A Comparison of Two CNOT Gate Implementations in Optical Quantum Computing

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Why Optical?

- Photons do not interact easily with the environment
- The transition to quantum networks and quantum communication is easy
- Optical equipment is easy to come by

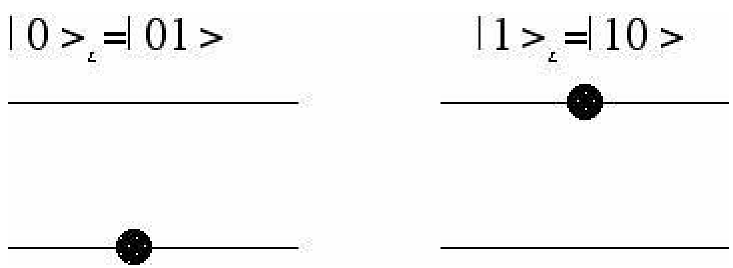
Two ways to encode information on a photon

- Polarization

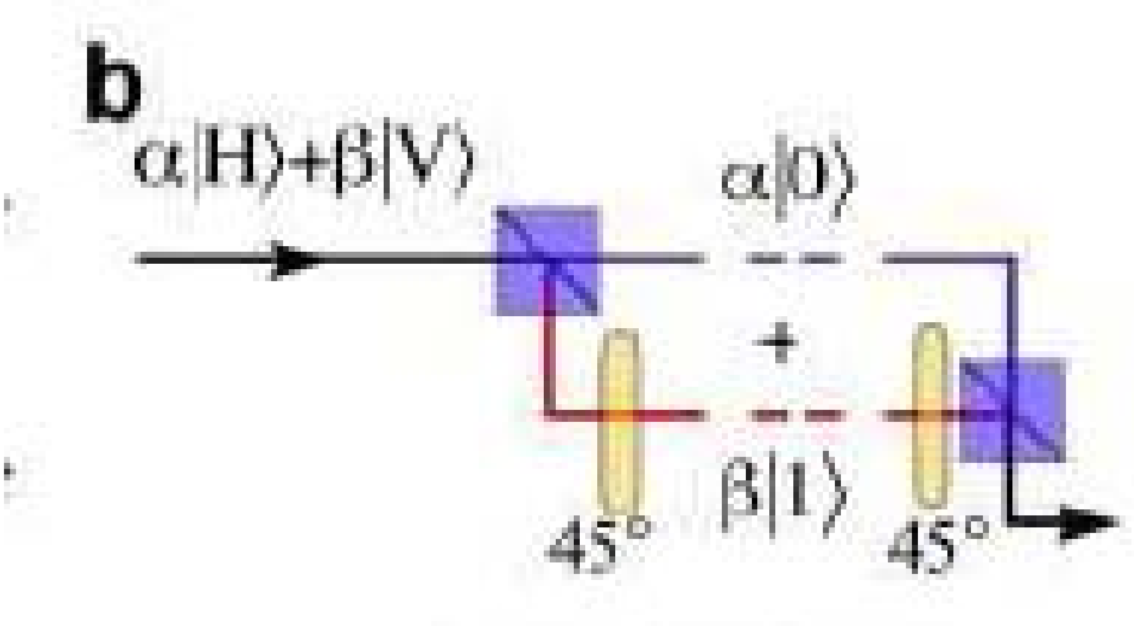
  |0>=|H>

  |1>=|V>

- Spatial Location – dual rail representation



Converting between the two encoding methods is easy.



GJ JLO'Brien, AG White, TC Ralph, D Branning -
Arxiv preprint quant-ph/0403062, 2004 - arxiv.org

Single Qubit Gates

- Phase shifts

  Go to the dual rail representation and add a piece of glass to one rail.

