Physics 311
General Relativity

Lecture 18:

Black holes. The Universe.

Today's lecture:

- Schwarzschild metric: discontinuity and singularity
- Discontinuity: the event horizon
- Singularity: where all matter falls
- Spinning black holes
- The Universe – its origin, history and fate

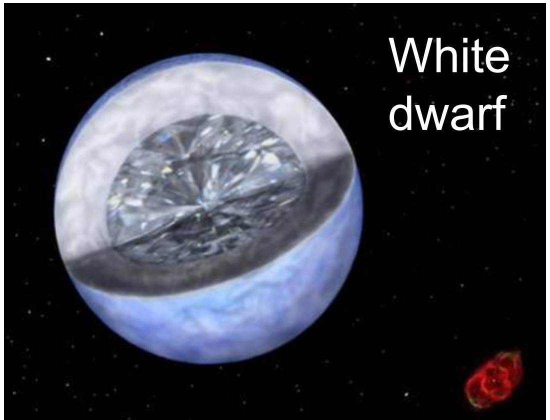
Schwarzschild metric – a *vacuum* solution

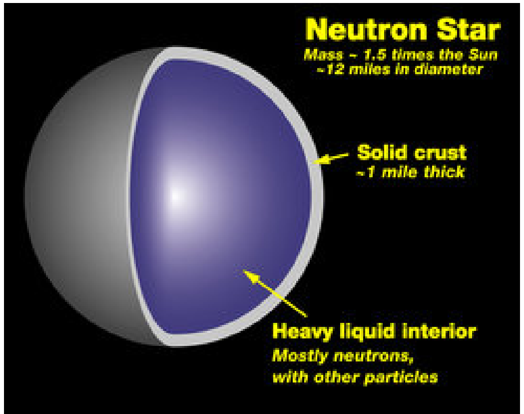
• Recall that we got Schwarzschild metric as a solution of Einstein field equation in vacuum – outside a spherically-symmetric, non-rotating massive body. This metric does not apply *inside* the mass.

• Take the case of the Sun: radius = 695980 km. Thus, Schwarzschild metric will describe spacetime from r = 695980 km outwards. The whole region inside the Sun is unreachable.

• Matter can take more compact forms:
  - white dwarf of the same mass as Sun would have r = 5000 km
  - neutron star of the same mass as Sun would be only r = 10km

• We can explore more spacetime with such compact objects!



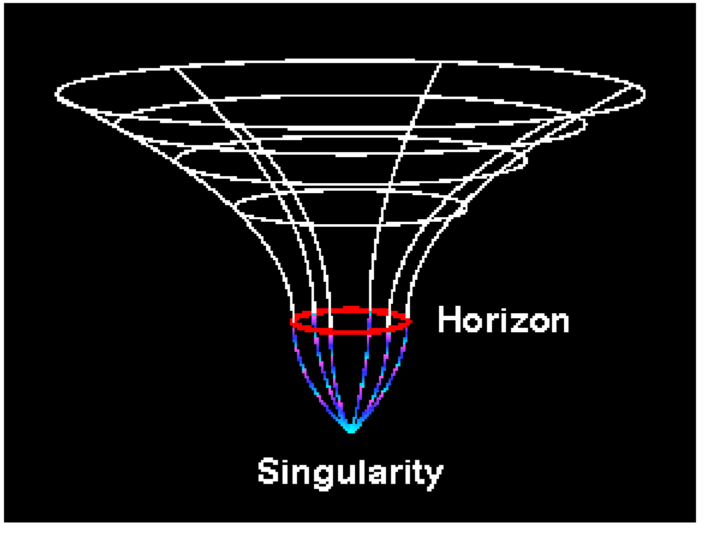


Black hole – the limit of Schwarzschild metric

• As the massive object keeps getting more and more compact, it *collapses* into a **black hole**. It is not just a denser star, it is something completely different!

• In a black hole, Schwarzschild metric applies all the way to r = 0, the black hole is *vacuum* all the way through!

• The entire mass of a black hole is concentrated in the center, in the place called the ***singularity***.



