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(last) (first)

Physics 311 December 10, 2007

FINAL EXAM (200 points)

The exam is **CLOSED BOOK**. However, you may refer to a two page (both sides each page) **HAND-WRITTEN** crib sheet of notes, formulas, etc. You may also use a calculator. **SHOW ALL YOUR WORK**. Please write your name on each page of the exam.

Useful constants: $G = 6.67 \times 10^{-11}$ kg^{-1}m^{3}s^{-2}, $c = 3.00 \times 10^8$ m/s, 1 eV = 1.6×10^{-19} J.
When energy is expressed in eV (or keV, or MeV, …), it is customary to measure momentum in eV/c and mass in eV/c^2

EXTRA! EXTRA! CREDIT! CREDIT!
(5 points)
WHILE YOU WAIT

Physics ca. 2107. Listed below (in no particular order other than alphabetically) are some theories and concepts of modern physics. Circle the ones you think will survive (i.e. the validity of which will still be widely accepted) the next 100 years. Draw a line through those you think will fail. Leave untouched if you're not sure. Use your broad knowledge of physics, common sense and imagination! You can't go wrong here!

Anthropic Principle (the Universe we live in is the way it is because we live in it)

Big Bang

Black Holes

Dark Energy

Dark Matter

Energy Conservation

General Relativity Theory

Gravitational Waves

Inflation ("faster-than-light" expansion of early Universe)

Quantum Mechanics

Special Relativity Theory

Standard Model of Particle Physics

String Theory

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Problem 1. [50 pts] Beth is at the rear and Becky is at the front of a train traveling at a relativistic speed v=0.6c with respect to Alan, who is standing on a station platform. Beth sets off a flare when her watch reads $t' = 0$ (Event #1). She passes Alan when her watch reads $t' = 150$ meters (Event #2). The flash from the firecracker reflects off of a mirror held by Becky (Event # 3). Beth sees the reflected flash when her clock reads $t' = 300$ meters.

a) [8 pts] What is the magnitude of the spacetime interval between Events 1 and 3? Is the interval timelike, spacelike, or lightlike? Explain your reasoning.

b) [8 pts] What is the magnitude of the spacetime interval between Events 2 and 3? Is the interval timelike, spacelike, or lightlike? Explain your reasoning.

c) [7 pts] Determine the length of the train in Beth's frame? Is it less than, greater than, or equal to the *proper length* of the train? Explain your reasoning.

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d) [17 pts] What is the spatial separation of Events 2 and 3 in Alan's frame? Is this equal to the length of the train in Alan's frame? If not, what is the length of the train in Alan's frame? Explain your reasoning.

e) [10 pts] Sketch a spacetime diagram for Alan's frame, showing the worldlines of Alan, Beth, and Becky and the light flash from the flare, plus the four events.

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Problem 2. [40 pts] Two spaceships (the Enterprise and the Constellation) are on a hear-head-on collision course. Each spaceship is 100 m long in its rest frame, and they move with equal and opposite velocities of 0.99c with respect to the planet Vulcan. The spaceships take part in a tournament whose objective is to be ***the first*** to plug the exhaust pipe at the aft of the other ship thus rendering it a "sitting duck". The Enterprise skipper, Captain Kirk, thinks, "Spock's ship will be Lorentz-contracted. The front of my ship will reach the aft of his long before his will. My victory is certain!" Spock, the skipper of the Constellation, is equally confident of victory.

a) [10 pts] What is the speed of the Constellation in the Enterprise frame? Show your work.

b) [7 pts] What is the length of the Constellation in the Enterprise frame? Show your work.

c) [13 pts] In the Enterprise frame, what time (in meters) elapses between the Enterprise's front reaching the aft of the Constellation and the front of the Constellation reaching the aft of the Enterprise? Explain your reasoning.

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d) [10 pts] Who wins the tournament according to Kirk? Spock? The Vulcans? Explain the problem with the objective of the tournament.

