Homework #3

- L-8 (25 points)
- L-16 (25 points)
- 4-1 (20 points)
- Extra credit problem (30 points): Show that Lorentz transformations of 4-vectors are *similar* to rotations. (Recall that the magnitude of any 4-vector is invariant under Lorentz transformations - just like the magnitude of a real vector is invariant under its rotations.) Derive the expression for the rotation angle as a function of γ and $\boldsymbol{v}$. Hint: examine the Lorentz transformation matrix M shown in class. Notice that $\boldsymbol{v} < 1$. Your "rotation" matrix will contain *hyperbolic* functions rather than simple trigonometric functions - a manifestation of the special metric of Special Relativity.

Physics 311
Special Relativity

Lecture 8:

Twin Paradox continued: time travel and relativity of simultaneity

Plan of the lecture

- Time travel? One way – yes, round trip – no!
- Another look at the Twin Paradox: Lorentz length contraction
- Time dilation: paradox again?
- Simultaneity is relative, and that’s the solution.
- Proper time and the interval: a universal language.

Time dilation – time travel?

- The Rocket twin ages less. In fact, if the speed of the rocket is very near the speed of light, the whole trip to far-away and back may be over in a few minutes of the Rocket time. A time machine????

Time travel, but one-way

• Yes indeed, for the Rocket traveler, a round-trip to a faraway world at near speed of light is equivalent to going to some distant future. The distance to the remote world of our choosing will determine the distance in time! 1000 light years and back will take us a bit over 1000 x 2 = 2000 years forward.

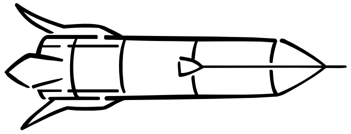
• But: it's a one way trip. No going back for the Rocket twin.

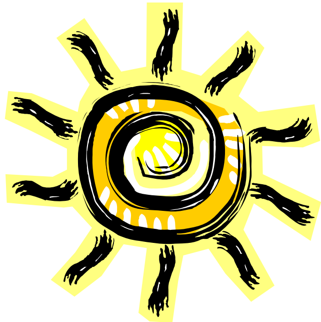
• It has been speculated that if superluminal (faster-than-light) travel is possible, it would take the traveler backwards in time!

• Tachyons: a putative class of particles which are able to travel faster than the speed of light. The word tachyon derives from the Greek ταχυσ (*tachus*), meaning "speedy". Tachyons have a strange property: when they lose energy, they gain speed. Consequently, when tachyons gain energy, they slow down. The *slowest* speed possible for tachyons is **the speed of light**.

Twin Paradox again

- Let's carefully consider what happens in both frames – the Earth and the Rocket.
- The Earth twin would see the spaceship flying at about $\boldsymbol{v} = 0.98c$, passing by Earth at $x = 0$ and $t = 0$, then by Canopus at $x = 99$ and $t = 101$ (times and distances in years). The Earth twin is 101 years older when the spaceship reached Canopus.
- Due to length contraction, the spaceship appears shorter by the $1/\gamma$.
- But, apart from pure curiosity, this doesn't appear important...



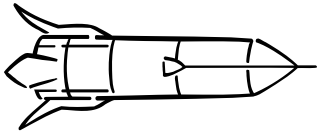


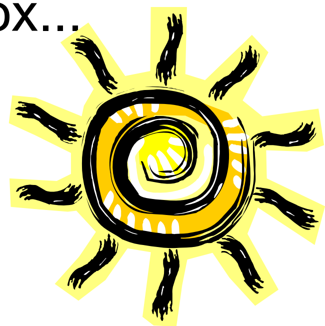
In the Rocket frame

• In the Rocket frame, the Earth, along with Canopus, is flying at 0.98c. To the Rocket twin, the Earth zooms by at this high speed and goes away. Some time later the Canopus zooms by. The Earth by that time is far away. But how far?

• Due to Lorentz length contraction, the Earth, Canopus **and** the distance between them are all shrunk by the factor of $1/\gamma$. Now, this is important! The distance between Earth and Canopus, the distance that the Earth twin travels in the Rocket frame, is only $99/\gamma \approx 19.6$ light years!

