Quantum Computation using Circuit Cavity QED

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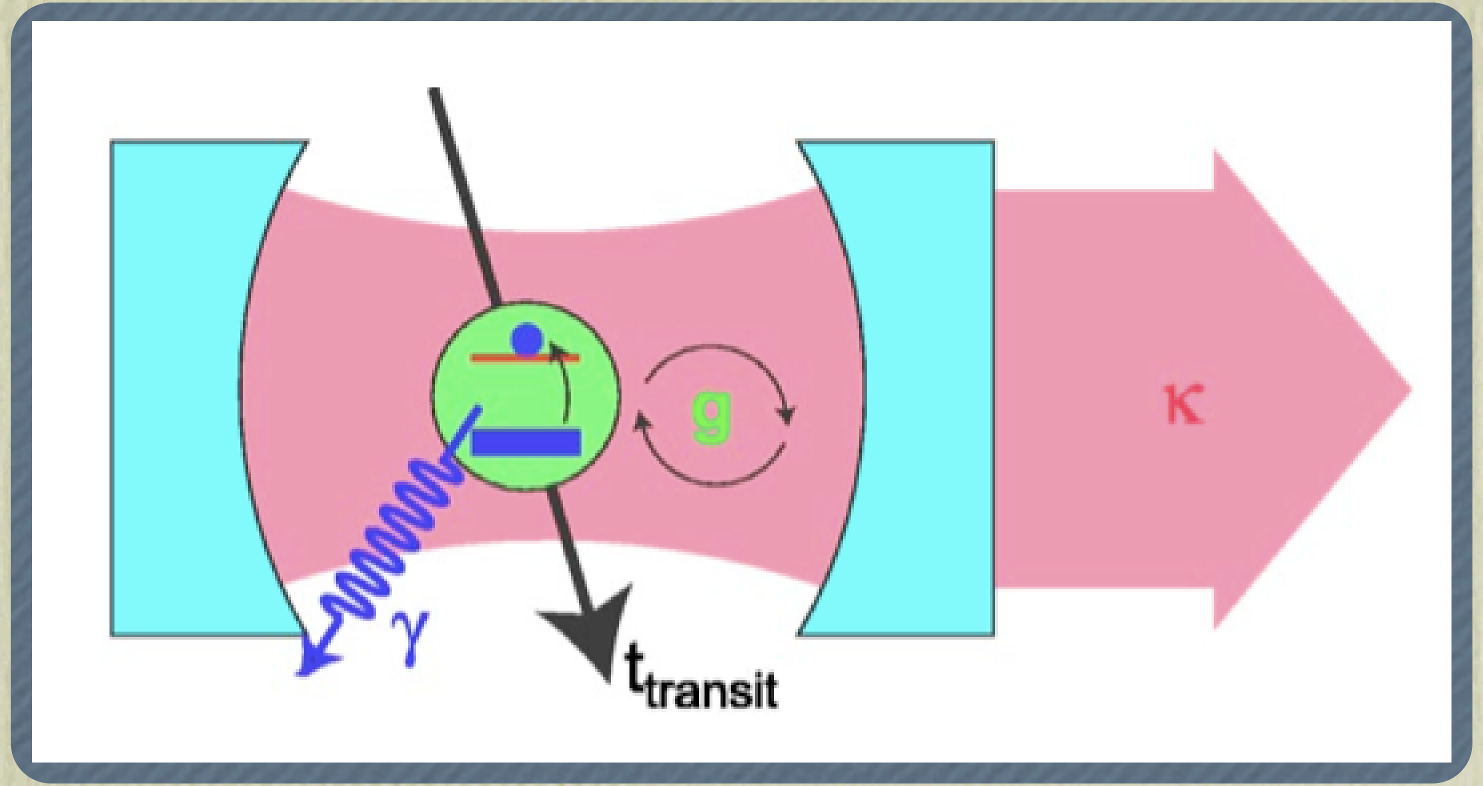
Outline

- Cavity QED: optical and microwave.
- Zero and large detuning
- Proposed architecture: TLRs & CPBs
- Qubit readout, control & entanglement
- Summary & epilogue