$$P = \begin{pmatrix} e^{i\Delta} & 0 \\ 0 & 1 \end{pmatrix}$$

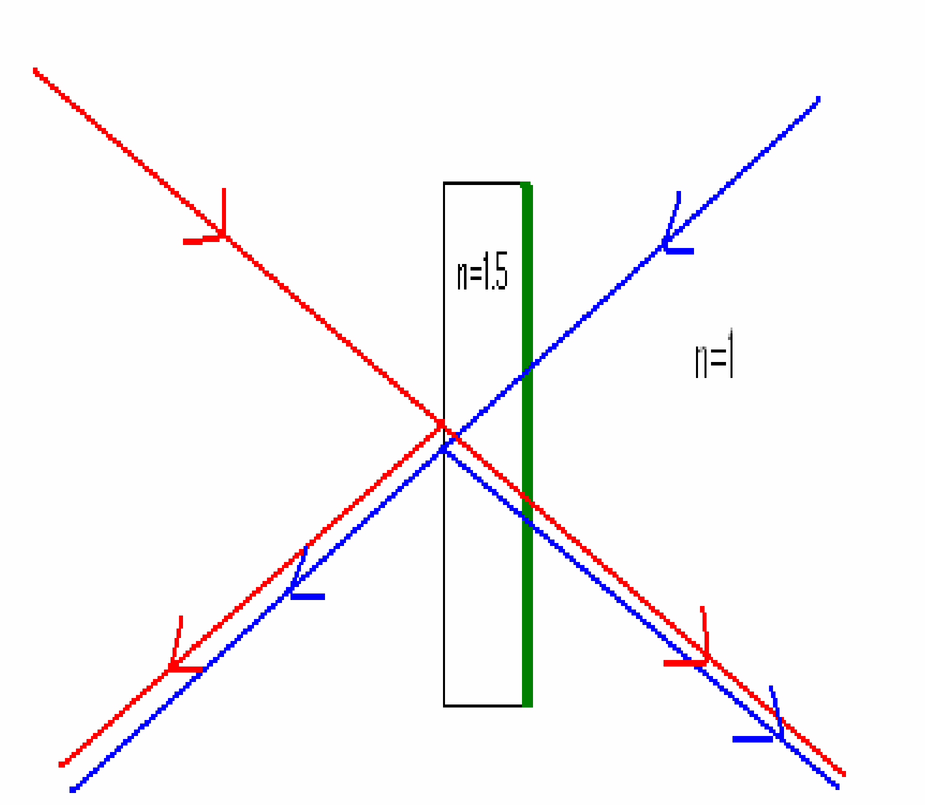
- Hadamard

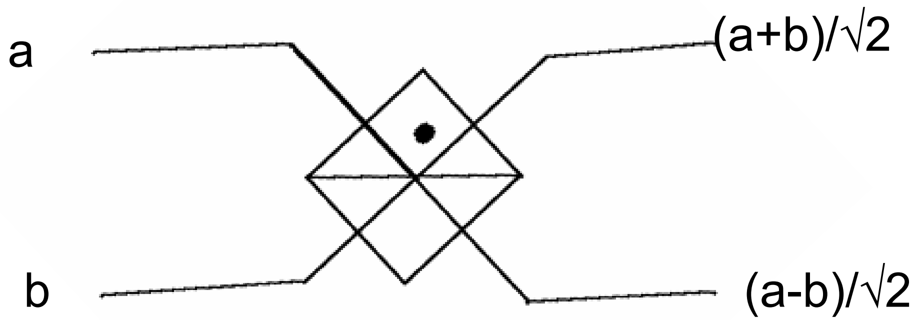
  Nothing but a beamsplitter!

$$H = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Why is a beamsplitter a Hadamard gate?

- An AR coating ensures all reflection: happen on the same face
- Going to a higher index of refraction causes a sign flip





$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} a+b \\ a-b \end{pmatrix}$$

Nielson and Chuang use a different sign convention here.

- Phase shifters and Hadamards are enough to do any single qubit operation
- But what about two qubit gates?

  Photons do not interact so how can we get entaglement?

The Kerr Effect

- In linear media the polarization of the material is proportional to the electric field

$$P = \chi^{(1)} E$$

- Some materials have significant higher order terms

$$P = \chi^{(1)} E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \cdots$$

The Kerr Effect

- The Kerr effect depends on the third order term
- It causes the index of refraction of a material to change depending on the intensity of the electric field present in the material

The Kerr Effect

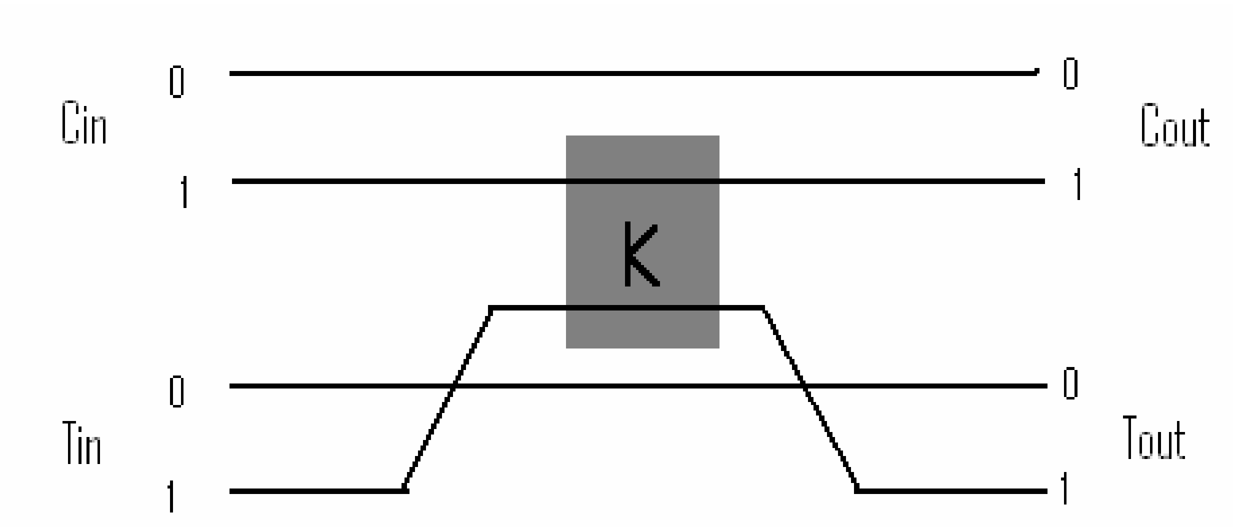
- This leads to interactions of the form
- If the material is length L we have phase shifts given by
- If we choose chi*L=pi we have

