

Physics 311

Special Relativity

Lecture 2:

Unity of Space and Time

Inertial Frames

OUTLINE

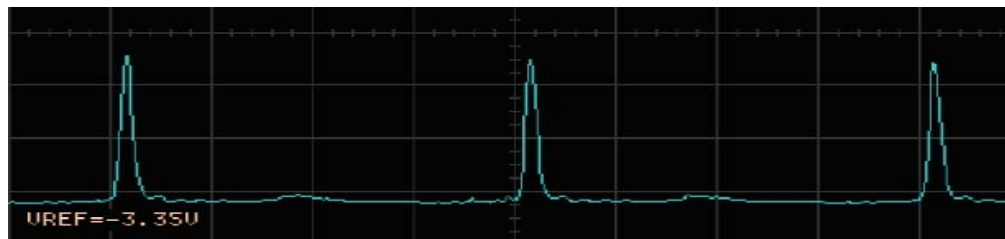
- Same unit to measure distance and time
- Time dilation without Lorentz transformations
- Inertial frames
- Tidal accelerations

Administrative issues

- ♦ Office hours: Tues. 9:30 am – 11:30 am and by appointment
- ♦ A copy of "Spacetime Physics" put on reserve in the Physics/Astronomy Library
- ♦ Lecture slides available on the web (PPT and PDF, see the "Schedule" page)
- ♦ Still debating the location for tutorials...

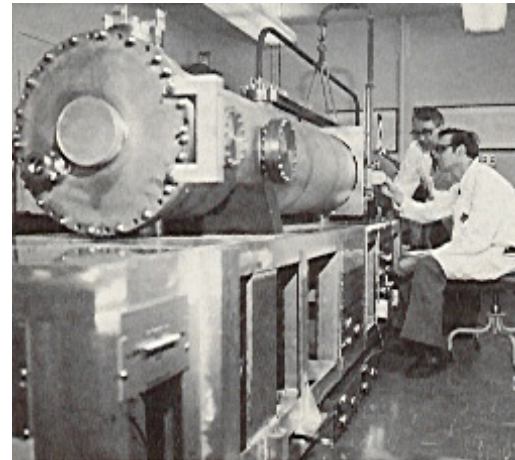
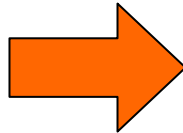
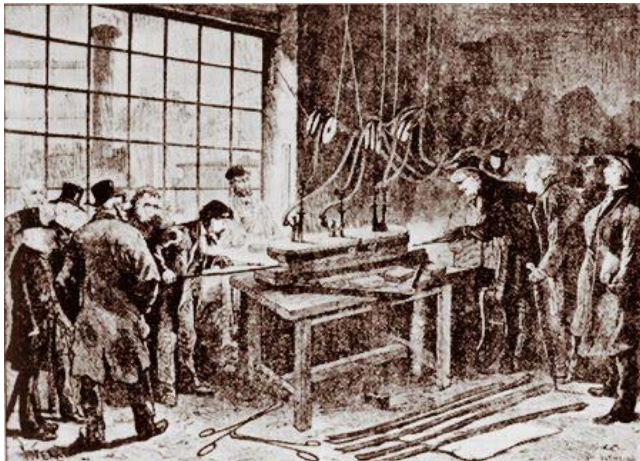
SAME UNIT FOR SPACE AND TIME

- Have seen in the previous lecture that using different units to measure different dimensions leads to confusion
- Therefore, we've used same units (meters) for both time and space. Conversely, can use seconds (nanoseconds, minutes, years) to measure time and space.
- Examples:
 - Distances between stars are measured in years of light travel (light-years)
 - Distances in high-speed electronics are measured in nanoseconds ($1 \text{ ns} \approx 1 \text{ ft}$) "Pass me a 16 ns cable, will you?"
 - "The Mall is just 10 minutes by car"
 - "Laser pulses arrive every 12.5 ft."