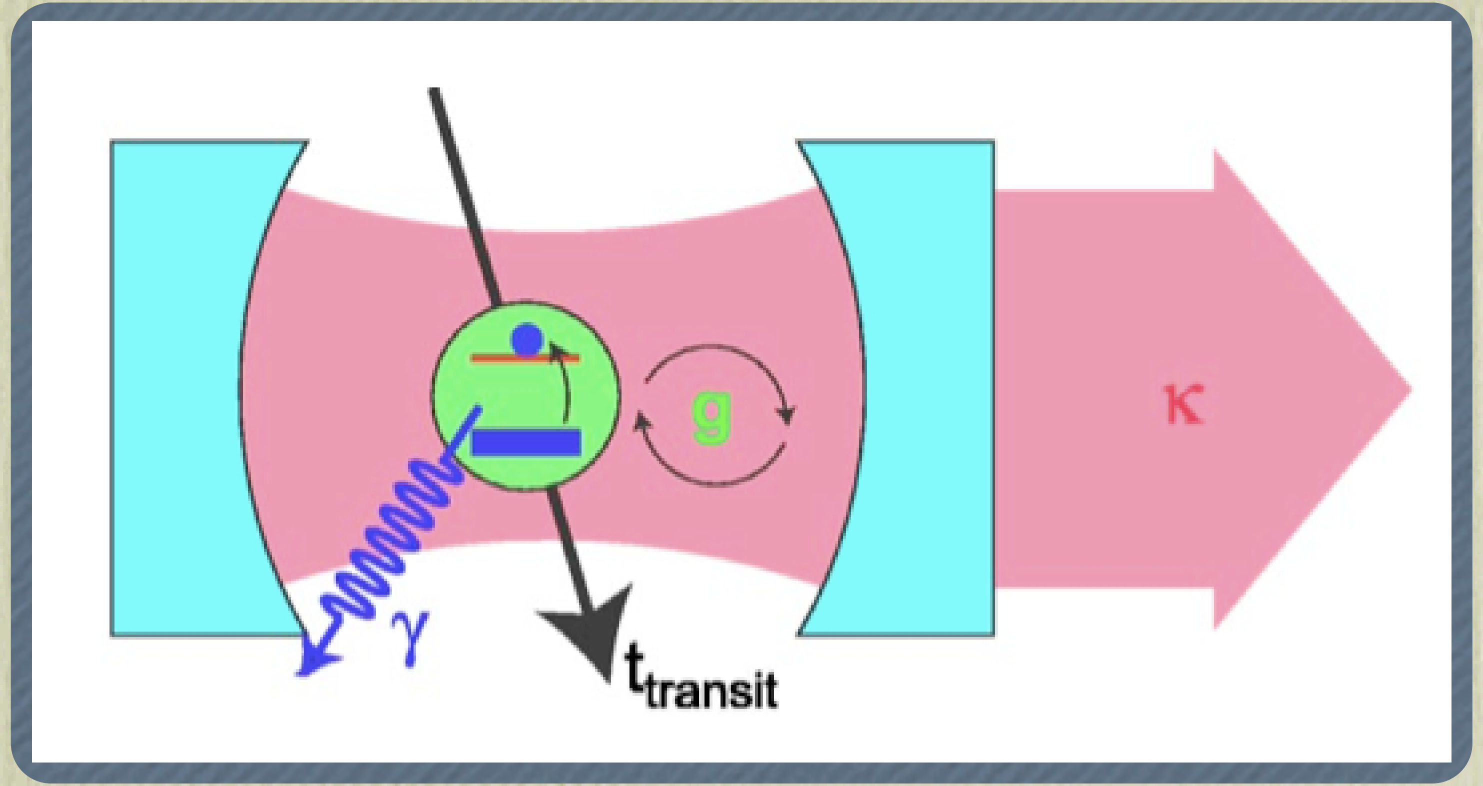
Cavity QED



Blais et al.
PRA **69**, 062320 (2004)

- Optical CQED ➔
  - Drive cavity with laser
  - Monitor changes in transmission due to atoms falling through cavity
- Microwave CQED ➔
  - Determine state of atoms after passing
  - Get information about photons inside the cavity

Cavity QED



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Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\Omega}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + \sigma^+ a) + H_\kappa + H_\gamma.$$

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cavity frequency

atomic transition frequency

atom-cavity coupling

coupling to continuum

coupling to other decay modes

Solution

Eigenstates

$$|\overline{+\,,n}\rangle = \cos\ \theta_n|\downarrow,n\rangle + \sin\ \theta_n|\uparrow,n+1\rangle$$

$$|\overline{-\,,n}\rangle = -\sin\ \theta_n|\downarrow,n\rangle + \cos\ \theta_n|\uparrow,n+1\rangle$$

$$\theta_n = \frac{1}{2}\tan^{-1}\left(\frac{2g\sqrt{n+1}}{\Delta}\right)$$

Energies

$$E_{\overline{\pm,n}} = (n+1)\hbar\omega_r \pm \frac{\hbar}{2}\sqrt{4g^2(n+1)+\Delta^2}$$

$$E_{\uparrow,0} = -\frac{\hbar\Delta}{2}$$

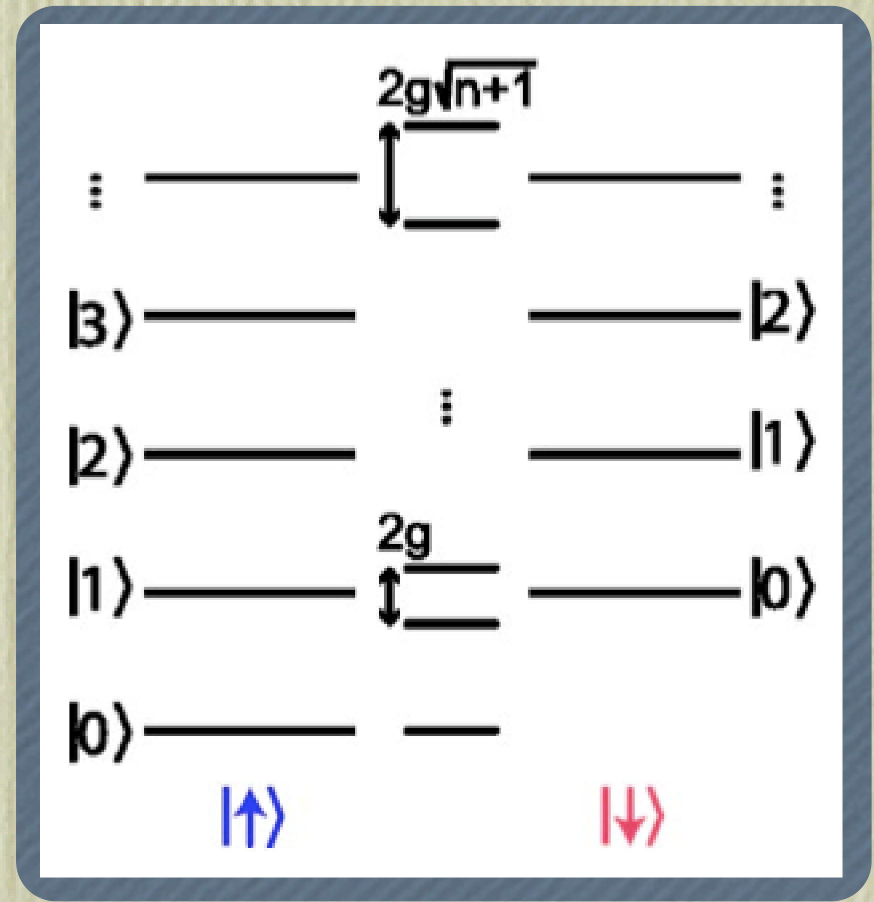
$$\omega_r = \Omega - \Delta$$

$\Downarrow$

Resonant &

Non-resonant regimes

Zero detuning (resonance)