$$H_{Kerr} = \chi a^{\mp} a b^{\mp} b$$

$$K = e^{i\chi L a^{\mp} a b^{\mp} b}$$

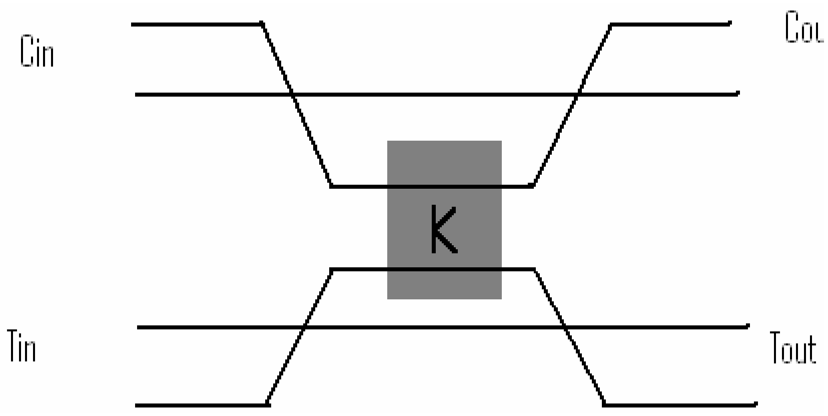
$$K = e^{i\pi a^{\mp} a b^{\mp} b}$$

- 2 photons in the Kerr medium causes a 180 degree phase shift. 0 or 1 photons do nothing
- Now build this gate



Conditional Sign Flip

- This gate has the following matrix representation



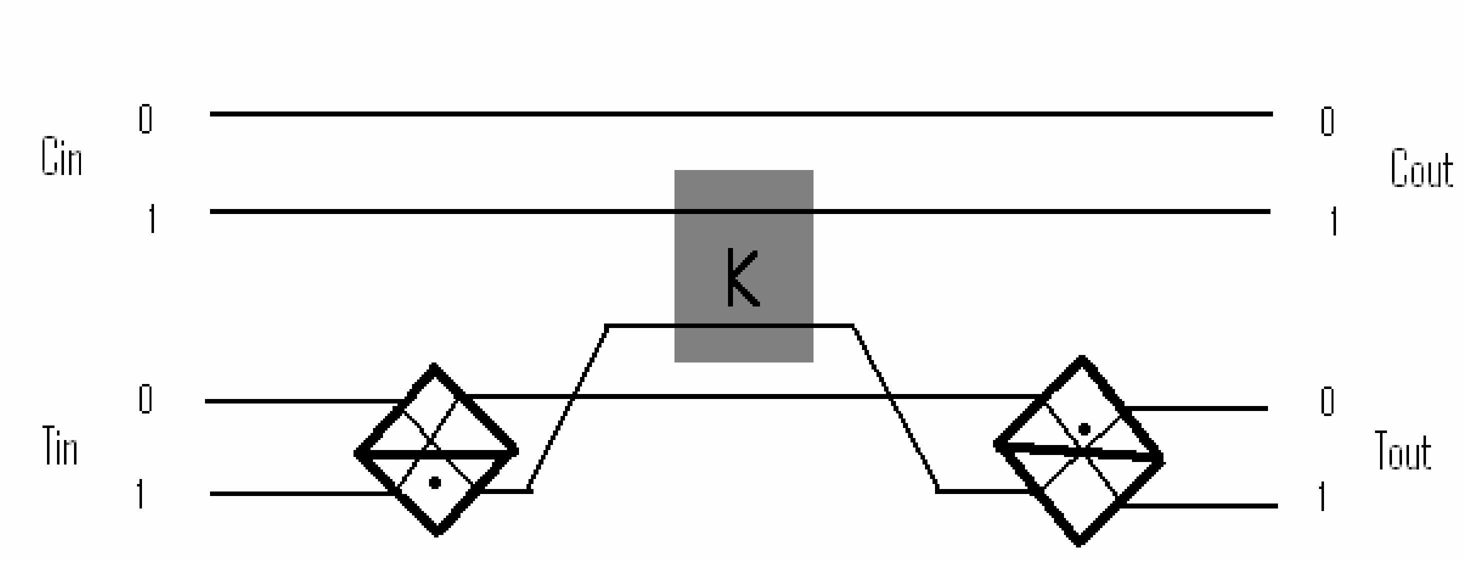
$$K = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Now we can build a CNOT