• Thus, to the Rocket twin the from Earth to Canopus is only 19.6 light years, which at the speed of 0.98c can be done in just under 20 years! The Rocket twin will be only 20 years older upon arrival to Canopus! The Lorentz contraction alone can resolve the Twin Paradox...





...but what about time dilation?

• The Earth twin sees the Rocket time going slower. It is going slower by the same factor γ, so the 101 years of the Earth time are only 20 years of Rocket time.

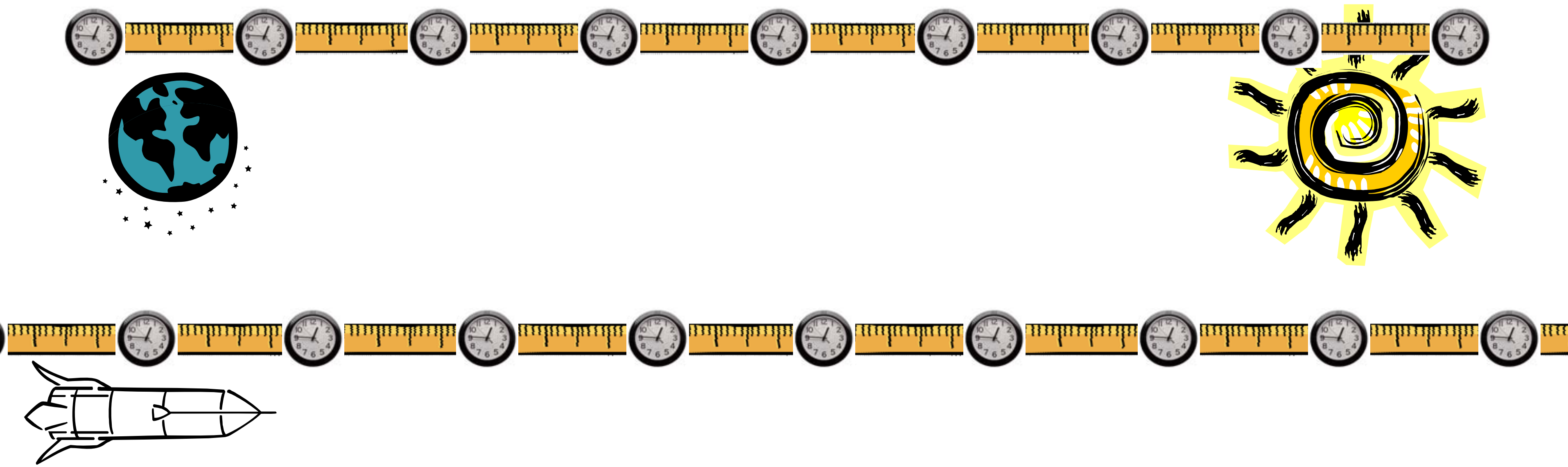
• Now let's hop into the Rocket frame. We've seen that the Earth has to travel just about 19.6 light years, which takes about 20 years of Rocket time. Yet, for the Rocket observer the Earth time is going slower by the factor γ. So when the Earth is 19.6 light year away, the Earth clock as seen by the Rocket observer reads $20/\gamma = 3.96$ years! The Earth twin ages less!!!

• Haven't we just undone the solution for the twin paradox???

• The answer is in carefully looking in what the two twins *see* and what actually *is*.

Simultaneity is relative!

• Let's look at the latticeworks of clocks associated with the Earth and the Rocket frames. The clocks are 1 light year apart in each frame.



• The Earth observer sees the Rocket latticework shrunk by $1/\gamma$, so instead of 1 light year, the Rocket clocks are separated by about 0.2 light years!