Rabi frequency g/π

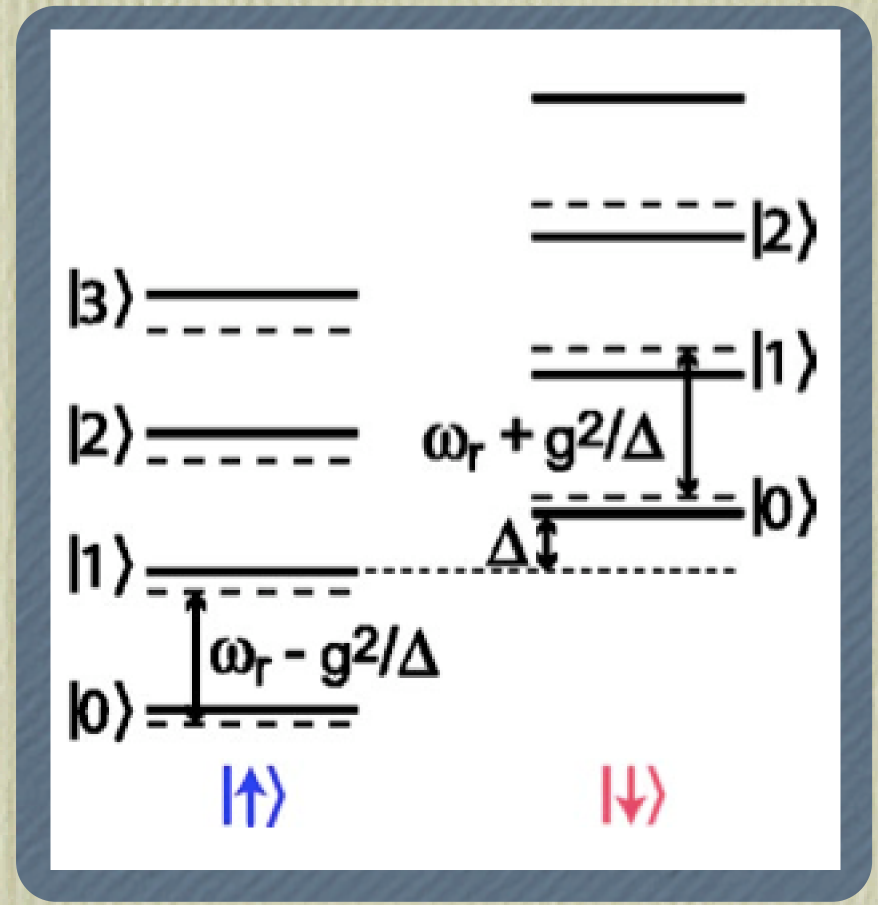
Decay rate (width) $(\kappa+\gamma)/2$

Coupling $g = \mathcal{E}_{\text{rms}} d/\hbar$

Transition dipole

Resolution of the states requires strong coupling : $\boxed{g \gg \kappa, \gamma}$

Large detuning



$$|\overline{-,0}\rangle \sim -(g/\Delta)|\downarrow,0\rangle + |\uparrow,1\rangle,$$

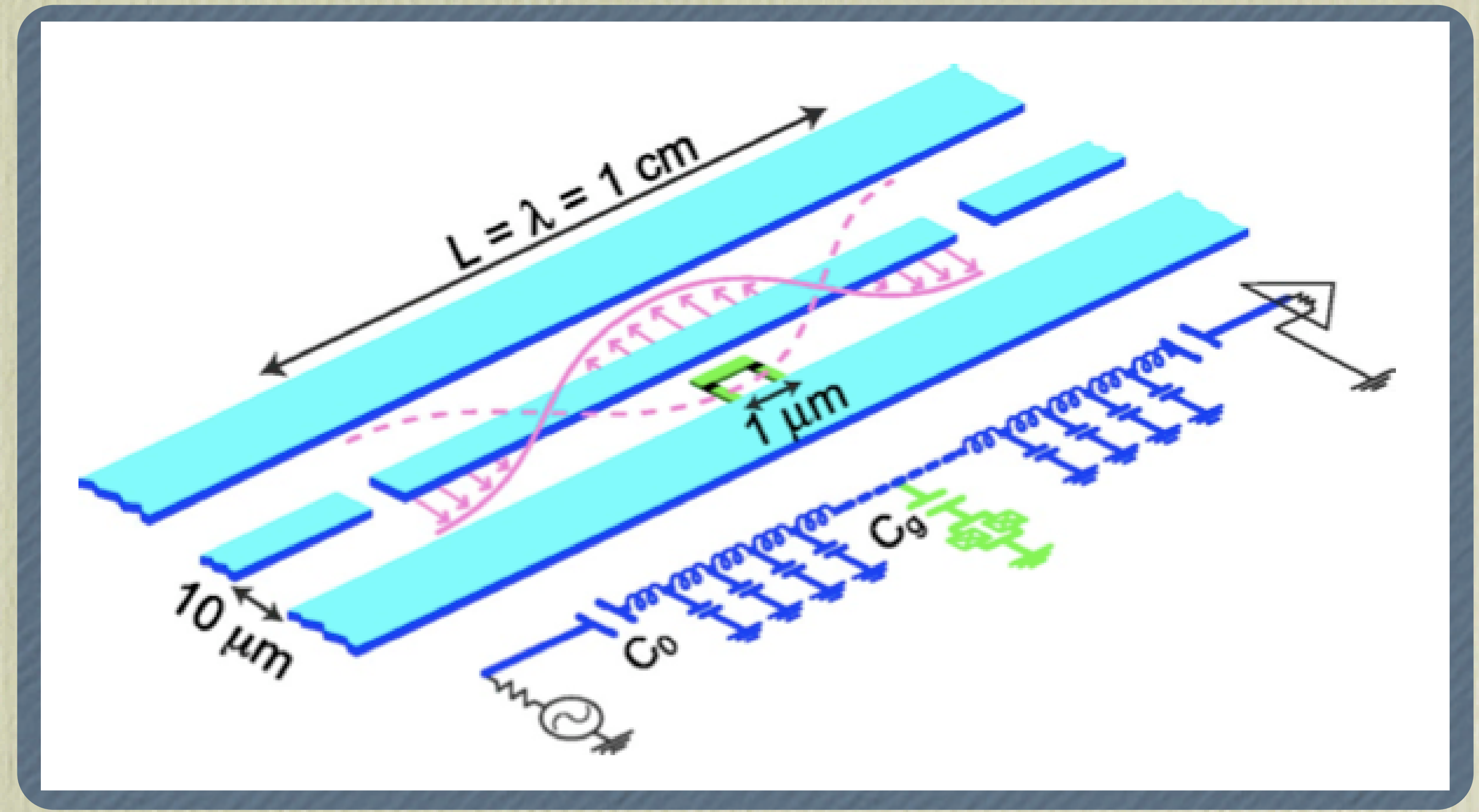
$$|\overline{+,0}\rangle \sim |\downarrow,0\rangle + (g/\Delta)|\uparrow,1\rangle.$$