$$U_{CNOT}=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix}$$

$$=\underbrace{\begin{pmatrix}1 & 1 & 0 & 0\\ 1 & -1 & 0 & 0\\ 0 & 0 & 1 & 1\\ 0 & 0 & 1 & -1\end{pmatrix}}_{I\otimes H}\underbrace{\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}}_{K}\underbrace{\begin{pmatrix}1 & 1 & 0 & 0\\ 1 & -1 & 0 & 0\\ 0 & 0 & 1 & 1\\ 0 & 0 & 1 & -1\end{pmatrix}}_{I\otimes H}$$

Schematically this would look like…



Or can we?

- In most materials the third order susceptibility is on the order of 10^{-18} m^2/W
- These interactions are extremely unlikely in the single photon regime
- Some techniques can enhance the nonlinearity significantly ($\sim 10^{-2}$)

Is there another way?

It was shown in 2001 that entanglement could be created using linear optics and projective measurements.

(E. Knill, R. Laflamme, and G.J. Milburn, *Nature* **409**, 46 (2001).)

How exactly does this work?

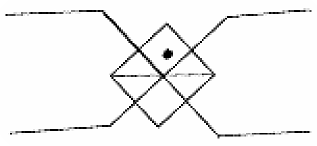
- *This method was demonstrated experimentally in 2003 so lets look at what they did*

"Demonstration of an all-optical quantum controlled-NOT gate"

O'Brien, Pryde, White, Ralph, Branning Nature **426**, 264-267 (20 November 2003)

Without a Kerr medium how do we get photons to interact?

Photon Bunching

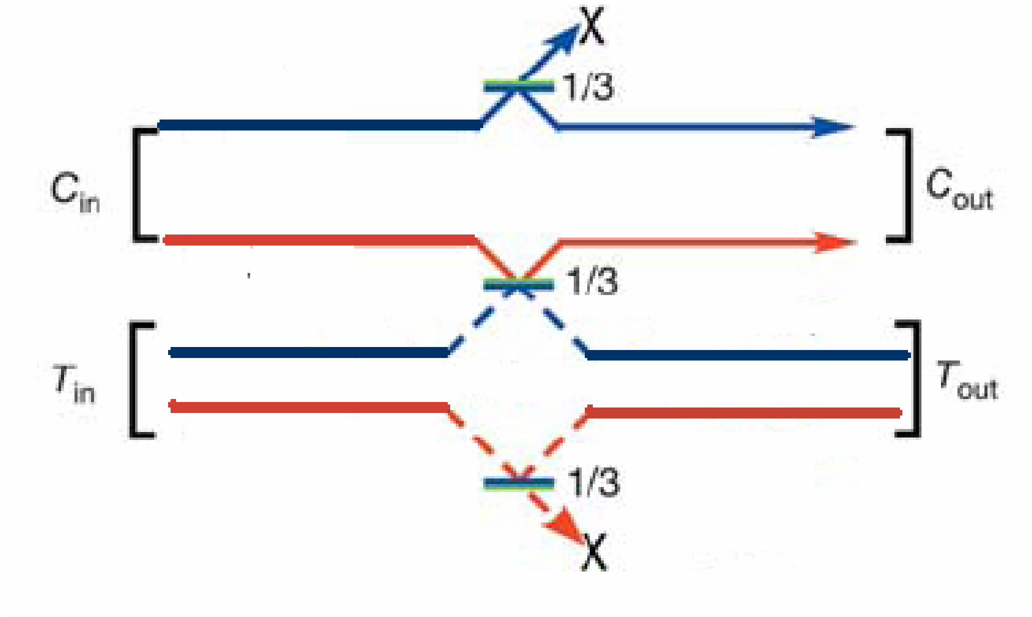


$$a^+ \to \sqrt{\frac{1}{2}}\left(c^+ + d^+\right) \quad b^+ \to \sqrt{\frac{1}{2}}\left(c^+ - d^+\right)$$