Problem 3. [50 pts] An η meson, whose rest energy is 548 MeV, is moving in the Lab frame with a total energy of 685 MeV in the +x-direction. It decays into two photons.

a) [8 pts] What is the momentum of the η before the decay? Show your work.

b) [8 pts] What is the velocity of the η before the decay? Show your work.

c) [12 pts] Given that one of the photons is detected going in the +x-direction (this is one of many possible scenarios), find its energy and the energy and direction of the other photon.

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d) [15 pts] Given that the directions of the two photons make equal angles with the x-axis (on opposite sides), find the energy of each photon and the angle its direction makes with the x-axis in the lab frame. What are the energies and directions of the two photons in the η meson rest frame? Explain your reasoning and show your work.

e) [7 pts] What is the *system* mass after the decay. Explain your reasoning.

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Problem 4. [50 pts] The Schwarzschild metric for timelike event separation in the equatorial plane is $d\tau^2 = (1-2M/r)dt^2 - \frac{1}{1-2M/r}dr^2 - r^2 d\phi^2 = \frac{1}{\alpha^2}dt^2 - \alpha^2 dr^2 - r^2 d\phi^2$. Shelly's toe is at $r = \frac{25}{8}M = 3.125M$, with $M = 6000$ m. An accurate tape measure stretched radially outward from Shelly's toe to Shelly's nose measures a distance of 1.5 m.

a) [8 pts] What is the physical meaning of the coordinate *r*? Relate it to a property of the spherical shell it labels.

b) [10 pts] What is the difference *dr* in coordinate radius *r* between Shelly's toe and Shelly's nose? Explain your reasoning.

c) [12 pts] Let Event 1 be the event "a match flares up at Shelly's toe". The time of Event 1 is coordinate time $t = 0$ and time measured by Shelly's watch $t^{Sh} = 0$. Simultaneously, in Shelly's frame, Shelly winks (Event 2). At what time as measured by Shelly's watch (in units of meters) does the light flash from the match reach Shelly's nose (Event 3)? What is the coordinate time *t* of Event 3? Explain your reasoning.

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d) [12 pts] Fargo is at rest at a very large r, where the metric is very close to that of flat spacetime. Fargo *sees* Shelly's wink (Event 4) and then Fargo *sees* the flash from the match (Event 5). What is the time between Events 4 and 5 measured by Fargo's watch? Explain your reasoning.

e) [8 pts] Find the mass of the black hole in *units of kg*.

Problem 5. [10 pts] State the basic assumptions of the theory of special relativity. How must these assumptions be modified or qualified in the presence of gravity? Discuss.

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Problem 6 (Extra Credit). [50 pts]

a) [20 pts] Observers A, B, and C each travel from Event 1 to Event 2 in flat spacetime. In a certain inertial frame (Frame F) A, B, and C have the following motions:

A travels with constant speed from Event 1 to Event 2.
B is at rest for some time after Event 1 and then travels with constant speed to Event 2.
C travels in the $-x$ direction at constant speed for some time after Event 1 and then travels with constant speed to Event 2.

Draw the worldlines of the three observers on a space-time map. Consider the time that elapses on each observer's wristwatch between Events 1 and 2, T_A, T_B, and T_C. Rank T_A, T_B, and T_C from largest to smallest. If it is not possible to rank them from the information given, state that explicitly. Explain your reasoning.

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b) [30 pts] Observers X, Y, and Z each travel from Event 3 to Event 4 in Schwarzschild spacetime.

X is at rest relative to the gravitating body.
Y jumps radially upward at Event 3 and drops freely to Event 4.
Z, who is attached to a bungee cord, falls radially downward from Event 3, and bounces back radially upward to Event 4.

Consider the time that elapses on each observer's wristwatch between Events 3 and 4. T_X, T_Y, and T_Z. Rank T_X, T_Y, and T_Z from largest to smallest. If it is not possible to rank them from the information given, state that explicitly. Explain your reasoning.