Event horizon

- Let's look at the functional form of Schwarzschild metric again:

$$ds^2 = [1-(2m/r)]dt^2 - [1-(2m/r)]^{-1}dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2$$

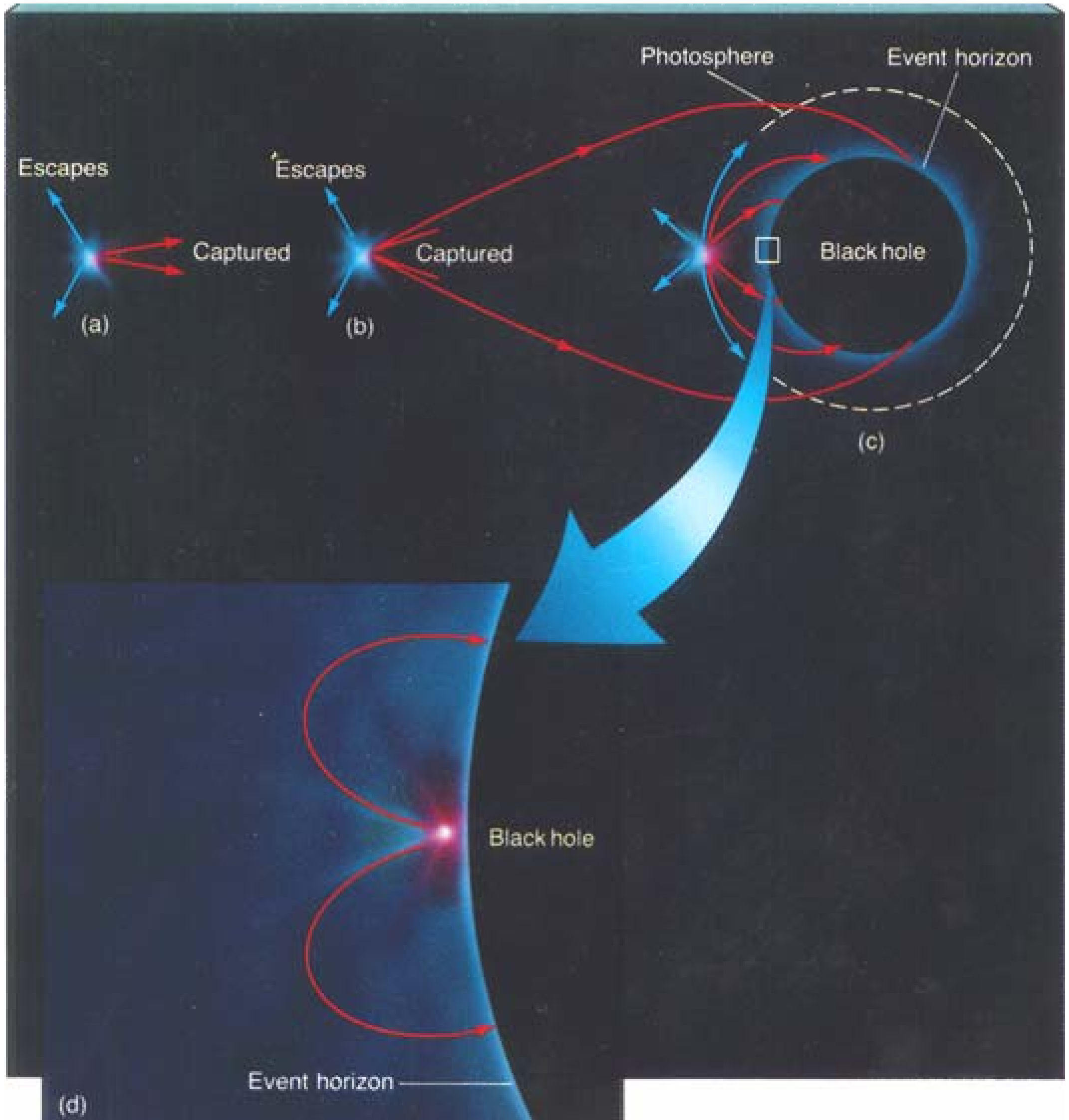
- We want to study the radial dependence only, and at fixed time, i.e. we set $d\phi = d\theta = dt = 0$. Then all we have left is:

$$ds^2 = [1-(2m/r)]^{-1}dr^2 \text{ , or}$$

$$dr^{shell} = dr/[1-(2m/r)]^{1/2}$$

- At $r_S = 2m$ there is a discontinuity (denominator $[1-(2m/r)]^{1/2}$ turns into zero). This radius r_S is known as the Schwarzschild radius, and the spherical shell defined by this radius is called the ***event horizon***.
- You've all (probably) heard about this feature of black holes, that the escape velocity is equal to the speed of light for them. It is at the event horizon that this statement is true.
- However, event horizon is *not* the surface of a black hole! Black hole has no "surface" as we understand it – an interface between vacuum and stuff. **It's all vacuum!**

Light trajectories near event horizon



Crossing the event horizon

• For a remote observer (whose coordinates are r and t), velocity of an object free-falling into a black hole is:

$$dr/dt = -[1-(2m/r)](2m/r)^{1/2}$$

• (Derivation of this equation is straightforward but lengthy; it can be found in Chapter 3, Section 5 of “Exploring Black Holes”). At $r = 2m$, i.e. on the event horizon, this velocity becomes zero – free-falling objects come to a stand-still!

