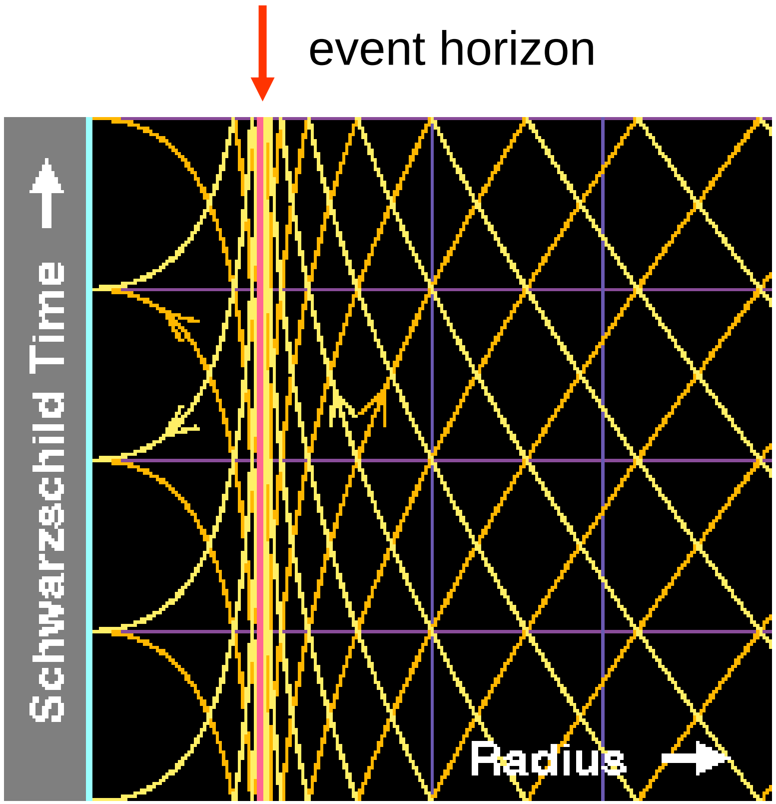
$$(dr/d\tau)^2 + (1 - 2m/r) = E^2$$ (timelike geodesics)

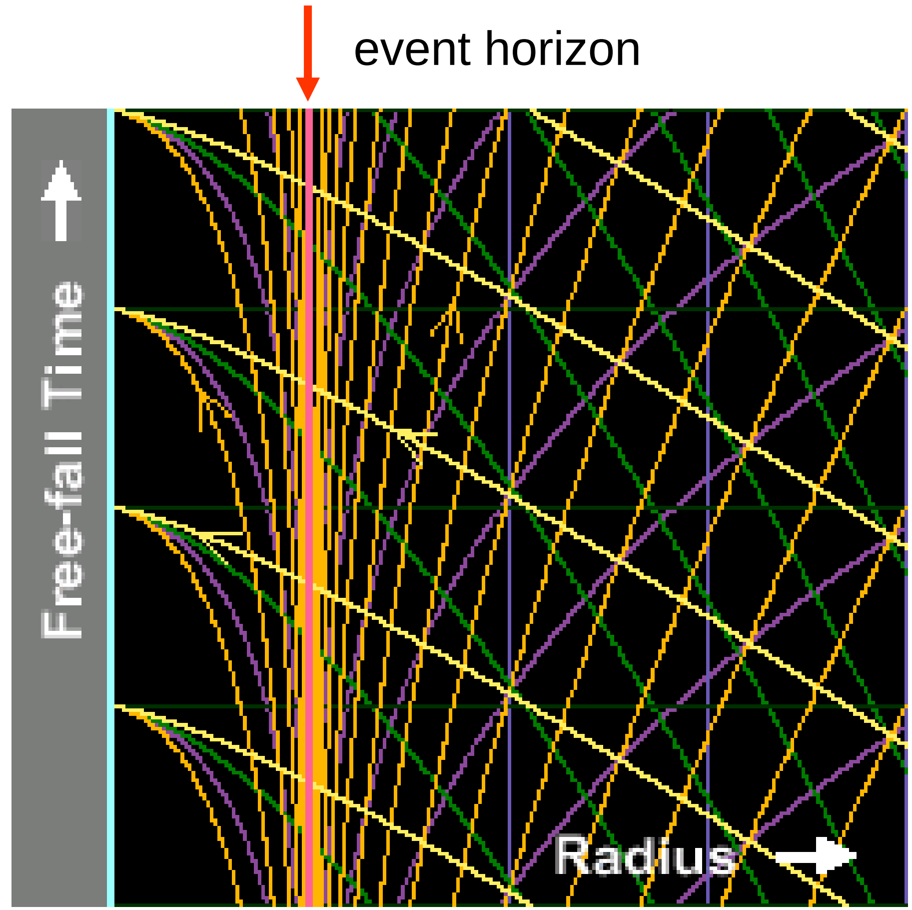
$$(dr/d\tau)^2 = E^2$$ (lightlike or *null* geodesics)