$$|11\rangle_{ab} = a^+ b^+ |00\rangle_{ab}$$

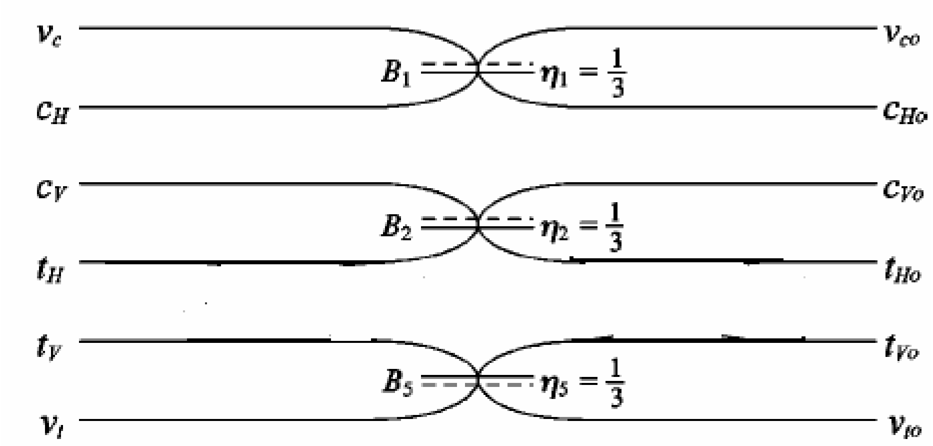
$$\to \frac{1}{2}\left(c^+ + d^+\right)\left(c^+ - d^+\right)|00\rangle_{cd} = \frac{1}{2}\left(\left(c^+\right)^2 - \left(d^+\right)^2\right)|00\rangle_{cd}$$

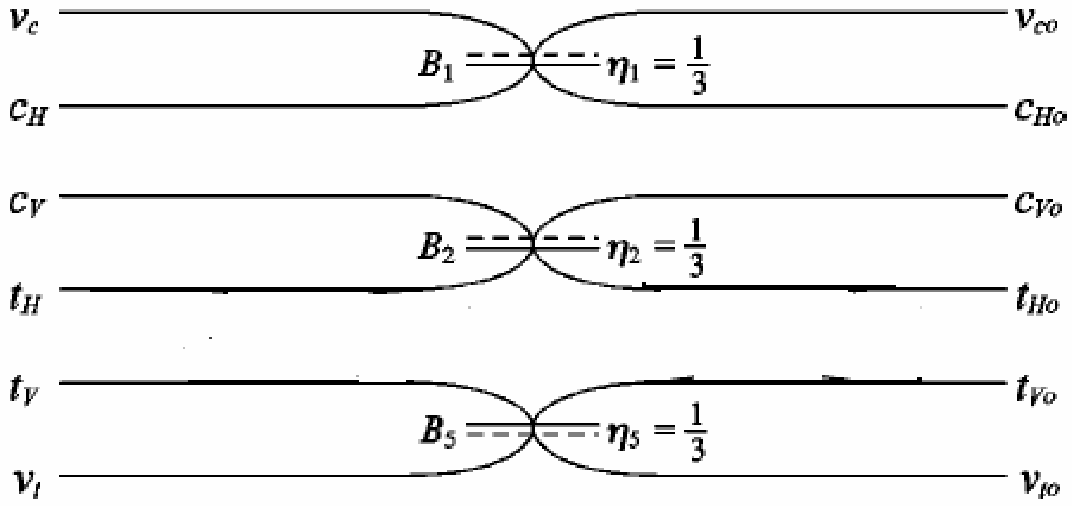
$$= \sqrt{\frac{1}{2}}\left(|20\rangle_{cd} - |02\rangle_{cd}\right)$$



- Clearly this gate does not succeed 100% of the time.
- But what happens when it does?

- Use a slightly different schematic
- Trace all paths to each output to get a system of operator equations