• To derive the free-fall velocity for the falling observer themselves, we can use:

$$dt^{shell} = [1-(2m/r)]^{1/2}dt \text{ , and}$$

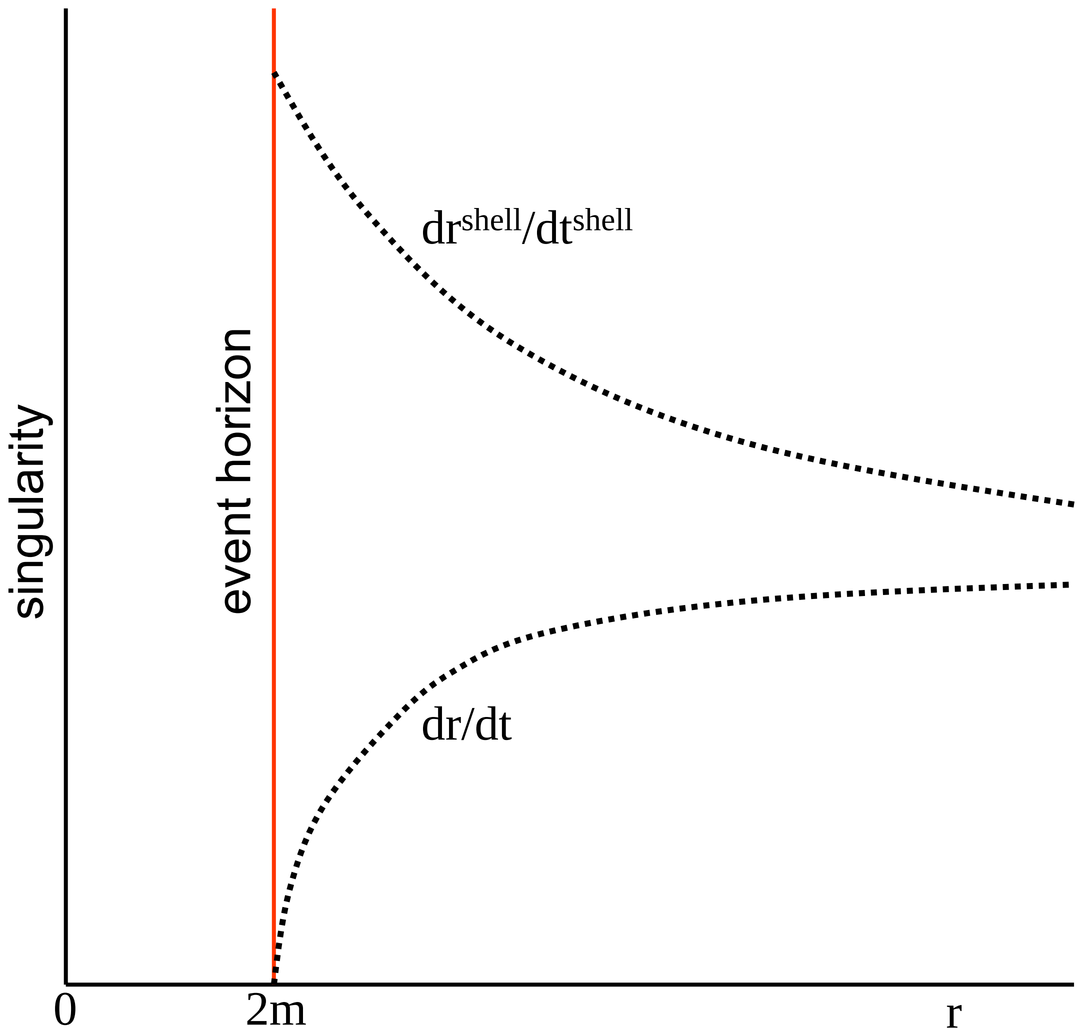
$$dr^{shell} = dr/[1-(2m/r)]^{1/2}$$

Then:

$$dr^{shell}/dt^{shell} = -(2m/r)^{1/2}$$

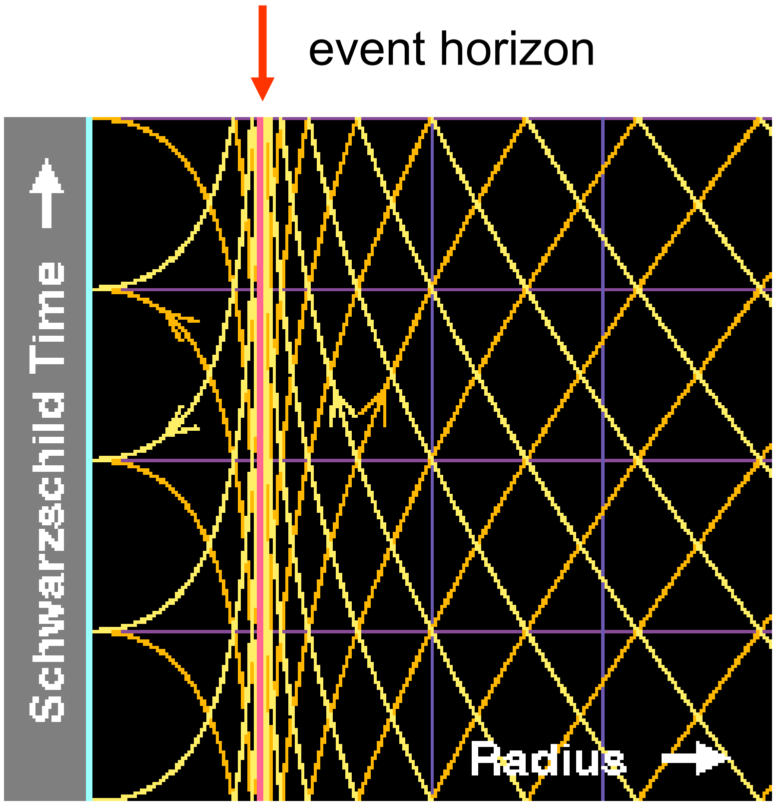
• This velocity is finite at the event horizon.

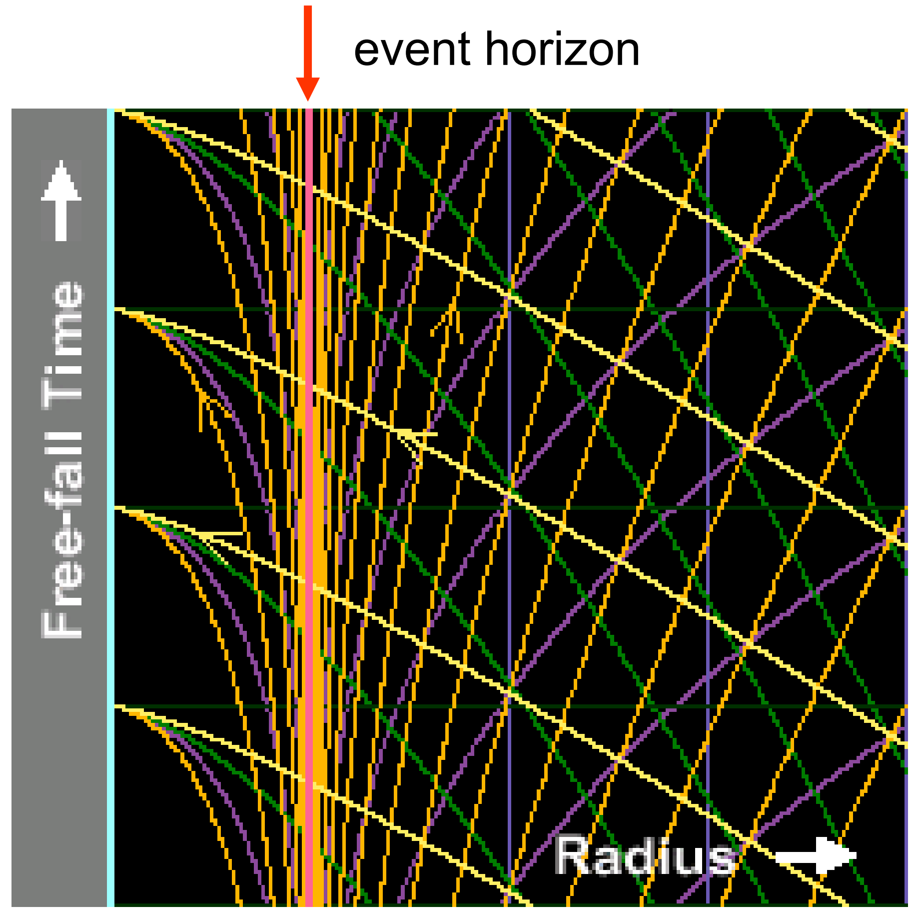
Crossing the event horizon



- For a faraway observer, objects never cross the event horizon.
- For the local, free-falling observer, the event horizon is crossed at a finite velocity, in a finite time.
- In fact, for the local observer nothing special happens while crossing the event horizon
- Collapsing into the singularity is one special event for the free-falling observer...