Time defines length (and vise versa)

- 1983: meter is defined as the distance light travels in $1/299,792,458$ of a second.
- (thus, speed of light is, **by definition**,
 $c=299,792,458$ m/s)
- By the way, one second is defined as the duration of 9,192,631,770 cycles of hyperfine transition in Cs atom



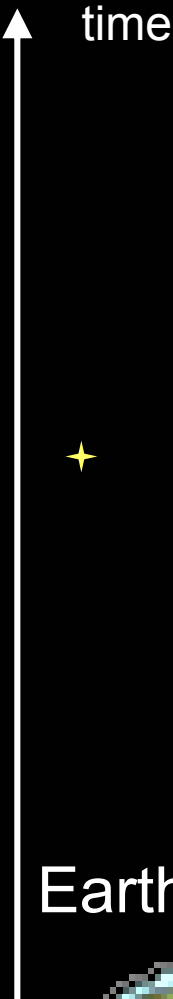
The Interval - again

Sample problem 1: a starship

- Starship leaves the Earth (Event 1) and goes at 95% of the speed of light to arrive at Proxima Centauri (Event 2) 4.3 light years away. What is the space and time separations between the two events in the Earth frame and the ship frame?

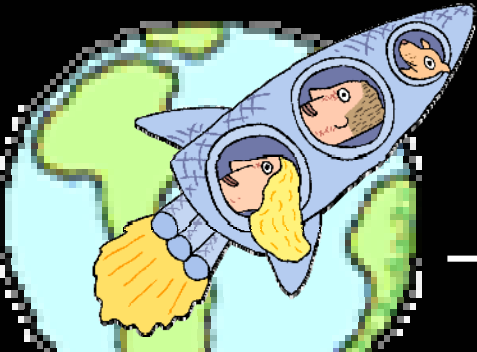
Earth frame

Proxima



Event 2

Earth



Event 1

space

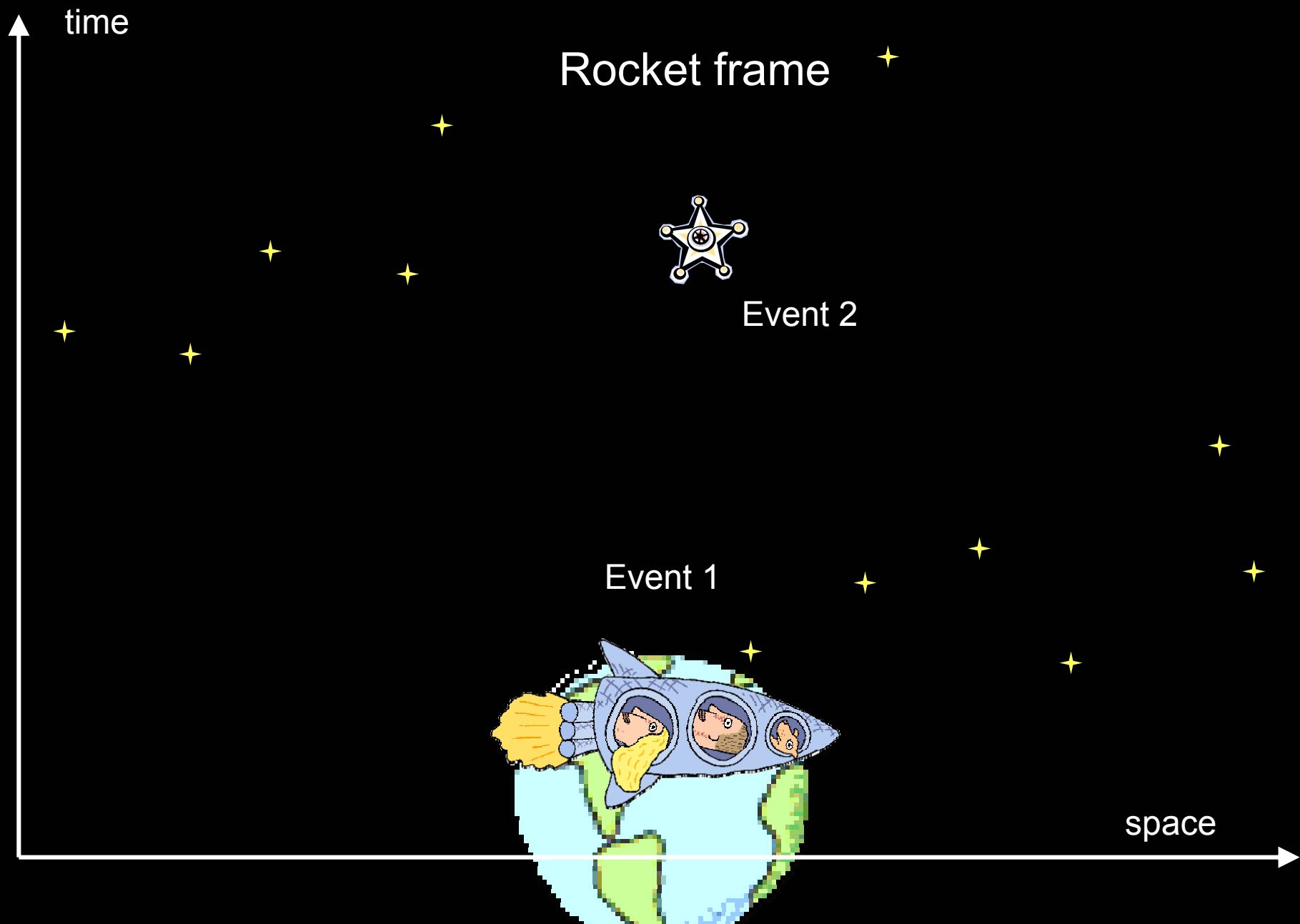


Sample problem 1: a starship

- Starship leaves the Earth (Event 1) and goes at 95% of the speed of light to arrive at Proxima Centauri (Event 2) 4.3 light years away. What is the space and time separations between the two events in the Earth frame and the ship frame?

Earth:

$$\Delta x_E = 4.3 \text{ light years}$$
$$\Delta t_E = (4.3 \text{ light years}) / (0.95 \text{ speed of light}) = 4.53 \text{ years}$$



Sample problem 1: a starship

- Starship leaves the Earth (Event 1) and goes at 95% of the speed of light to arrive at Proxima Centauri (Event 2) 4.3 light years away. What is the space and time separations between the two events in the Earth frame and the ship frame?