$$V_{Co} = \sqrt{\frac{1}{3}}\left(\sqrt{2}C_H - V_C\right)$$

$$C_{Ho} = \sqrt{\frac{1}{3}}\left(\sqrt{2}V_C - C_H\right)$$

$$C_{Vo} = \sqrt{\frac{1}{3}}\left(\sqrt{2}T_H - C_V\right)$$

$$T_{Ho} = \sqrt{\frac{1}{3}}\left(\sqrt{2}C_V + T_H\right)$$

$$T_{Vo} = \sqrt{\frac{1}{3}}\left(\sqrt{2}V_T + T_V\right)$$

$$V_{To} = \sqrt{\frac{1}{3}}\left(\sqrt{2}T_V - V_T\right)$$

$$V_C = \sqrt{\frac{1}{3}}\left(\sqrt{2}C_{Ho} + V_{Co}\right)$$

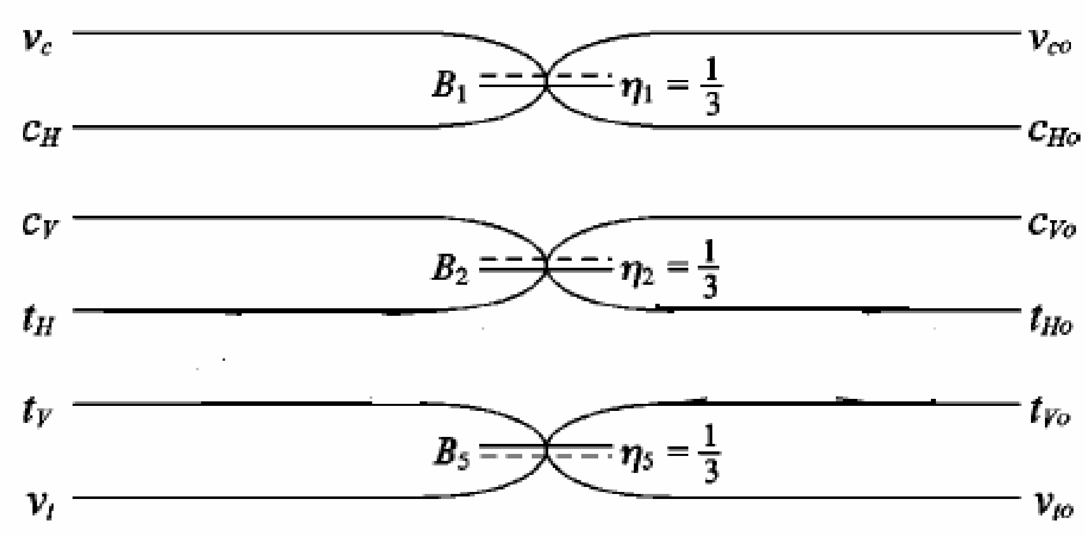
$$C_H = \sqrt{\frac{1}{3}}\left(\sqrt{2}V_{Co} - C_{Ho}\right)$$

$$C_V = \sqrt{\frac{1}{3}}\left(\sqrt{2}T_{Ho} + C_{Vo}\right)$$

$$T_H = \sqrt{\frac{1}{3}}\left(\sqrt{2}C_{Vo} - T_{Ho}\right)$$

$$T_V = \sqrt{\frac{1}{3}}\left(\sqrt{2}V_{To} - T_{Vo}\right)$$

$$V_T = \sqrt{\frac{1}{3}}\left(\sqrt{2}T_{Vo} + V_{To}\right)$$



Now take any two input operators and act on the vacuum.

$$\left|HH\right\rangle = \left|1010\right\rangle\left|00\right\rangle >= C_H T_H \left|0000\right\rangle\left|00\right\rangle$$

$$= \frac{1}{3}\left(\sqrt{2}V_{Co} - C_{Ho}\right)\left(\sqrt{2}C_{Vo} - T_{Ho}\right)\left|0000\right\rangle\left|00\right\rangle$$

$$= \frac{1}{3}\left(\sqrt{2}\left|1000\right\rangle\left|10\right\rangle + \left|1010\right\rangle\left|00\right\rangle + 2\left|0100\right\rangle\left|10\right\rangle + \sqrt{2}\left|0010\right\rangle\left|10\right\rangle\right)$$

But only one of these terms makes sense!

$$= \frac{1}{3}\left|1010\right\rangle\left|00\right\rangle = \frac{1}{3}\left|HH\right\rangle$$

Lets try another

$$\left|VV\right\rangle = \left|0101\right\rangle\left|00\right\rangle = C_V T_V \left|0000\right\rangle\left|00\right\rangle$$

$$= \frac{1}{3}\left(\sqrt{2}T_{Ho} - C_{Vo}\right)\left(\sqrt{2}V_{To} + T_{Vo}\right)\left|0000\right\rangle\left|00\right\rangle$$

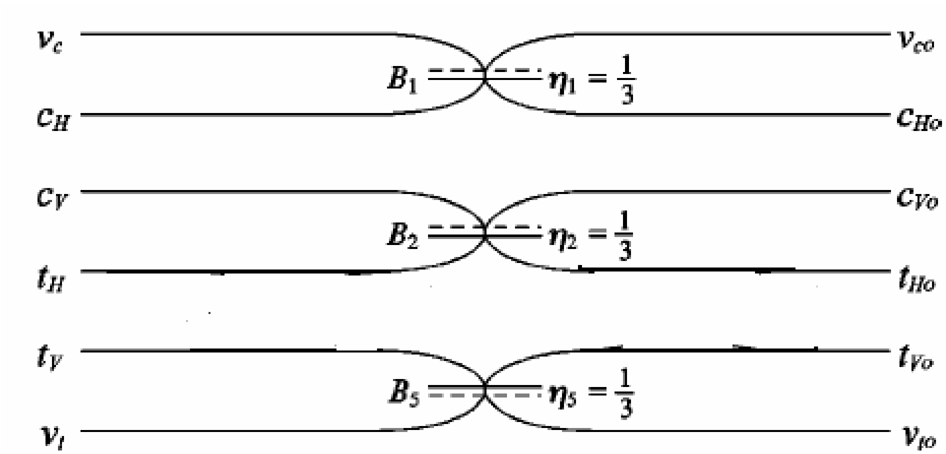
$$= \frac{1}{3}\left(-\left|0101\right\rangle\left|10\right\rangle - \sqrt{2}\left|0011\right\rangle\left|00\right\rangle + \sqrt{2}\left|0100\right\rangle\left|01\right\rangle + 2\left|0010\right\rangle\left|01\right\rangle\right)$$

Again only one of these terms makes sense.

$$= -\frac{1}{3}\left|0101\right\rangle\left|00\right\rangle = -\frac{1}{3}\left|VV\right\rangle$$

This is what is meant by projective measurements.