Geodesics of Schwarzschild metric





null geodesics *inward*

null geodesics *outward*

constant radius

constant time

geodesics of freefall

Singularity: where all matter goes

- To understand the nature of the singularity in the center of a black hole, we first have to look at how a star becomes a black hole.
- As you well know, stars are huge fusion reactors. In their cores, hydrogen nuclei combine to form helium, and tremendous amount of energy, in the form of photons, is released. These photons create immense outward pressure that counter-balances star’s gravity and keeps the star material from falling into the center.
- As hydrogen “fuel” burns out, helium begins to burn forming heavier elements, and keeping the photon pressure up. Then helium runs out, heavier elements begin to burn, until all light fuel runs out and fusion stops altogether...
- What happens as fusion winds down, and photon pressure can no longer keep the star’s shape and size, depends on the size of the star.

Black hole – a result of collapse

• We are interested in black holes here, so let's look at what happens to the heaviest of stars (at least 1.4 solar masses).

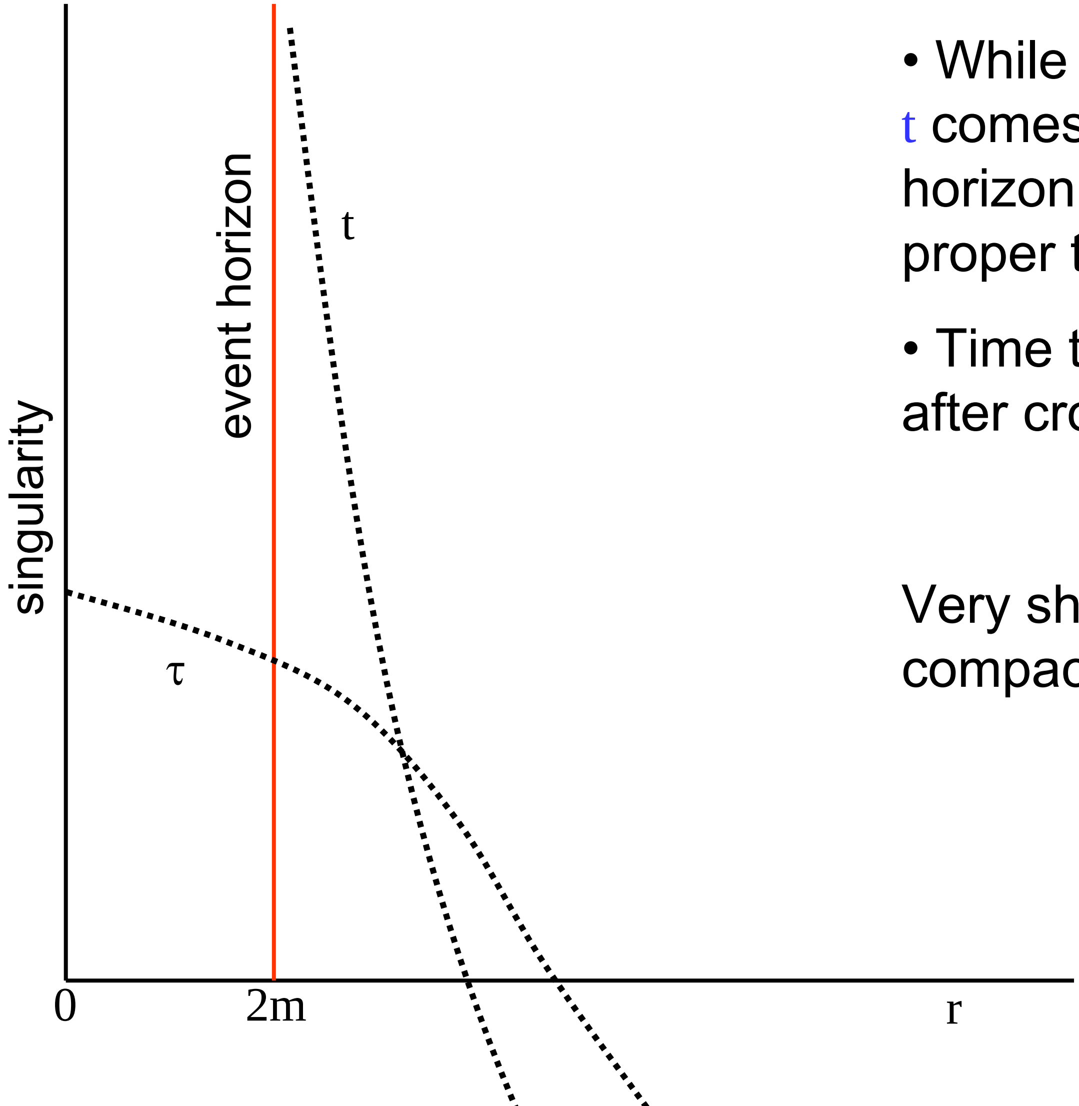
• As photon pressure reduces, the gravity overwhelms it and starts squeezing the star to smaller and smaller size.