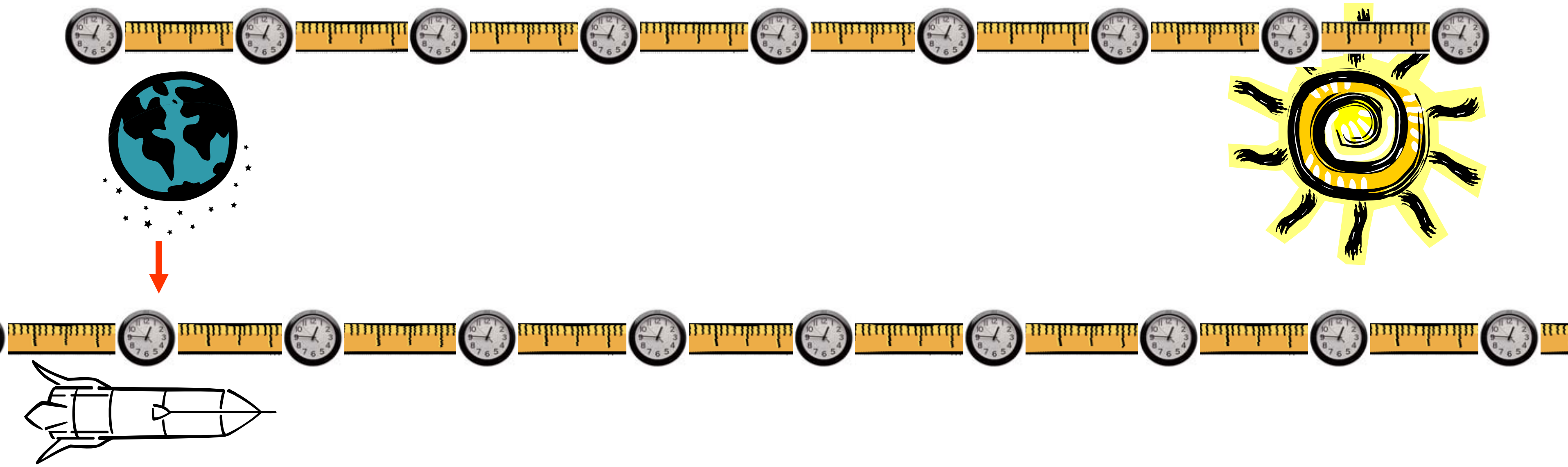
The events in the Earth frame

- The following two events are *simultaneous* to the Earth twin:

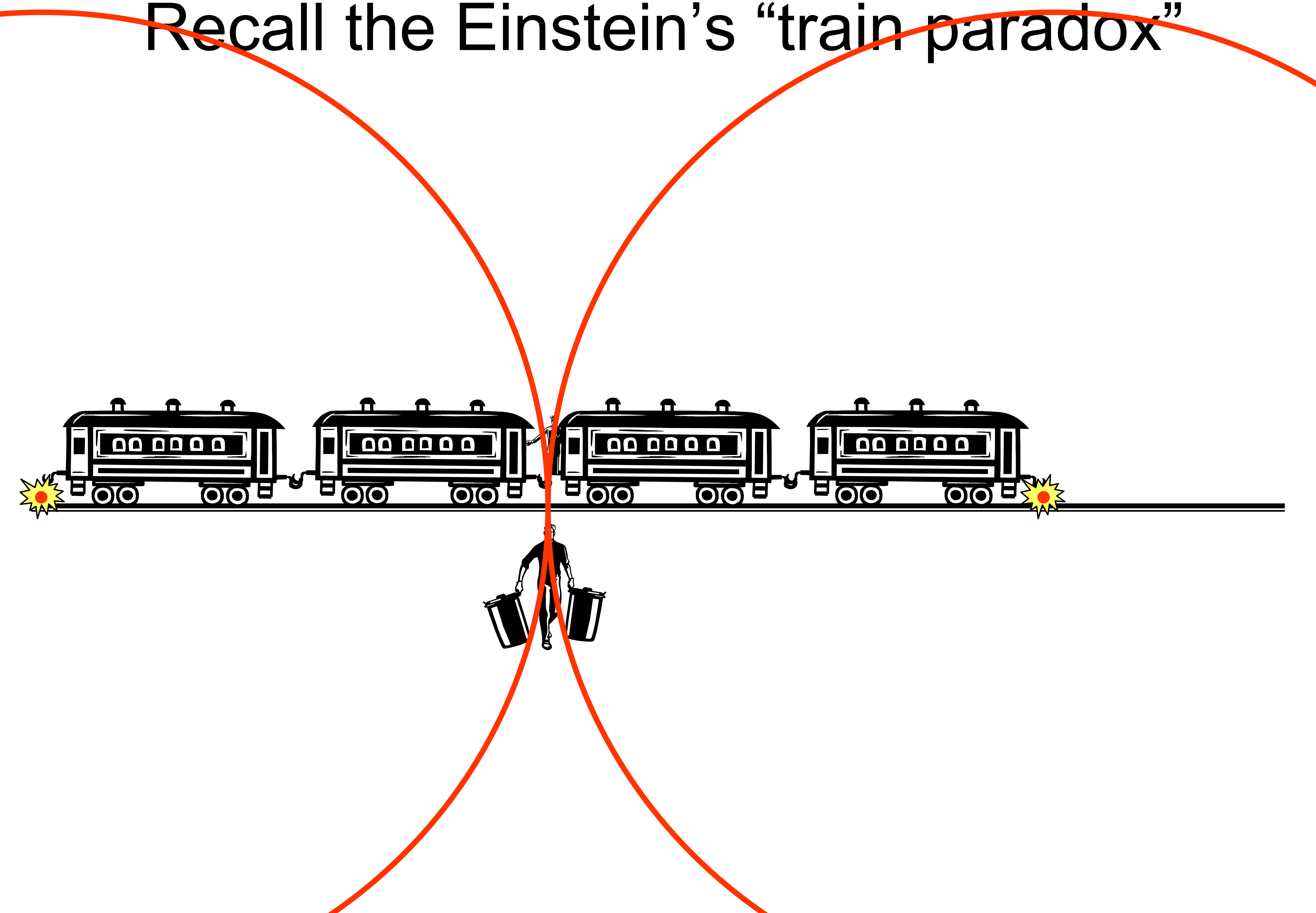
  Event 1: the spaceship passes by Canopus

  Event 2: the -497 clock passes by Earth

- The Earth twin read 20 years on the Rocket clock near Canopus.