(remember, geodesics is a path of a free particle, thus there is no spacelike geodesics)

- Here, E is called the energy of the geodesics.

Geodesics of Schwarzschild metric





null geodesics *inward*

null geodesics *outward*

constant radius

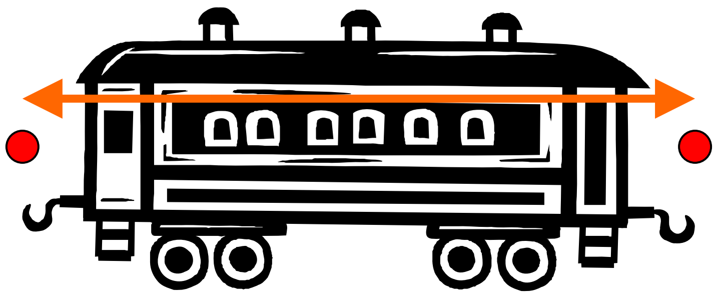
constant time

geodesics of freefall

Tidal accelerations

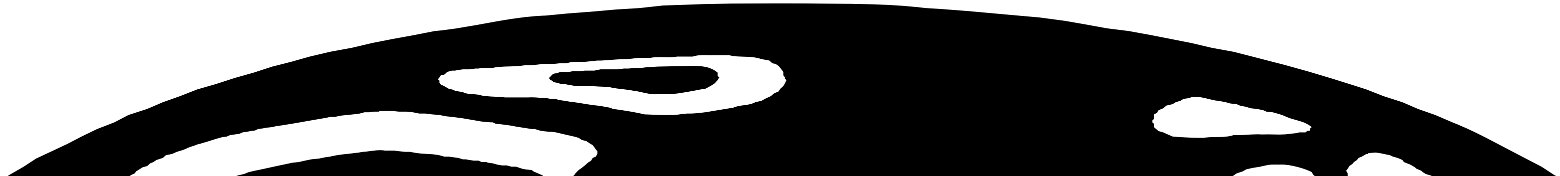
• Recall what happens to test masses as our reference frame is in the freefall near Earth.

• The test masses are *free particles,* so they move along the geodesics (of Schwarzschild metric in this case). Tidal accelerations is nothing more than a manifestation of the curvature of spacetime!