• Two processes take place: star becomes denser, and its angular velocity increases (to conserve the angular momentum).

• At some point, the radius of the star approaches $r_s = 2m$ – the Schwarzscild radius. At this point, the event horizon forms, the star becomes a black hole, and all the matter that formed the star falls to the very center – the singularity.

• Why does the matter fall to the center? Because there's nothing inside a black hole that could prevent it!

Observer falls into a black hole in a finite proper time



- While for the remote observer time t comes to a standstill at the event horizon, for the falling observer proper time τ keeps moving along.
- Time to plunge into the singularity after crossing the event horizon:

$$\tau = 15.4 \times 10^{-6}\ M/M_{sun}\ [s]$$

Very short! (but black holes are very compact, so it is to be expected...)

Spinning black holes. Kerr metric.

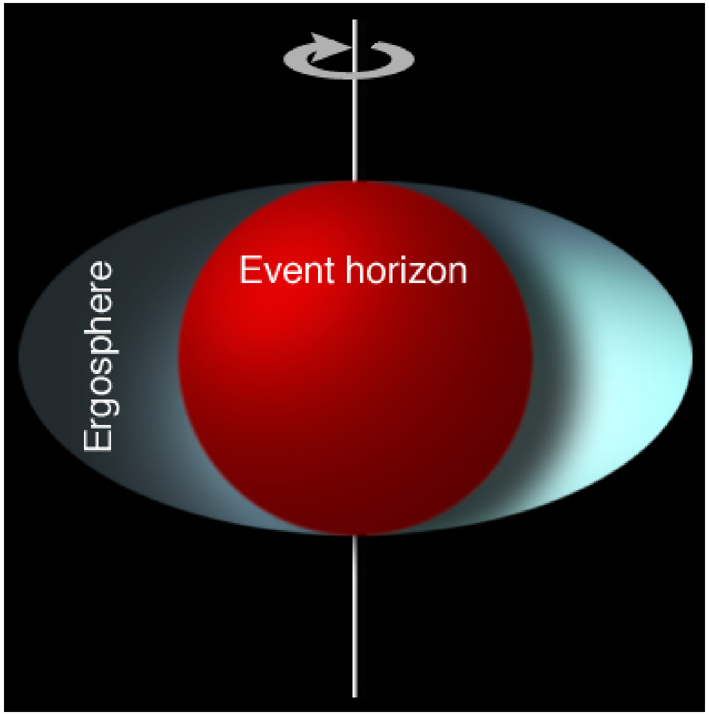
• For a spinning black hole, vacuum solution of Einstein field equation is given by Kerr metric:

$$ds^2 = \rho^2\left(\frac{dr^2}{\Delta} + d\theta^2\right) + (r^2 + a^2)\sin^2\theta d\phi^2 - dt^2 + \frac{2mr}{\rho^2}(a\sin^2\theta d\phi - dt)^2$$

where $\rho^2 = r^2 + a^2\cos^2\theta$ and $\Delta = r^2 - 2mr + a^2$ and a is the *specific angular momentum*.

• Special case of $a = 0$ yields Schwarzschild metric.

• Ergosphere: region outside the event horizon where matter is forced to rotate with the black hole and thus gain energy. This matter can still escape and carry away some of black hole's energy.