Earth: $\Delta x_E = 4.3$ light years
 $\Delta t_E = (4.3 \text{ light years}) / (0.95 \text{ speed of light}) = 4.53$ years

Starship: $\Delta x_S = 0$, but what about Δt_S ? Use the interval!

$$\begin{aligned} s^2 &= (\Delta t_S)^2 - (0)^2 = (\Delta t_E)^2 - (\Delta x_E)^2 \\ &= (4.53)^2 - (4.3)^2 \text{ (years)}^2 \end{aligned}$$

$$\Delta t_S = 1.42 \text{ years}$$

Time dilation without Lorentz transformations!!!

Spacetime

- The invariance of the spacetime interval is really, really deep. Space by itself and time by itself are not fundamental (noted first by Hermann Minkowski). Space and time are different for different observers. Spacetime is the same for all observers.
- BUT: while space and time are thus on equal footing, they are NOT the same entity; their nature is different, as manifested, albeit formally, by the “minus” sign in the expression for the interval:

$$s^2 = (\Delta t)^2 - (\Delta x)^2$$



Sample problem 2: SLAC

- An electron traveling at 0.999999999995 speed of light (51 GeV) passes through detector 1 (Event 1), then detector 2 located 10 meters away (Event 2). What is the space and time separations between the two events in the Lab frame and the electron frame?

Lab:

$$\Delta x_L = 10 \text{ met}$$

$$\Delta t_L = (33.356 - 33.356)$$

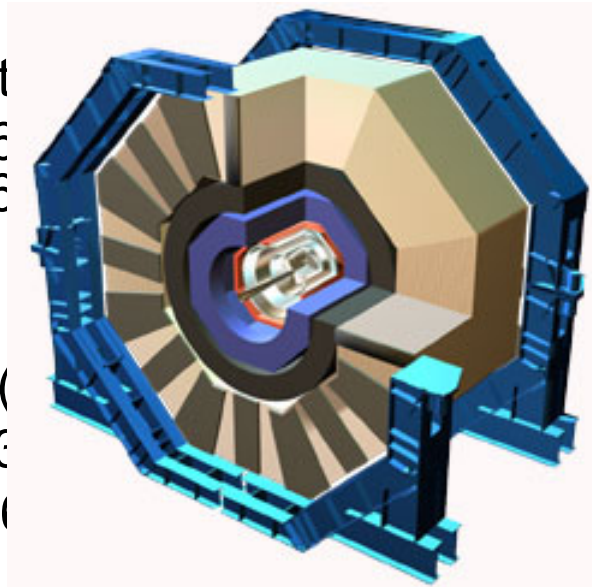
Electron:

$$\Delta x_e = 0$$

$$s^2 = (\Delta t_e)^2 - (\Delta x_e)^2$$

$$= (33.356 - 33.356)^2$$

$$\Delta t_e = 0.33356$$



$$3.3564 \text{ ns}$$

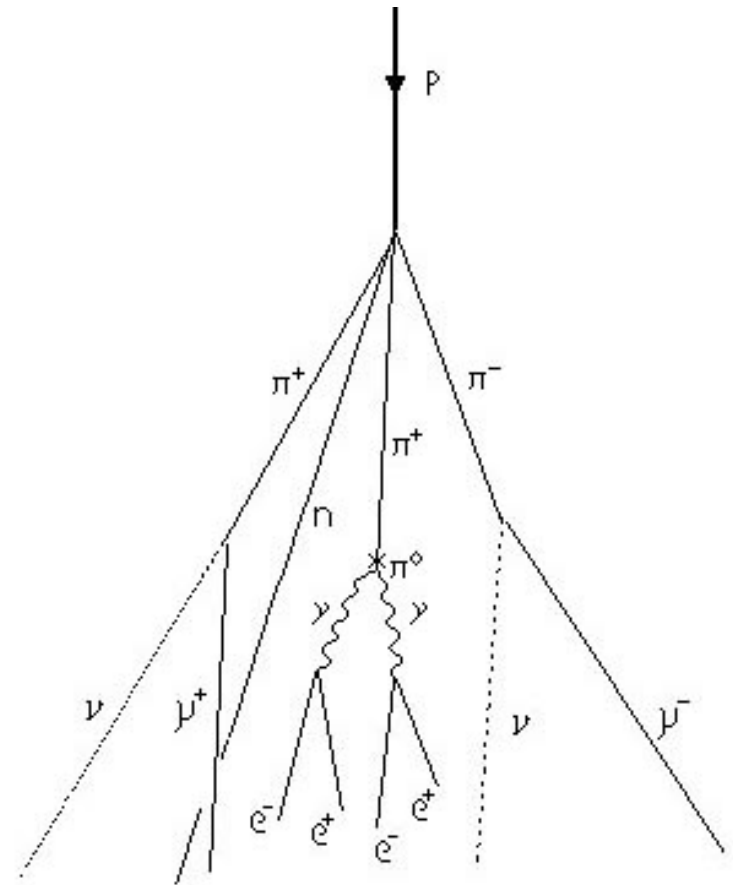
$$\text{eed of light) =}$$

$$3.3564)^2 (\text{ns})^2$$

- This effect is responsible for longer lab frame lifetimes of unstable particles moving very-very fast

OBSERVING SHORT-LIVING PARTICLES

- Many elementary particles are extremely short-lived. They quickly decay into product particles (which often subsequently decay). However, when moving fast, these short-lived particles can be observed in the laboratory due to time dilation.
- An important example is the observation of muons (μ -mesons, close relatives of the electrons) originating from the cosmic rays hitting the upper atmosphere. The muon lifetime is about $2.2 \mu\text{s}$, so, traveling at about speed of light, the muons will go about 660 meters. Cosmic ray labs are able to detect muons that have traveled as much as 10-20 km...



OBSERVING SHORT-LIVING PARTICLES

A muon is created in the upper atmosphere at the altitude of 10000 meters at a high energy, such that it is moving vertically at 0.999 speed of light. What is the probability that the muon decays before it reaches an alpine cosmic ray lab located at the altitude of 3000 meters? Compare that to the non-relativistic probability of its decay.