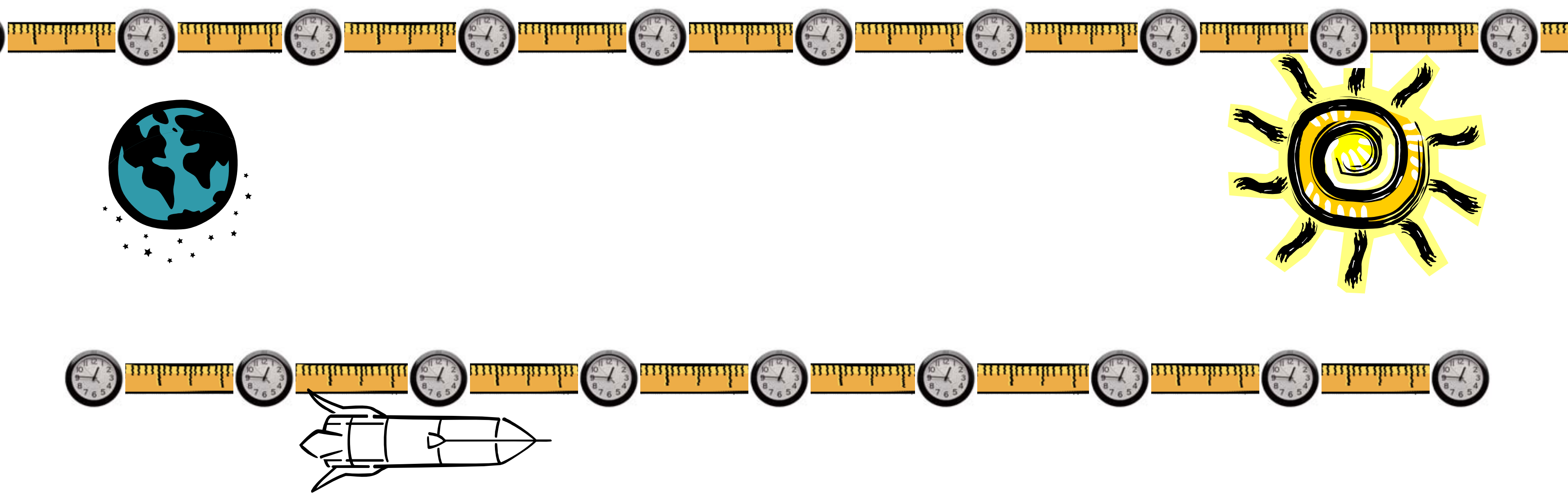


Recall the Einstein's "train paradox"



What was simultaneous in the Earth frame, is not so in the Rocket frame

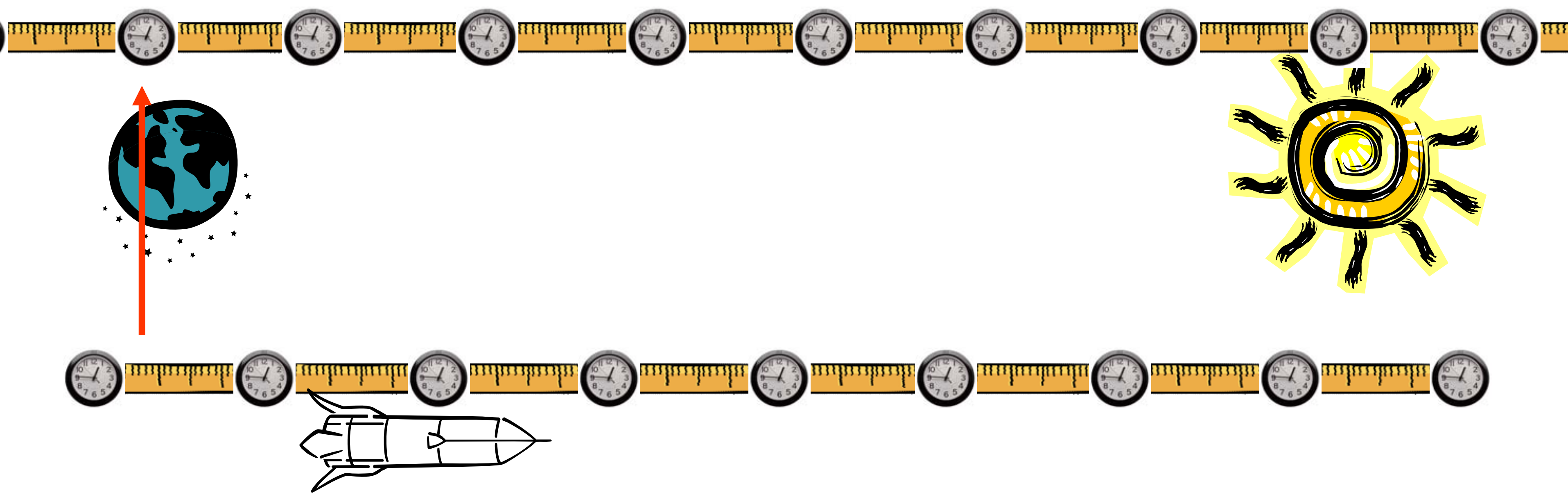
- The Rocket twin is moving fast with respect to the Earth twin. The Earth frame latticework appears contracted to the Rocket twin!



- When the Rocket twin measures the Event 1 (fly by Canopus), the Earth is only 19.6 light years away, flying by the clock -20 or so. It's a long way from the clock -497!