$$UHU^\dagger \approx \hbar\left[\omega_r + \frac{g^2}{\Delta}\sigma^z\right]a^\dagger a + \frac{\hbar}{2}\left[\Omega + \frac{g^2}{\Delta}\right]\sigma^z$$

$$U = \exp\left[\frac{g}{\Delta}(a\sigma^+ - a^\dagger\sigma^-)\right]$$

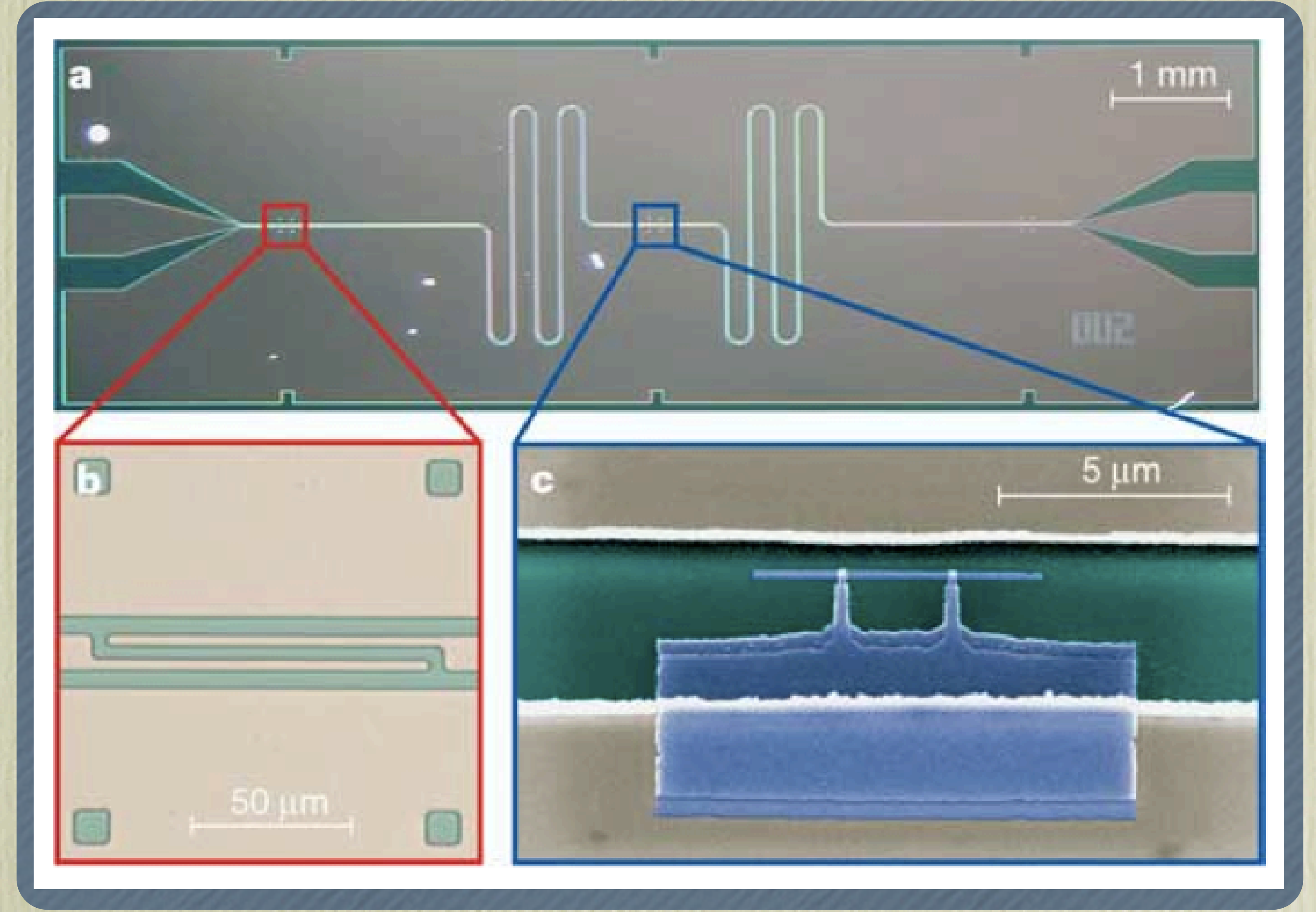
Atom 'modifies' cavity frequency ! → Qubit readout