Lab: $\Delta x_L = 7000$ meters

$$\Delta t_L = (7000 \text{ m}) / (0.999 \cdot 299792458 \text{ m/s}) = 7007 \text{ m} = 23.373 \mu\text{s}$$

Muon: $\Delta x_m = 0$

$$(\Delta t_m)^2 = (\Delta t_L)^2 - (0)^2 = s^2 = (\Delta t_L)^2 - (\Delta x_L)^2 = (7007)^2 - (7000)^2$$

$$\Delta t_m = 313 \text{ m} = 1.05 \mu\text{s}$$

Probability of decay is

$$(1 - \exp(\Delta t_m / 2.2 \mu\text{s})) = 0.38$$

Non-relativistically, probability of decay would be:

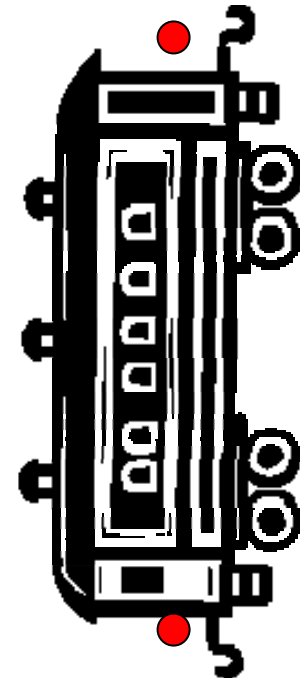
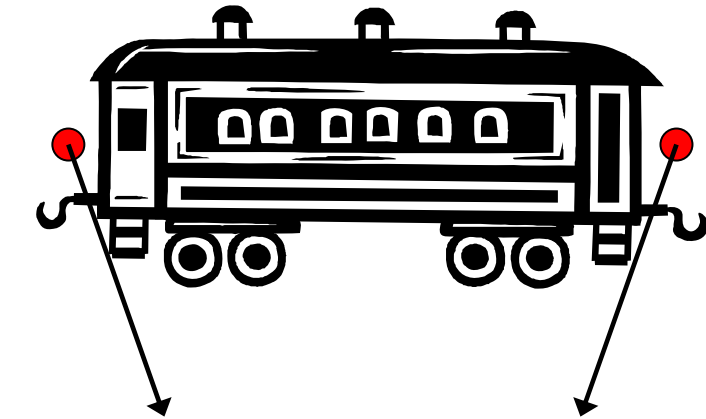
$$(1 - \exp(\Delta t_L / 2.2 \mu\text{s})) = 0.999975$$

THE INERTIAL FRAMES

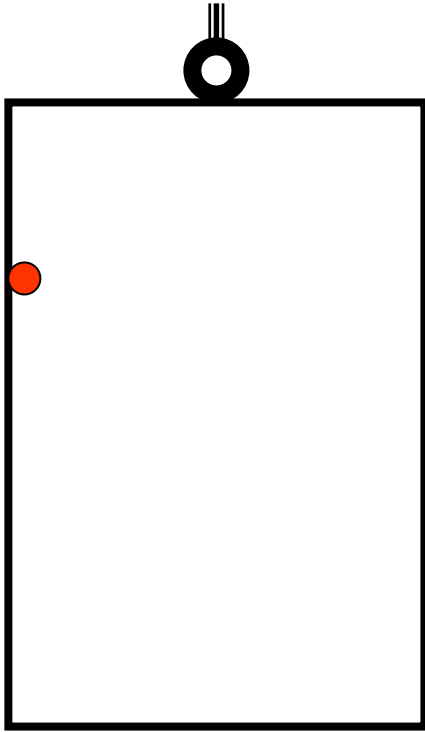
Inertial Frames

- Inertial (freefall, or free-float) frames: *every free particle preserves its state of motion (or rest) in such frames.*
- Inertial frames are *local* in both space and time:

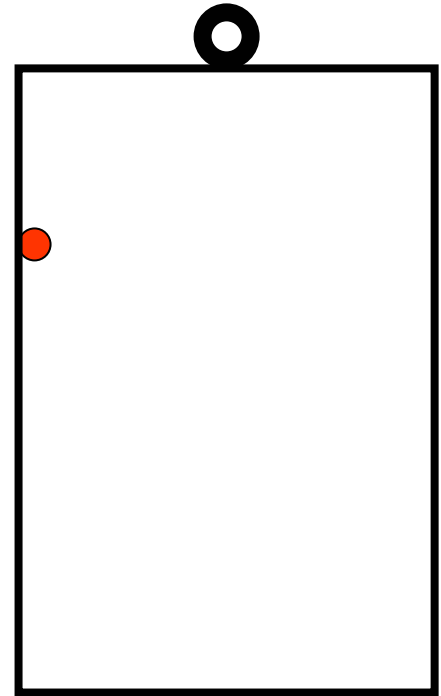
(by the way, in his papers on Relativity Einstein often used people riding trains to explain things or to present "paradoxes" – do you know why?)