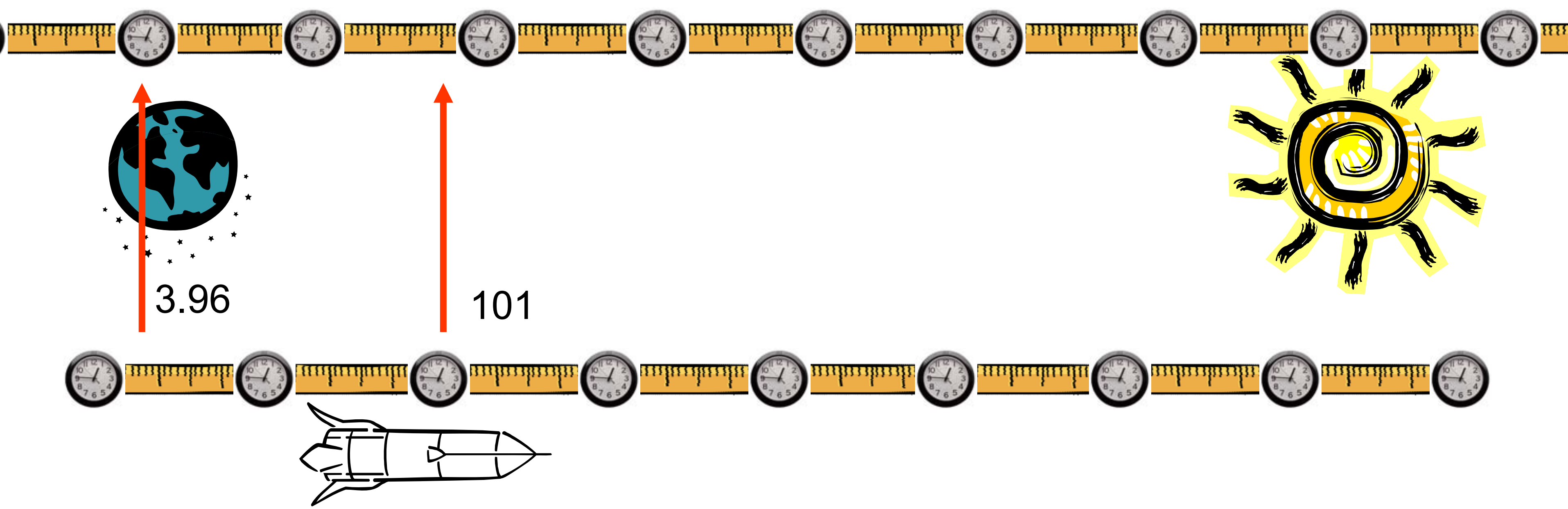
What was simultaneous in the Earth frame, is not so in the Rocket frame

- It gets even better: Flying by Canopus, the Rocket twin observes the Earth clock 99 (which is 19.6γ light years) due to length contraction. The Earth clock near Earth reads 3.96 years, as seen by the Rocket twin!