Circuit implementation



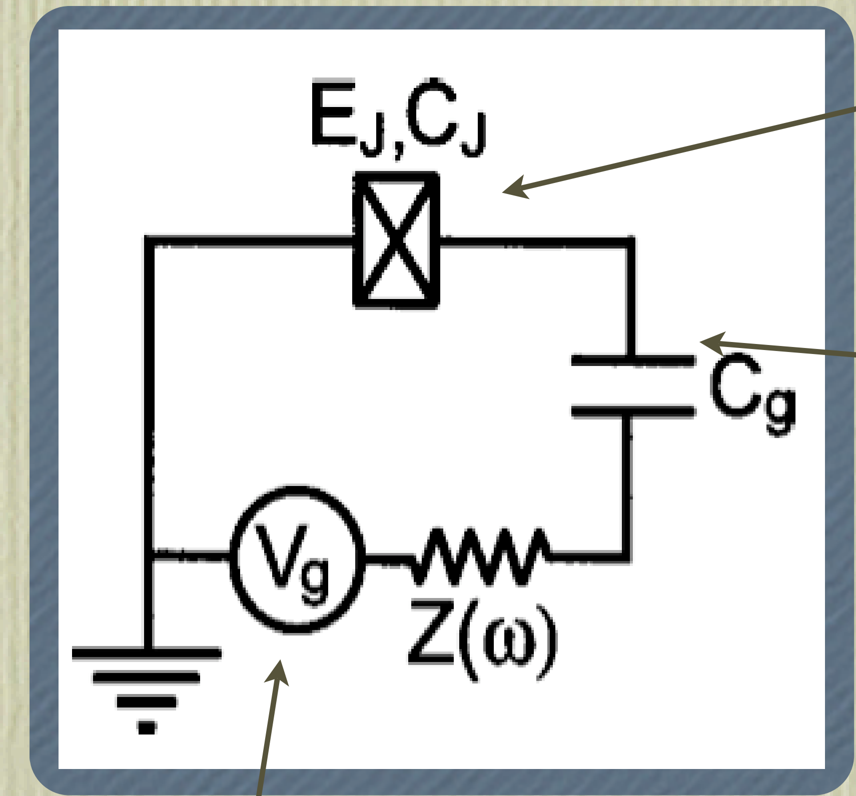
- Cavity: Transmission line resonator (TLR)
- Atom: Cooper pair box (CPB)

Circuit implementation



Wallraff et al.
Nature **431**, 162 (2004)

The 'atom'



Josephson junction (x2)

Superconducting Island

Gate voltage

SC Island Hamiltonian

$$H_Q = 4E_c \sum_N (N - N_g)^2 |N\rangle\langle N| - \frac{E_J}{2} \sum_N (|N+1\rangle\langle N| + \text{H.c.}),$$

Charging energy

total charge injected into the island by the source

\# Cooper pairs (only degree of freedom if the gap is large enough)

Josephson energy

The 'atom'

Effective Hamiltonian

$$\boxed{H_Q = -\frac{E_{\mathrm{el}}}{2}\bar{\sigma}^z - \frac{E_J}{2}\bar{\sigma}^x}$$

Charge regime

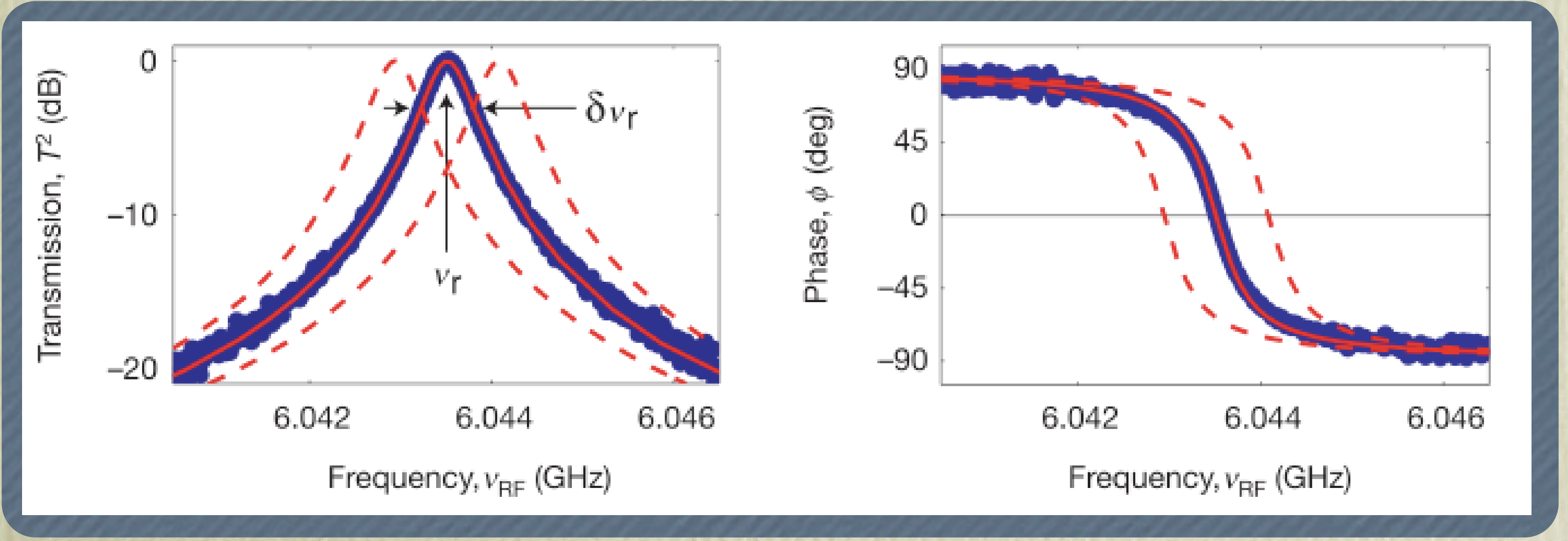
$$\boxed{4E_c \gg E_J}$$

$$E_{\mathrm{el}} = 4E_C(1-2N_g)$$

- Advantage: large effective dipole moment, can be 10 times larger than in a Rydberg atom.
- One can show that the resonator modes are quantized just like the cavity modes of CQED.
  The whole TLR-CPB system is described effectively by the Jaynes-Cummings Hamiltonian.

