Quantum Computing with Trapped Ion Hyperfine Qubits

General requirements

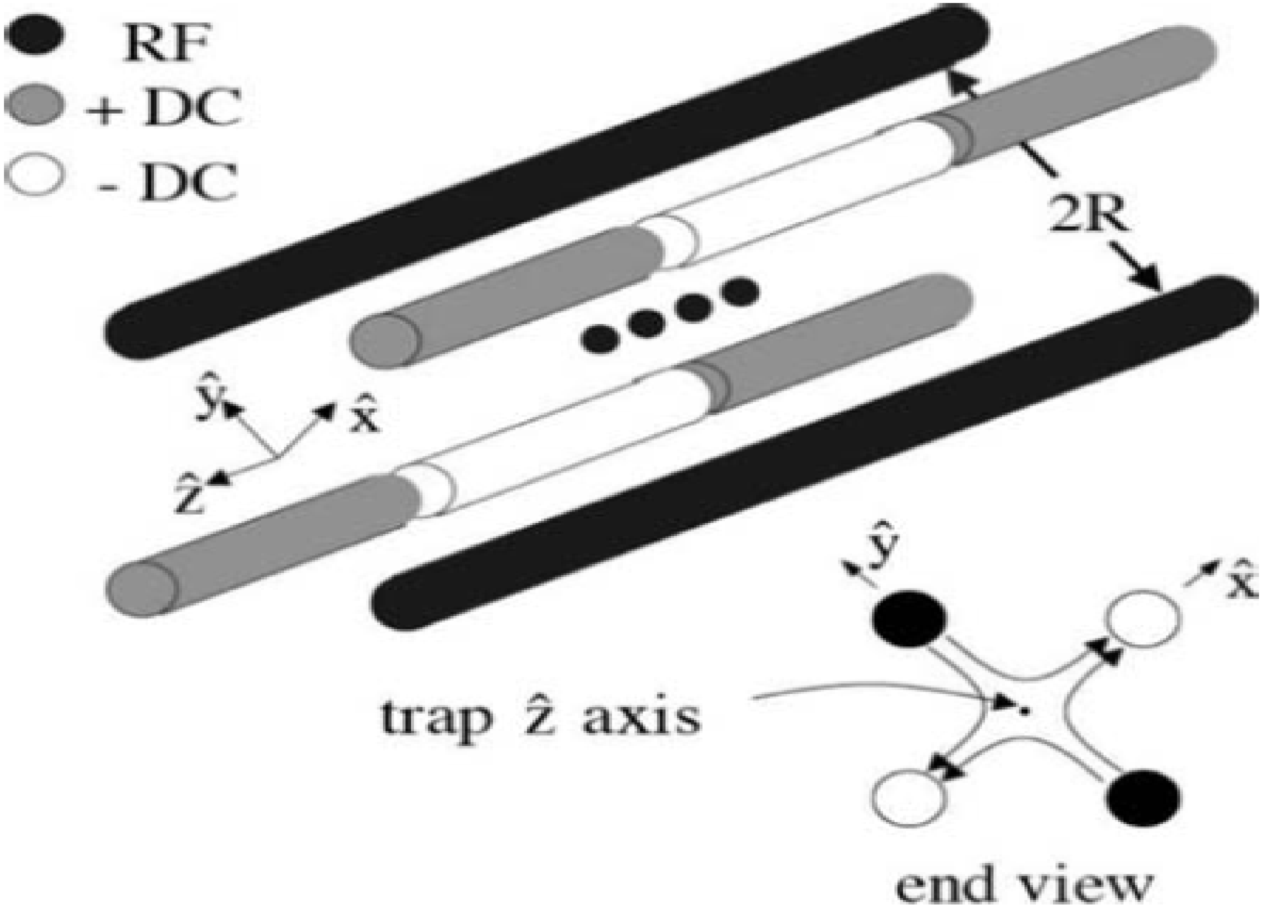
- A scalable system of well-defined qubits
- A method to reliably initialize the quantum system
- Long coherence times
- Existence of universal gates
- An efficient measurement scheme

Type of qubits for trapped ion

- Optical qubits - derived from ground state and an excited metastable state separated by an optical frequency
- Hyperfine qubits - derived from electronic ground-state hyperfine levels separated by a microwave frequency

Ion traps

- Quadrupole ion trap: using DC and radio frequency (RF) ~1 MHz oscillating AC electric fields
- Penning trap: using a constant magnetic field and a constant electric field



Ponderomotive pseudopotential

$$U_{x,y}(\mathbf{r}) = \frac{q^2}{2m\Omega_T^2}\langle E^2(\mathbf{r})\rangle \simeq \frac{q^2 V_0^2}{4m\Omega_T^2 R^4}(x^2+y^2),$$