d=20m

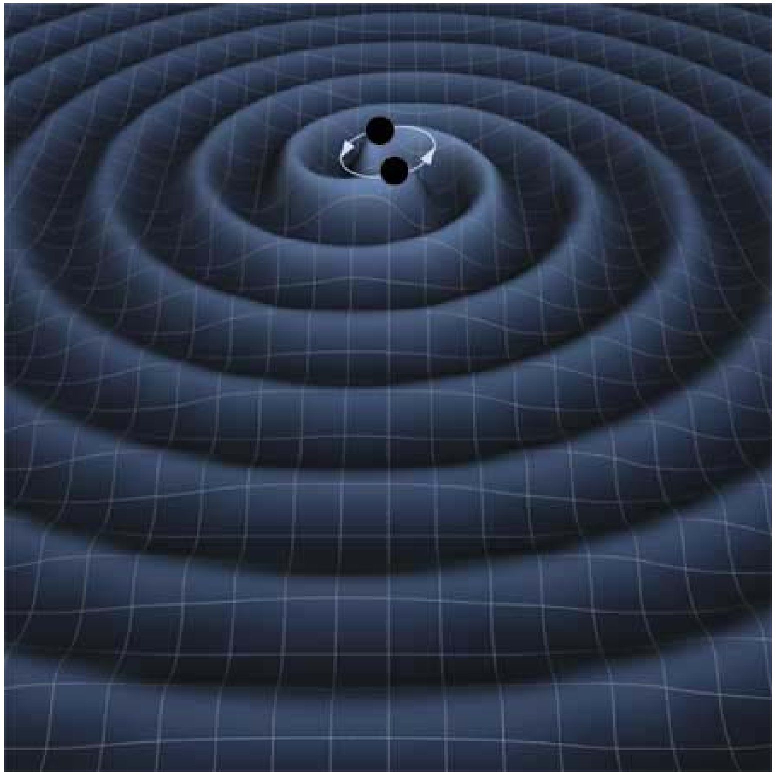
d<20m



Gravitational waves

• Time-dependence in Einstein field equation leads to spacetime curvature that varies with time.

• These time-variations of spacetime curvature are expected to propagate at speed of light and are called *gravitational waves.*



In this figure, two hypothetical black holes orbit each other at high rate.

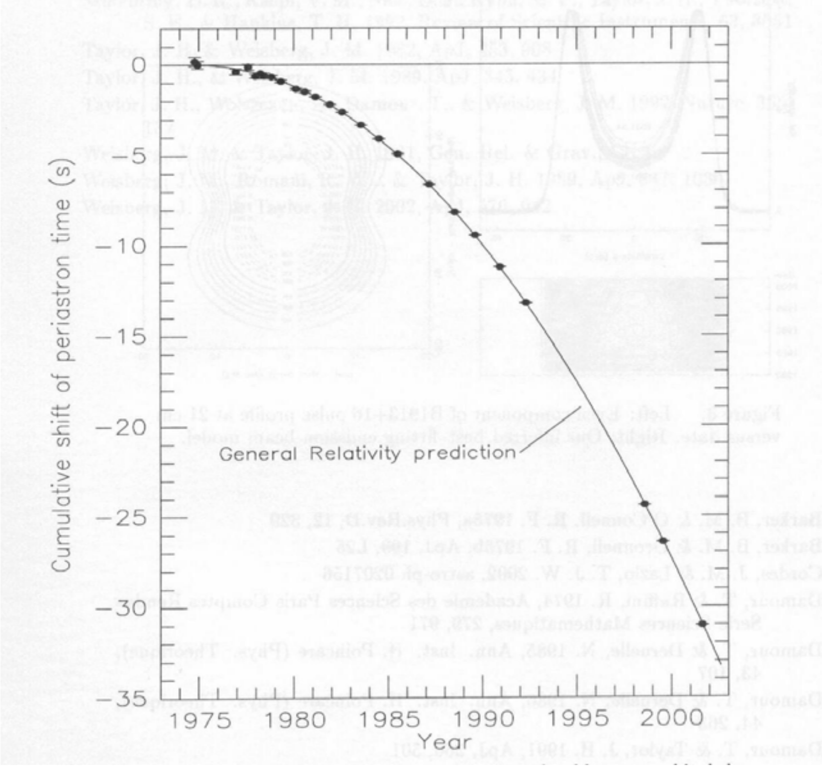
Each black hole creates its own curved spacetime around itself.

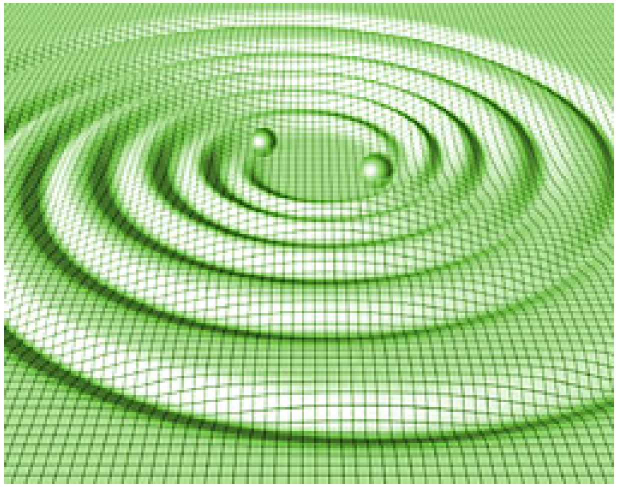
As the black holes rotate, the centers of their respective metrics move. This creates a wave pattern!