Free-Falling and Floating Freely



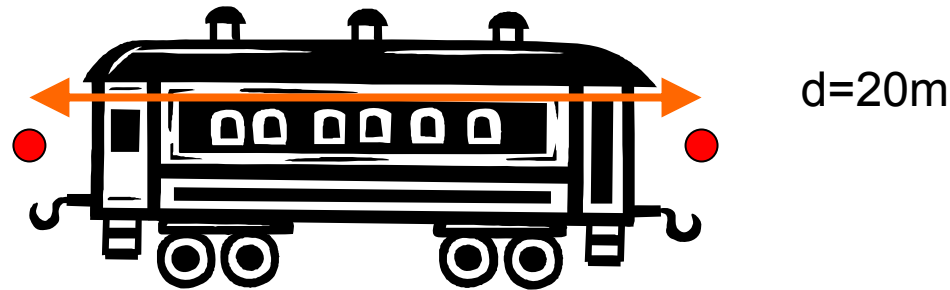
elevator at rest w.r.t. Earth



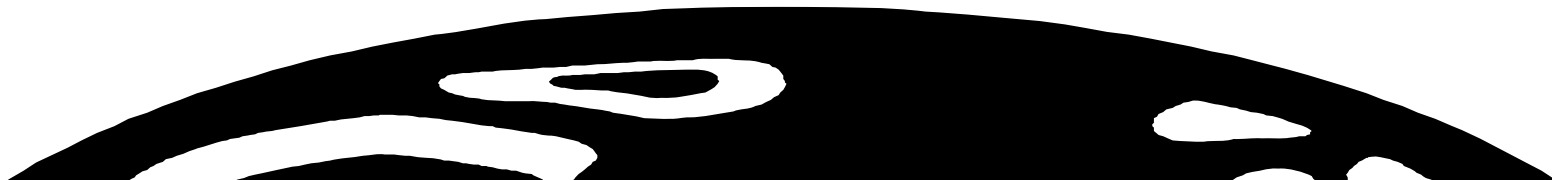
elevator in a free-fall (free-float)

Tidal accelerations

- In a real free-float frame (with non-zero extents in space and time) free objects will eventually change their motion, given sufficient time