Oscillation frequency of ion

$$\omega_{x,y} \simeq \frac{q V_0}{\sqrt{2}\Omega_T m R^2}$$

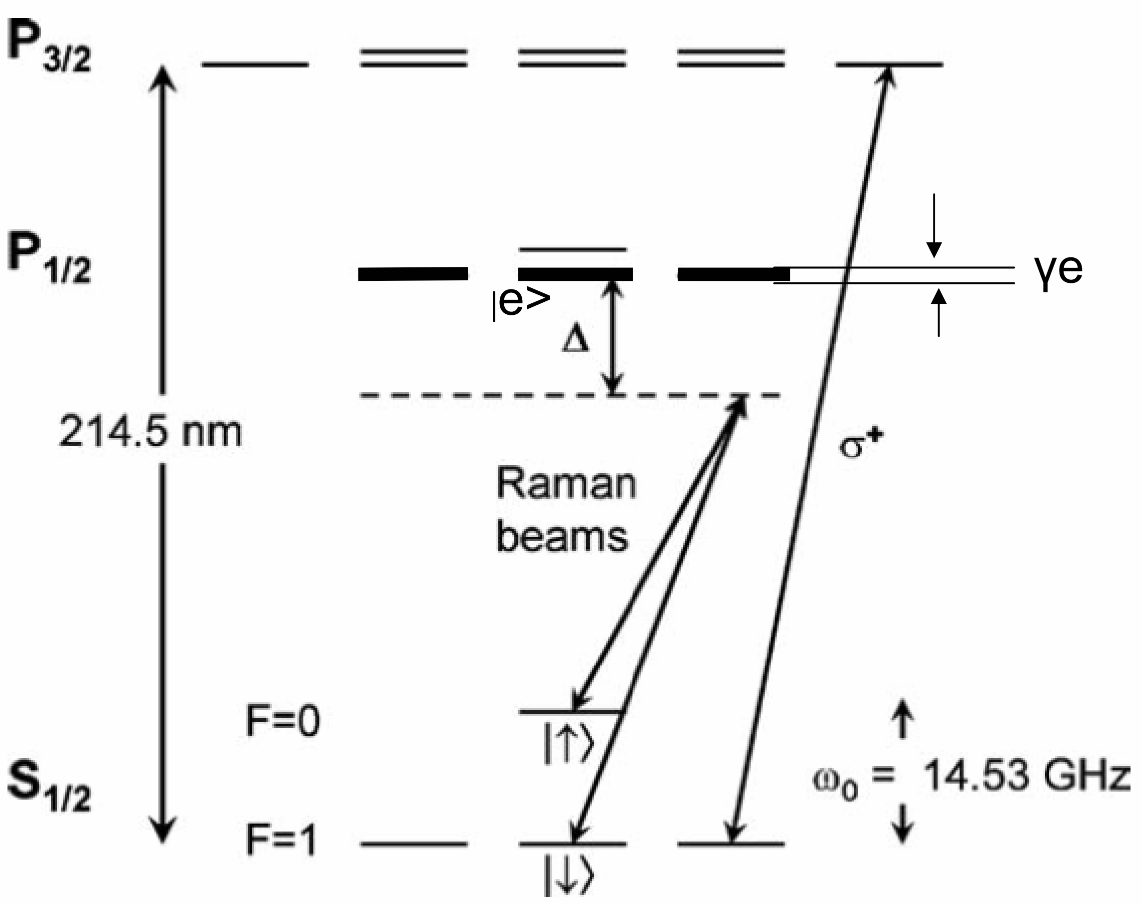
$$\omega_z \ll \omega_{x,y} \ll \Omega_T$$