Some common myths about black holes

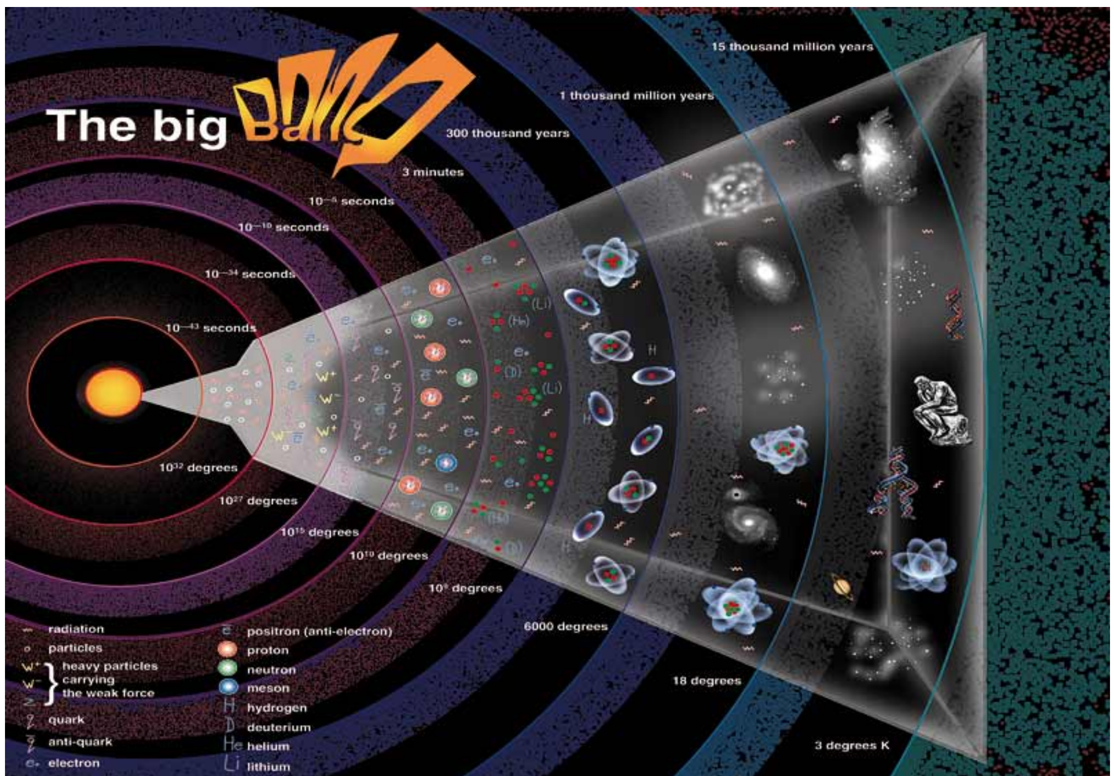
• ***Black hole sucks in everything around it.*** A black hole is not really much different from a regular star of the same mass. If our Sun were to suddenly become a black hole, it's gravitational field would look exactly the same! The planets will keep moving around as if nothing happened!

• ***Black holes emit no light, because light cannot escape. Thus, black holes are dark and invisible.*** It is true, light cannot escape from the *event horizon* (much less from inside a black hole). But the process of accretion creates such enormous accelerations, that a lot of energy is radiated by the falling matter before it reaches the event horizon. This energy surpasses the energy emitted by regular stars. Black holes are among the ***brightest*** objects in the sky.

• ***Since time comes to a halt at event horizon, it takes infinite time to form a black hole. So black holes do not exist!*** As you have seen, at the time the vent horizon is formed, all of the dying star's mass is inside – the matter is there already!

The birth of the Universe

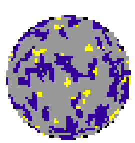
- “Big Bang” started the Universe out of a singularity.
- Period of faster-than-light “inflation” followed that created causally-disconnected regions of the universe. Inflation was mere creation of more space, not faster-than-light motion.

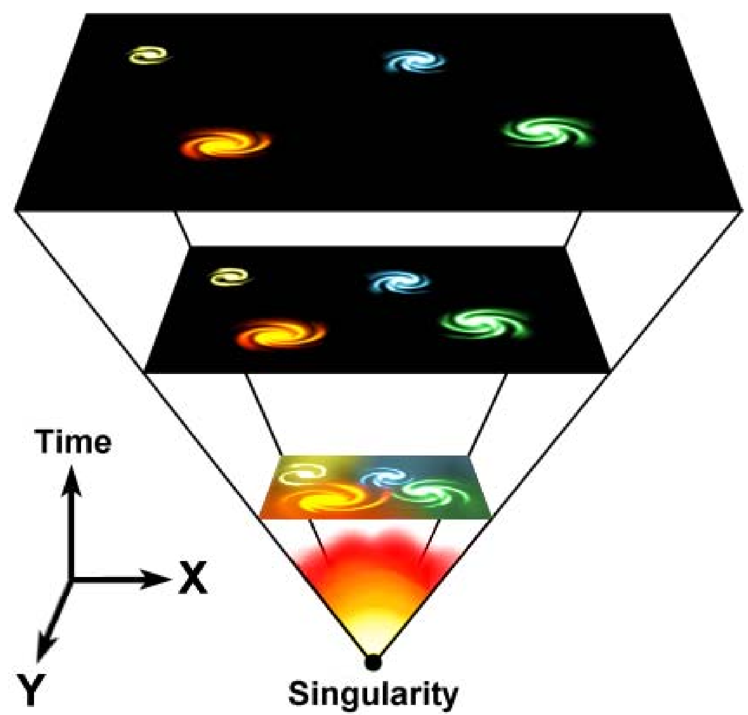


The expansion of the Universe

- The Universe expands, that much we know. What does this expansion mean, though? That galaxies fly apart? That stars in galaxies fly apart? That atoms in stars fly apart?