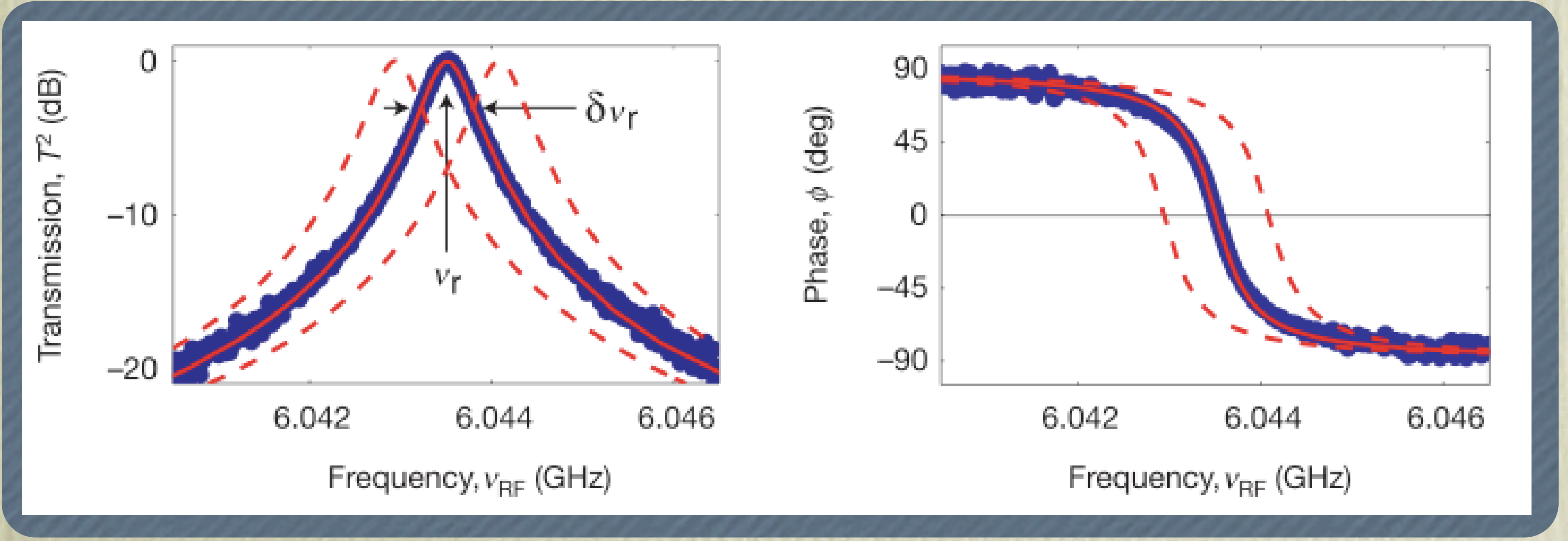
Qubit readout

- Drive the qubit-resonator system in the far-detuned regime, but close to the **cavity** resonance frequency.
- Measure amplitude and phase.
- Small shifts in the resonator frequency are sensitively measured as a phase shift.
- In this regime the mixing with the photon is minimized.