Energy of gravitational waves

• Gravitational waves carry away energy. This energy must come from somewhere. In other words, the source of gravitational waves must lose energy.

• Looking for this loss of energy is an *indirect* way of detecting gravitational waves.



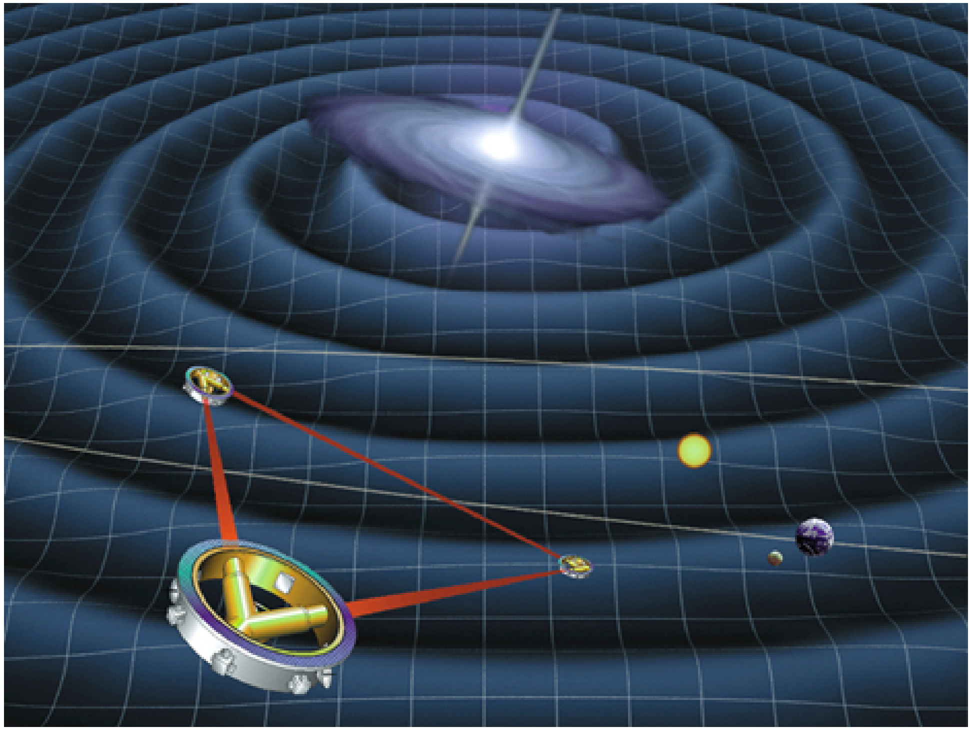


LIGO - Laser Interferometer Gravitational wave Observatory

- Two enormous Michelson interferometers look for tiny *relative* movements of their mirrors caused by gravitational waves.
- Current sensitivity ~ 10^{-18} meters (1000 times smaller than the proton!), yet not sensitive enough (would probably detect waves coming from our entire Galaxy collapsing...)

LISA - Laser Interferometer Space Antenna

- Three satellites flying 5 million kilometers apart, with laser beams “connecting” them.
- May be launched in 2012.
- Would have sensitivity 1,000,000 times better than LIGO



Recap

• Geodesics is a line in spacetime that follows the path of a free particle; geodesics is the (locally) shortest distance on a given metric.

• Geodesics is found by minimizing the path between two events.

• Tidal accelerations, light bending and gravitational red shift are all manifestations of particles following geodesics in curved spacetime.

• Gravitational waves arise from time-dependent metrics; they come about because of finite speed (speed of light) of the metric propagation through space.