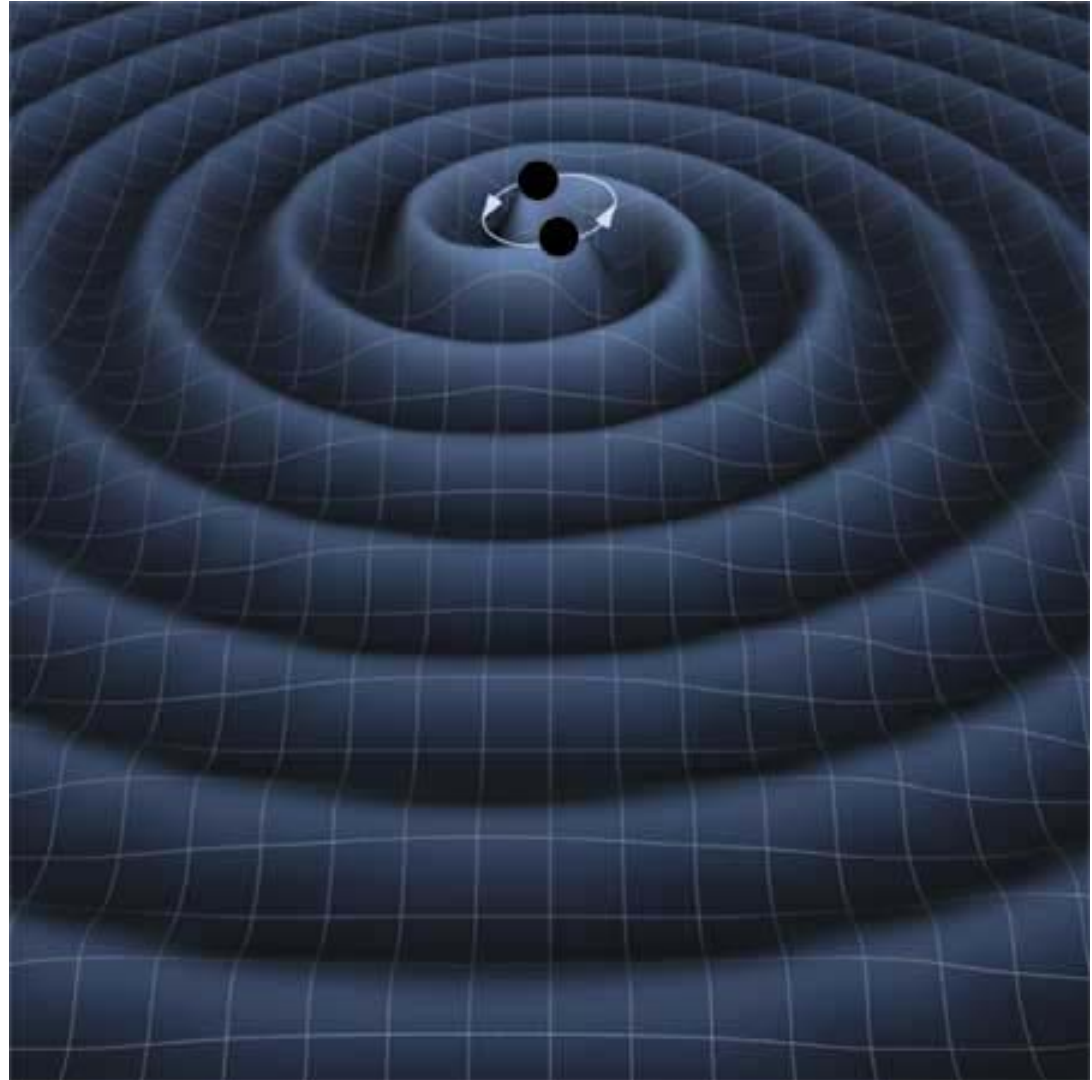
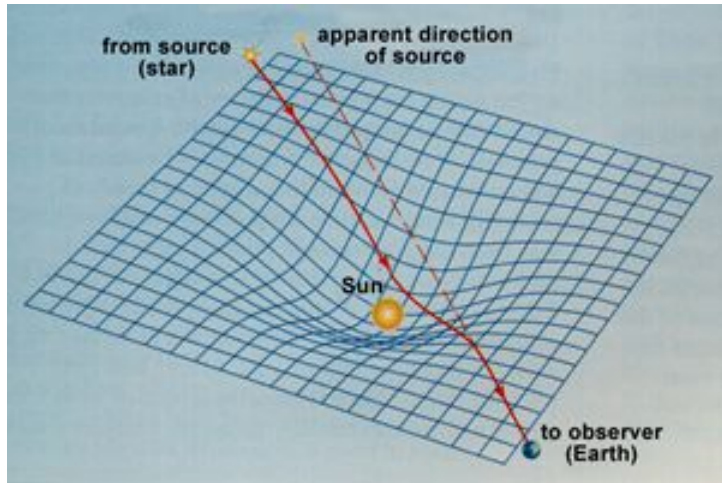
$d < 20\text{m}$



Sneak peak into General Relativity

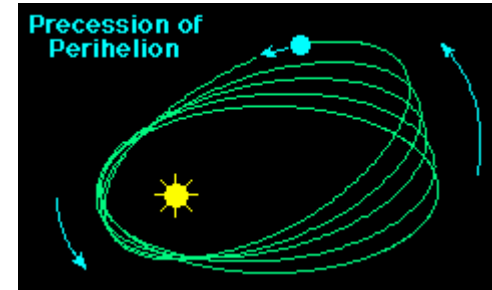
- Newtonian gravity is just another force
- Einstein says: gravity is different from all other forces, has to do with the spacetime curvature
- Get rid of local gravity, choose a free-float (inertial) frame!
- All that's left is *tidal accelerations* – relative motion of objects due to imperfectly local character of the reference frame

Curved spacetime: gravitational waves



Possible additional GR topics:

◊ Advance of the perihelion of Mercury



◊ Gravitational lensing



◊ Dark matter and dark energy searches



◊ Numerical Relativity