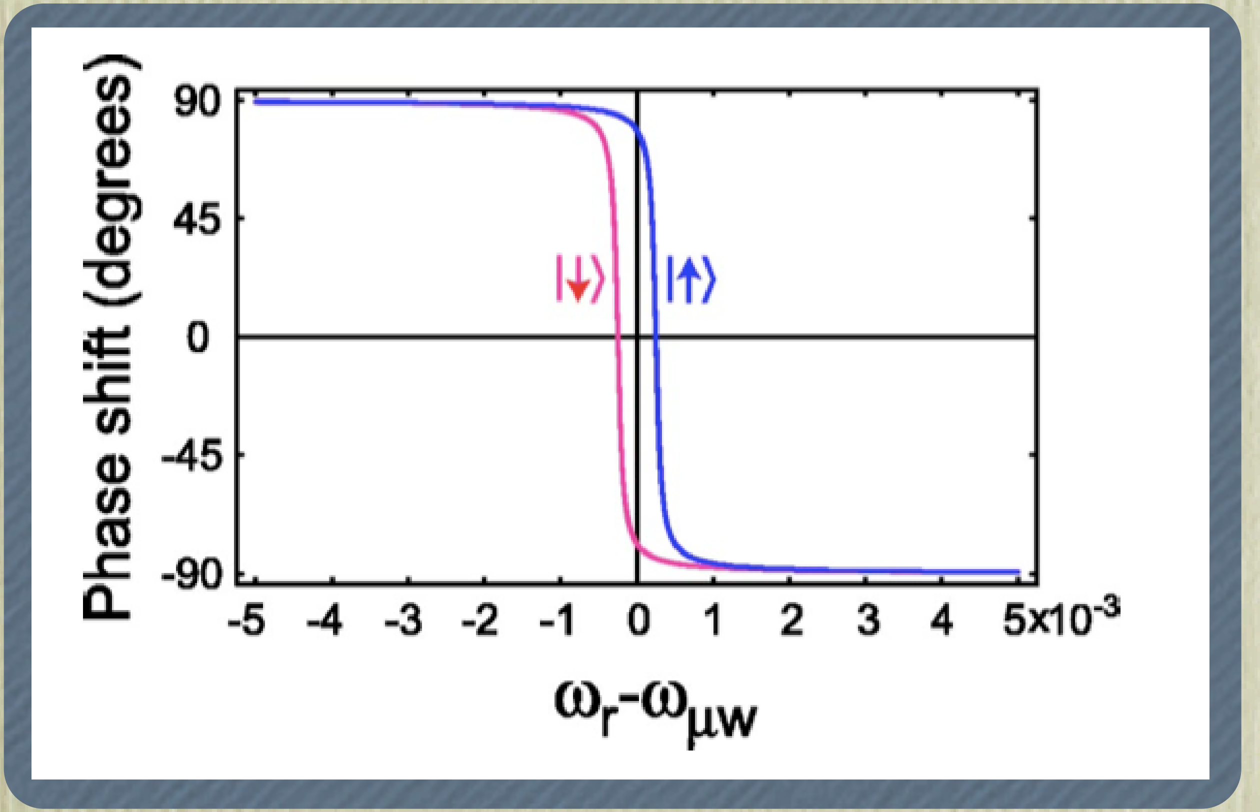
Qubit readout

- This measurement is the complement of the microwave CQED measurement, where the state of the cavity is inferred from the phase shift in the atomic beam.
- Most sensitive at charge degeneracy of the CPB, where dephasing is minimized.



Qubit control

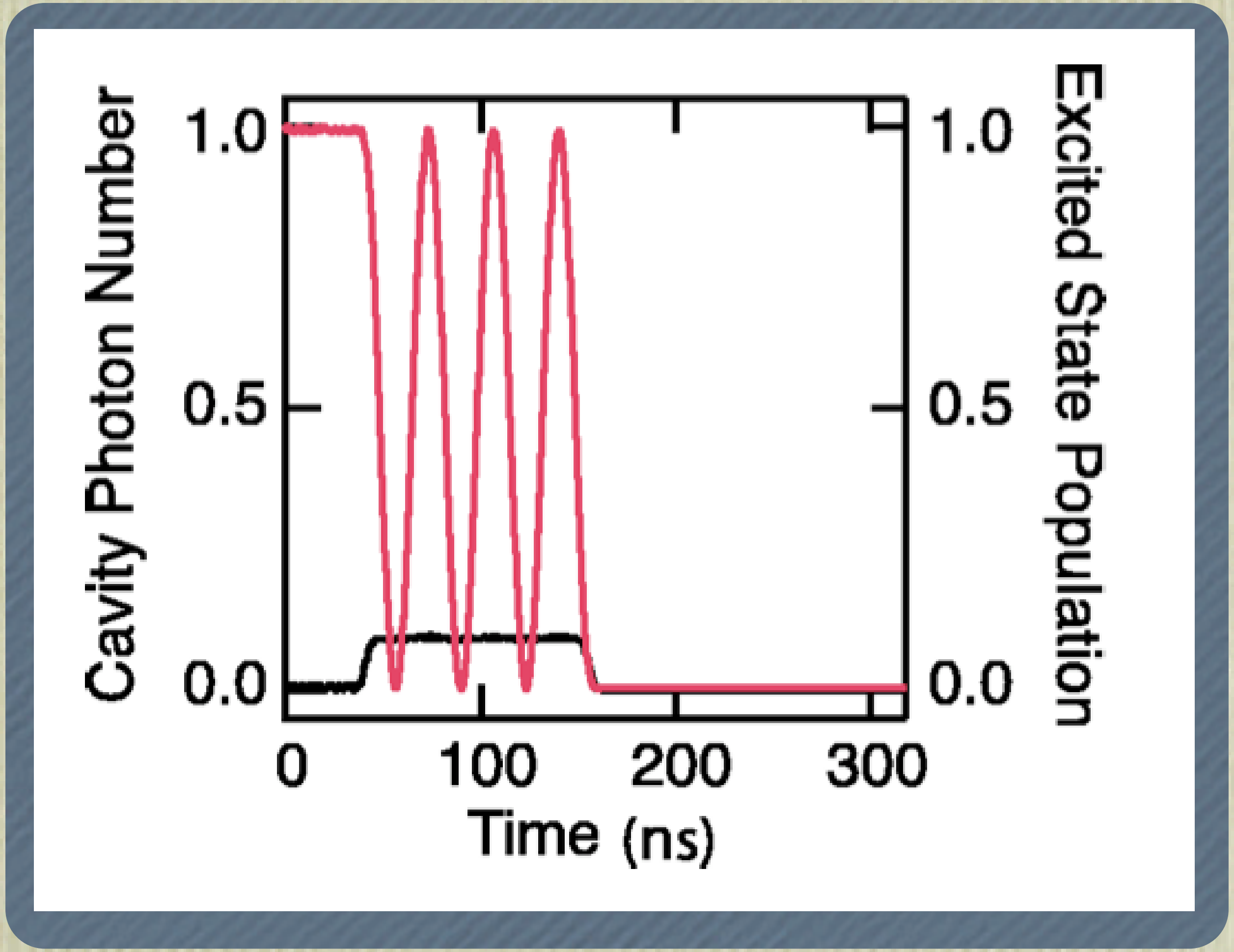
- Drive the qubit-resonator system in the far-detuned regime, but close to the **qubit** resonance frequency.



- Little entanglement with photon
- High fidelity rotation

Large detuning → little dependence of the phase on the state of the qubit.