Linear Conditional Sign Flip



$$= \begin{pmatrix} \frac{1}{9} & 0 & 0 & 0 \\ 0 & \frac{1}{9} & 0 & 0 \\ 0 & 0 & \frac{1}{9} & 0 \\ 0 & 0 & 0 & -\frac{1}{9} \end{pmatrix}$$

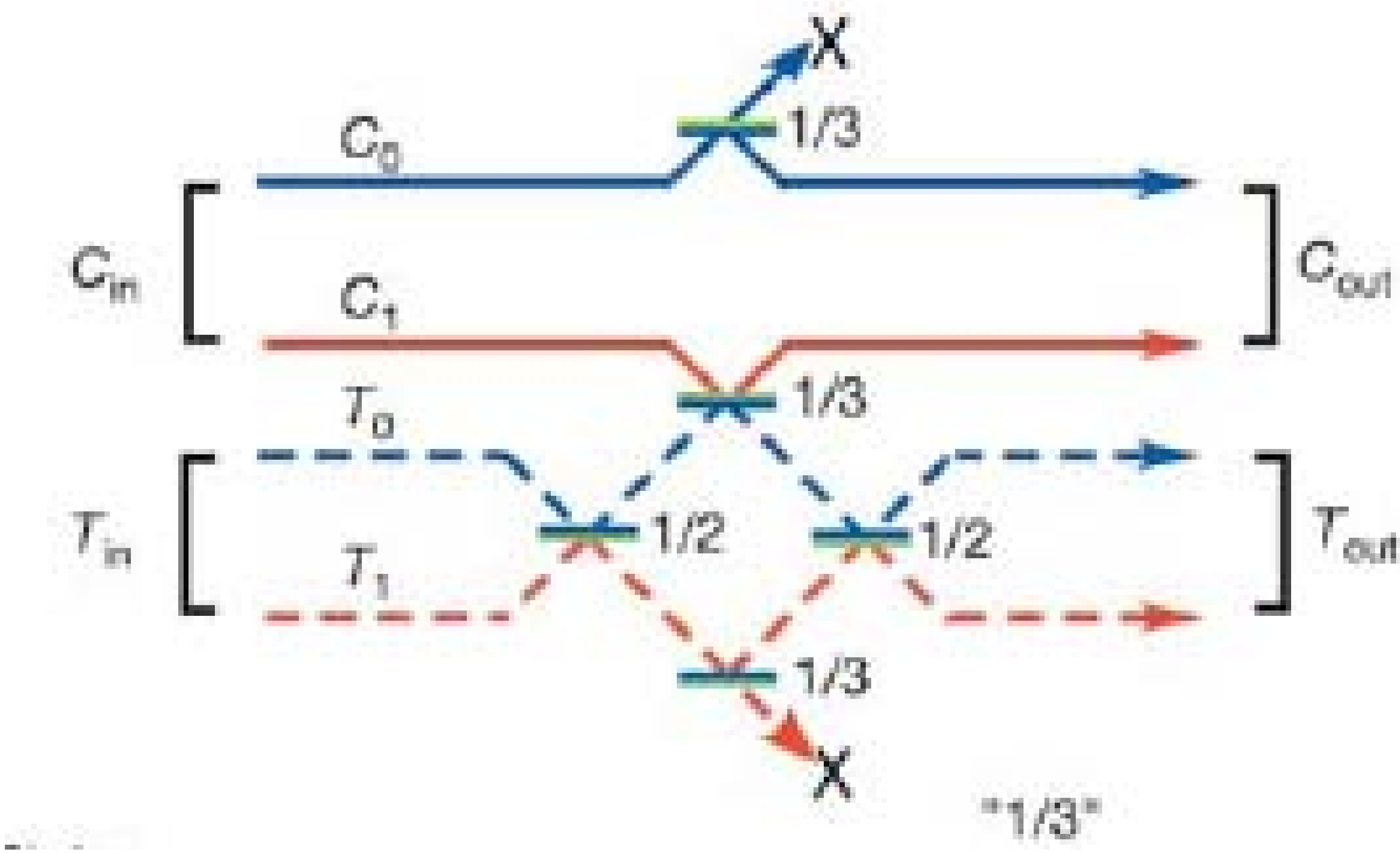
Now we are in familiar territory

- Simply mix the target qubit with a 50/50 beam splitter

$$U_{LinearCNOT} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} \frac{1}{9} & 0 & 0 & 0 \\ 0 & \frac{1}{9} & 0 & 0 \\ 0 & 0 & \frac{1}{9} & 0 \\ 0 & 0 & 0 & -\frac{1}{9} \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{9} & 0 & 0 & 0 \\ 0 & \frac{1}{9} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{9} \\ 0 & 0 & \frac{1}{9} & 0 \end{pmatrix}$$