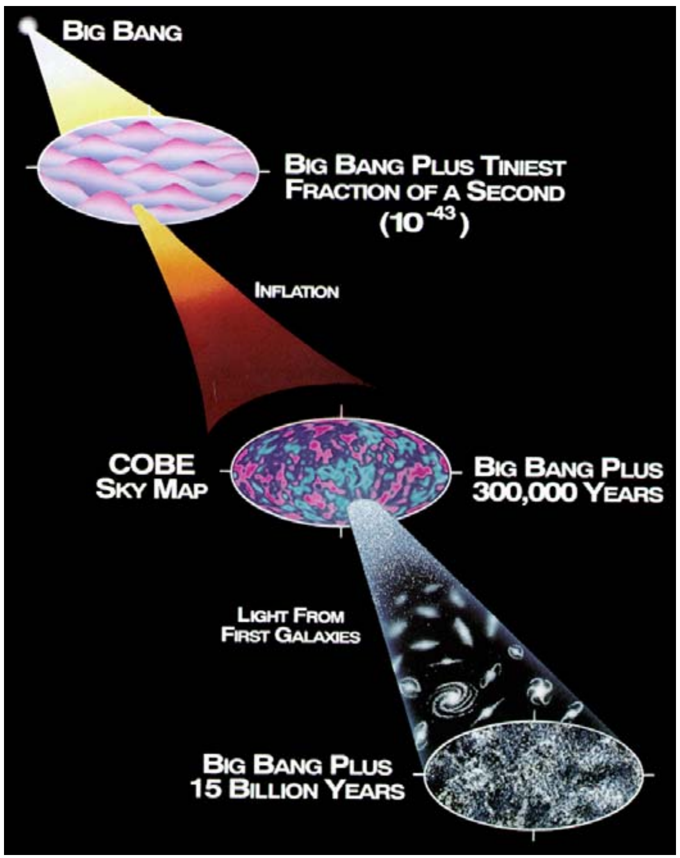
- NO! There is no “flying apart”! Space itself is being constantly created; gravity holds stars in galaxies together, and Coulomb forces hold atoms together.

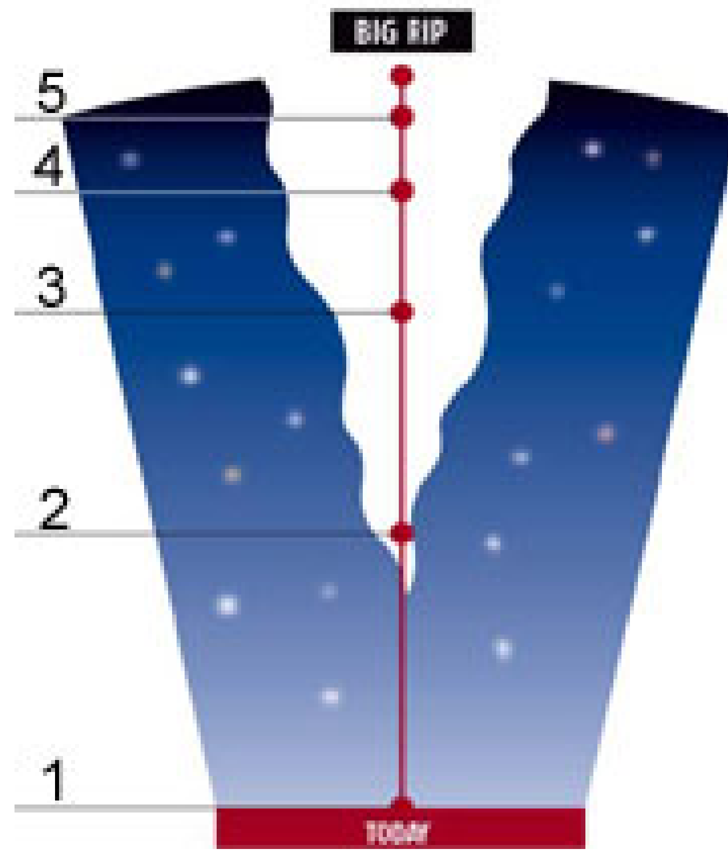




The fate of the Universe

- The Universe began with a bang. A Big Bang. Will it end with a bang, too? As far as today's theories predict, it will rather end with a Big RIP!





Recap

• Black holes arise as a limit solution to Schwarzschild metric. In black holes, vacuum solution applies to all space except the central singularity. Black hole is vacuum!

• Discontinuity at r = 2m forms the event horizon. Remote observer time goes to a halt at event horizon. Free-falling observer proper time keeps moving.

• Kerr metric and its extensions provide more realistic picture of rotating black holes.

• The Universe started in the Big Bang, is expending by creating more space, and its ultimate fate is probably still unknown...