Qubit control



Detuning between resonator and drive

↓

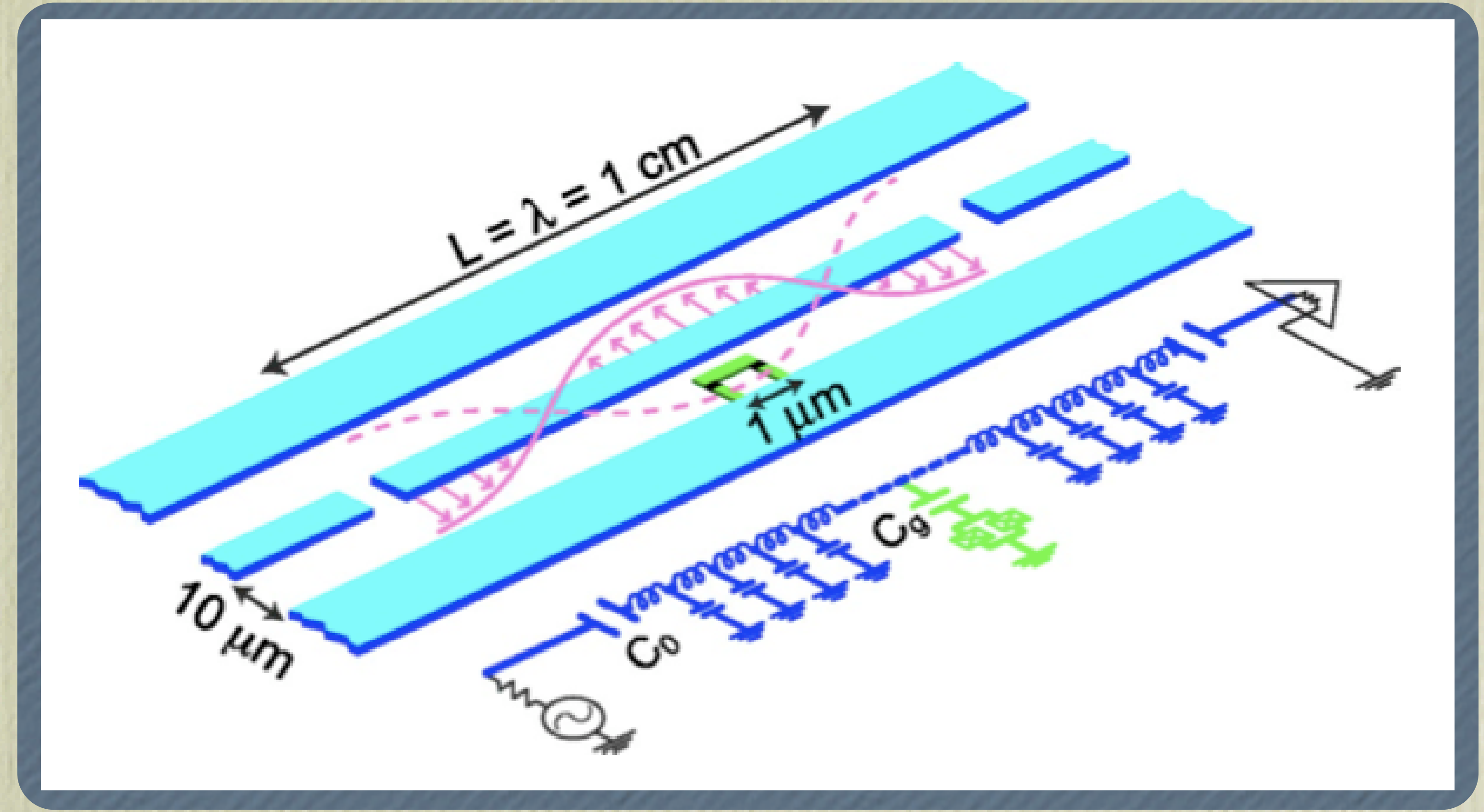
Small cavity photon number

↓

Fast qubit response to drive

There is a speed limit set by the cavity-qubit detuning Δ → If the drive is turned on or off faster than Δ^{-1} then the frequency spread of the drive may dephase the qubit

Multiple qubits



- It should be possible to place multiple qubits along the length of the TLR and entangle them together.
- Two qubits coupled to and in resonance with each other but detuned from the cavity can provide a CNOT gate.

Summary & conclusions

It is possible to construct a circuit implementation of cavity QED based on TLRs and CPBs, with many attractive features:

- Strong coupling is achieved
- Coherent superpositions of a single qubit state and a single photon can be generated
- The properties of the qubit can be determined in a transmission measurement
- Full in-situ control over the qubit parameters is achieved
- Two-qubit entanglement over cm distances 'should be possible'

Some numbers...

Parameter	Symbol	1D circuit
Dimensionless cavity pull	$g^2/\kappa\Delta$	2.5
Cavity-enhanced lifetime	$\gamma_\kappa^{-1}=(\Delta/g)^2\kappa^{-1}$	64 μs
Readout SNR	$\mathrm{SNR} = (n_{\mathrm{crit}}/n_{\mathrm{amp}})\kappa/2\gamma$	200 (6)
Readout error	$P_{\mathrm{err}}\sim 5\times\gamma/\kappa$	1.5%(14%)
One-bit operation time	$T_\pi > 1/\Delta$	>0.16 ns
Entanglement time	$t_{\sqrt{i\mathrm{SWAP}}}=\pi\Delta/4g^2$	~0.05 μs
Two-bit operations	$N_{\mathrm{op}}=1/[\gamma\, t_{\sqrt{i\mathrm{SWAP}}}]$	>1200(40)

Parameter	Symbol	3D optical	3D microwave	1D circuit
Resonance or transition frequency	$\omega_r/2\pi$, $\Omega/2\pi$	350 THz	51 GHz	10 GHz
Vacuum Rabi frequency	g/π, g/ω_r	220 MHz, 3×10^{-7}	47 kHz, 1×10^{-7}	100 MHz, 5×10^{-3}
Transition dipole	d/ea_0	~1	1×10^3	2×10^4
Cavity lifetime	$1/\kappa, Q$	10 ns, 3×10^7	1 ms, 3×10^8	160 ns, 10^4
Atom lifetime	$1/\gamma$	61 ns	30 ms	2 μs
Atom transit time	t_{transit}	⩾50 μs	100 μs	∞
Critical atom number	$N_0=2\gamma\kappa/g^2$	6×10^{-3}	3×10^{-6}	$\leqslant 6\times10^{-5}$
Critical photon number	$m_0=\gamma^2/2g^2$	3×10^{-4}	3×10^{-8}	$\leqslant 1\times10^{-6}$
Number of vacuum Rabi flops	$n_{\mathrm{Rabi}}=2g/(\kappa+\gamma)$	~10	~5	$\sim 10^2$