Linear Optical CNOT Gate



Experimental Results

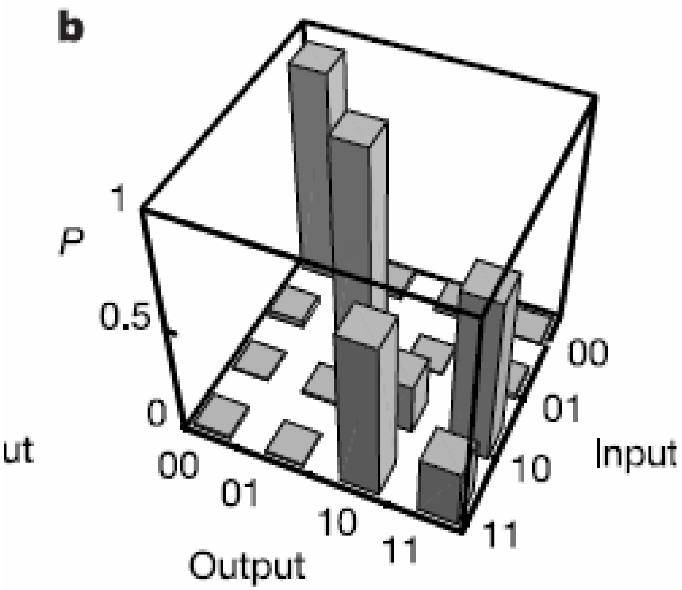


Table 1 Experimentally determined probabilities for the logical basis operation

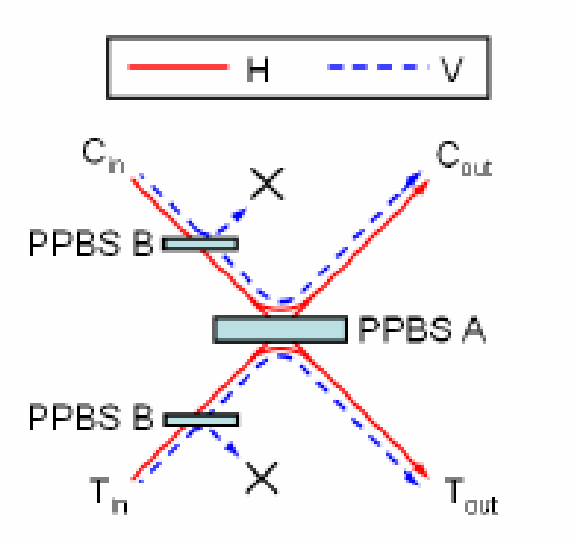
Input $\|CT\rangle$	$P_{\|00\rangle}$	$P_{\|01\rangle}$	$P_{\|10\rangle}$	$P_{\|11\rangle}$
$\|00\rangle$	0.95(2)	0.023(3)	0.024(3)	0.0006(5)
$\|01\rangle$	0.031(3)	0.94(2)	0.0019(8)	0.022(3)
$\|10\rangle$	0.005(1)	0.011(2)	0.23(9)	0.75(2)
$\|11\rangle$	0.011(2)	0.0005(1)	0.72(2)	0.26(1)

Sources of Error

- Lack of reliable single photon sources and number resolving photon detectors
- Timing/Path Length errors – Photons must arrive at BS's and detectors simultaneously
- BS ratio errors – The beam splitters are not exactly 50/50 or 30/70
- Mode matching – The photon wave functions do not completely overlap on the BS's
- Path Length Errors

Simplification

- In 2005 a Japanese group simplified the setup to the following. They only had 2 path lengths to stabilize instead of 4



PRL **95,** 210506 (2005)

More Results

- They achieved the following results. But the gate was still only successful in 1/9 attempts

(a)	$\langle 0_z 0_z \vert$	$\langle 0_z 1_z \vert$	$\langle 1_z 0_z \vert$	$\langle 1_z 1_z \vert$
$\vert 0_z 0_z \rangle$	0.898	0.031	0.061	0.011
$\vert 0_z 1_z \rangle$	0.021	0.885	0.006	0.088
$\vert 1_z 0_z \rangle$	0.064	0.027	0.099	0.810
$\vert 1_z 1_z \rangle$	0.031	0.096	0.819	0.054

How much better can we do?

- Sure these gates have 80-90% fidelity…WHEN THEY WORK!
- Can we improve on 1/9
- In 2005 it was shown that the best that can ever be done is a success rate of 1/4

J Eisert Phys. Rev. Lett. 95, 040502 (2005)

Linear or Non-Linear? That is the question.

- If nonlinear interactions can be made to occur with near unit probability nonlinear may be the best option
- Linear optics is much easier right now
- The nonlinear approach may never come close to the 1/9 success rate of linear optics.