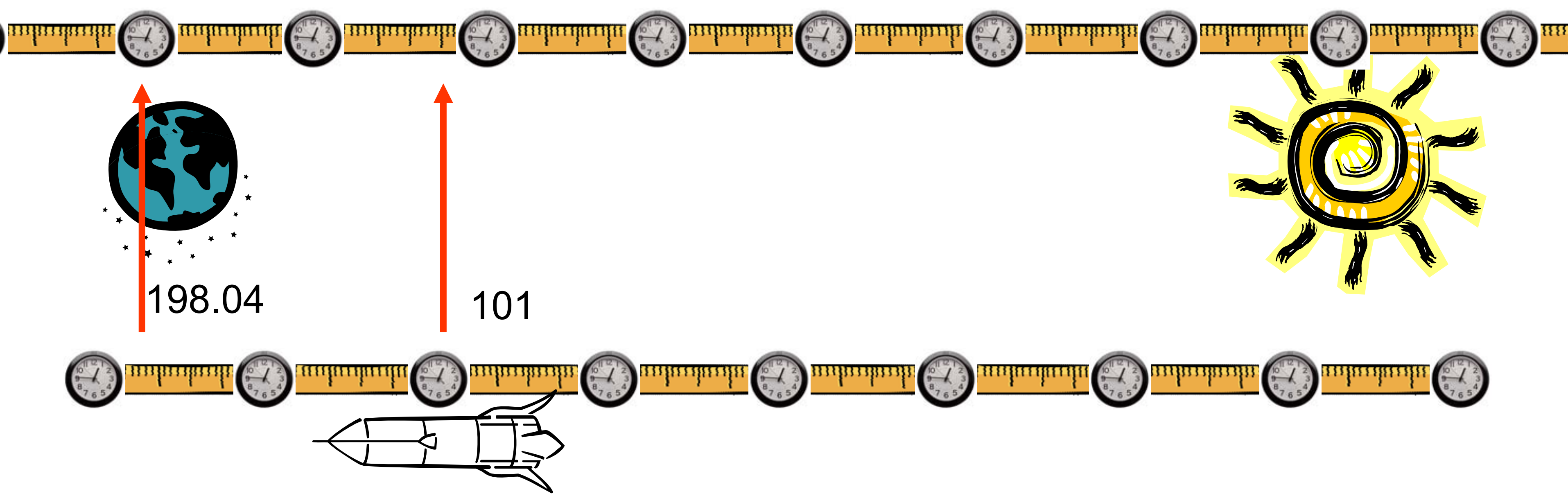
Clocks synchronized in one frame are not synchronized in the other!

- Let's say all the clocks in the Earth frame strike noon at the same time. They are synchronized. But the Rocket observer will see them out of sync! They will strike noon one by one, going front to back. So in fact the reading of 3.96 years on the Earth clock near Earth corresponds to reading of 101 years on the Earth clock near Canopus! The clock near Earth is lagging behind.



The return Rocket observer

- As the Rocket passes by Canopus, the Rocket twin jumps to another Rocket flying back to Earth at 0.98c. In this frame, the Rocket twin will again see the Earth travel the distance of 19.6 light years in 20 years of the Rocket time, but in only 3.96 years of the Earth time.
- BUT: The return Rocket frame reading of the Earth clock is different. When the return Rocket frame passes by Canopus, they read 198.04 years of Earth time near Earth, still 101 years at Canopus.



The Twin paradox resolved – once again!

- This is what the Rocket twin observes:

  - On the way out, Earth travels 19.6 light years in 20 years of Rocket time. The Earth clocks are out of sync; the clock near Canopus reads 101 years as the Rocket passes by Canopus, while the clock near Earth reads 3.96 years.

  - At Canopus, the Rocket twin jumps to another Rocket going back to Earth. The Earth clocks still are out of sync, but while the Earth clock near Canopus still reads 101 years, the clock near Earth is at 198.04 years.

  - On the way back, Earth travels 19.6 light years in 20 years of the return Rocket time, but in just 3.96 years of the Earth time. The Earth clock near Earth thus now reads

    198.04 + 3.96 = 202 years – the Paradox is resolved!

All observers agree on result, disagree on the reason

- In all three reference frames, the order at which events take place makes logical sense. From the point of view of each observer, there is no paradox.
- However, for each observer, there is some misbehavior in the other frames' measuring devices. Namely, the length measurement is incorrect due to Lorentz length contraction, and time measurement is incorrect due to time dilation.
- Moreover, the relativity of simultaneity leads observers to believe that clocks in other frames are not synchronized.

Proper time: the proper way to measure

- Look at the proper time in the Rocket frame. It takes 20 years to go to Canopus and another 20 years to return. Total time is 40 years.
- Now look at the proper time in the Earth frame. It takes 101 years for the Rocket to reach Canopus and another 101 years to return. Total time: 202 years.
- The proper time is universal. The Rocket twin goes along carrying the wristwatch that measures the Rocket proper time. The Earth twin also goes along wearing the Earth wristwatch, with the Earth proper time. When they finally meet, they compare their clocks. Whatever they find – 40 years for the Rocket twin, 202 long years for the Earth twin – is the fact of Nature. The spacetime intervals between events remain unchanged in all frames.