Ion trap demo

Trapped ion hyperfine qubits

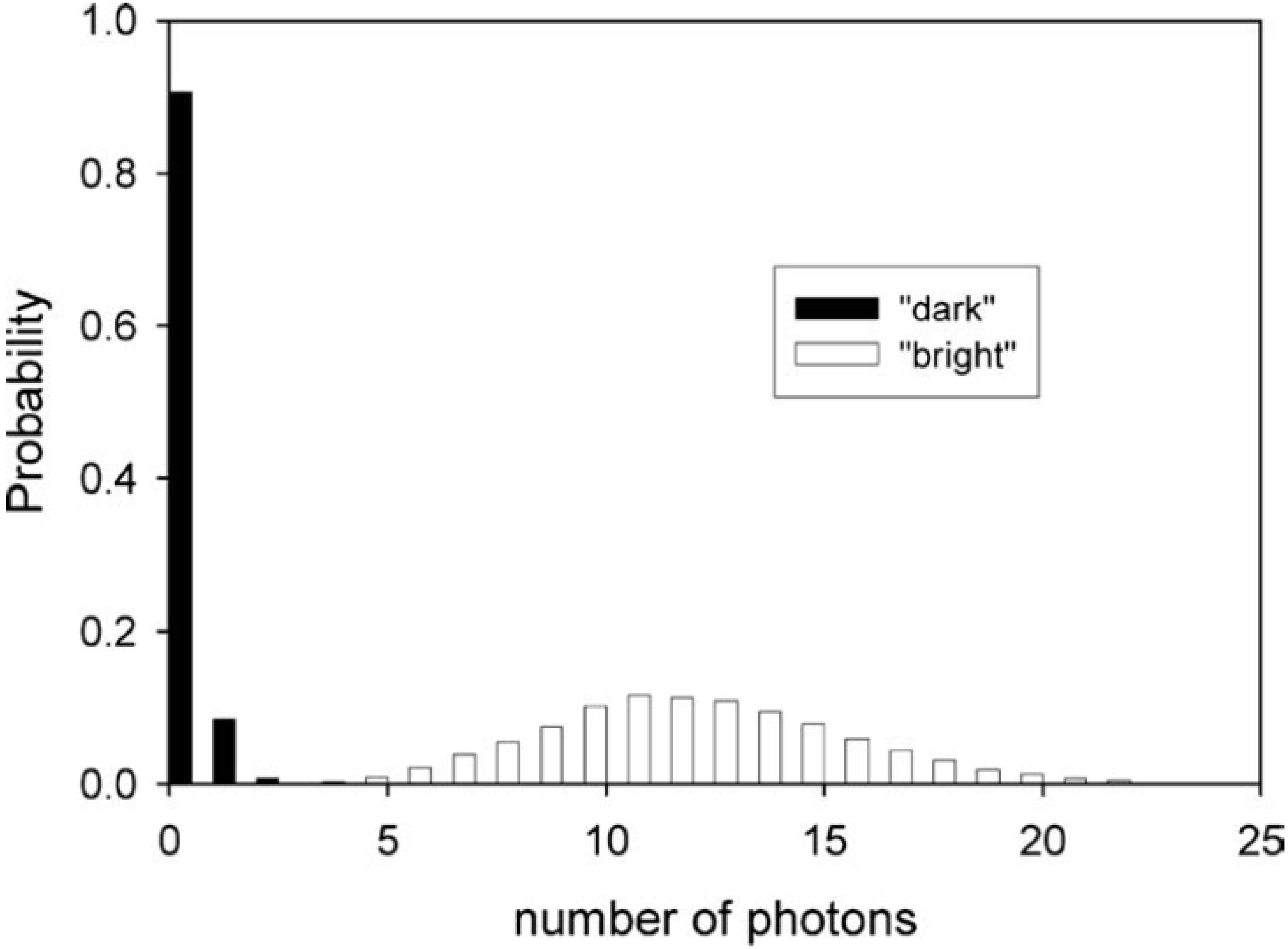
- Electric field perturbations are small
- Magnetic field perturbations can be reduced by coherence between two internal levels
- Extremely long radioactive lifetime

Electronic (internal) energy levels (not to scale) of the $^{111}\text{Cd}^{+}$ion.



Initialization and detection of qubits

- Standard optical pumping to initialize HF qubits to either $|\downarrow\rangle$or $|\uparrow\rangle$
- Polarized laser beam resonant with spacing level scatter either $|\downarrow\rangle$ or$|\uparrow\rangle$


1.0
0.8
0.6
0.4
0.2
0.0
Probability
"dark"
"bright"
0
5
10
15
20
25
number of photons

HF Qubit Rotations: Single Qubit Gates

- Microwave with frequency ω_{HF}

  Big wavelength ~cm

  - good for joint rotations of all qubits

  - difficult for individual qubits rotation.

- Stimulated Raman Transitions (STR)

  - two laser fields with detuning Δ from excited state and differing in frequency by ω_{HF}, $\Delta >> \gamma_e$

  - SRT Rabi frequency: $\Omega_{SRT}=g_1 g_2^* / \Delta$

  - individual qubits rotation can be achieved

Interactions between HF qubits: entangling qubit gates

- Interaction Hamiltonian

$$H_I = -(\hat{\mu}_{\uparrow,\mathrm{e}} + \hat{\mu}_{\downarrow,\mathrm{e}}) \cdot \mathbf{E}(\hat{\mathbf{x}}).$$

- *Motion-sensitive stimulated Raman transitions*
- *Spin-dependent optical forces*

Motion-sensitive stimulated Raman transitions

$$H_I = -(\hat{\mu}_{\uparrow,\mathbf{e}} + \hat{\mu}_{\downarrow,\mathbf{e}}) \cdot \mathbf{E}(\hat{\mathbf{x}}).$$

Rotating wave approximation

$$H = \eta \Omega_{\mathrm{SRT}} (\sigma_+ \mathrm{e}^{\mathrm{i}\eta(a + a^\dagger)} + \sigma_- \mathrm{e}^{\mathrm{i}\eta(a + a^\dagger)}),$$

$\eta = \delta \mathbf{k} \cdot \mathbf{x_0}$: Lamb–Dicke parameter “motional sideband” at frequency

$\omega_{\mathrm{HF}} \pm k\omega$

$$\Big\downarrow \; k\text{=-1}$$

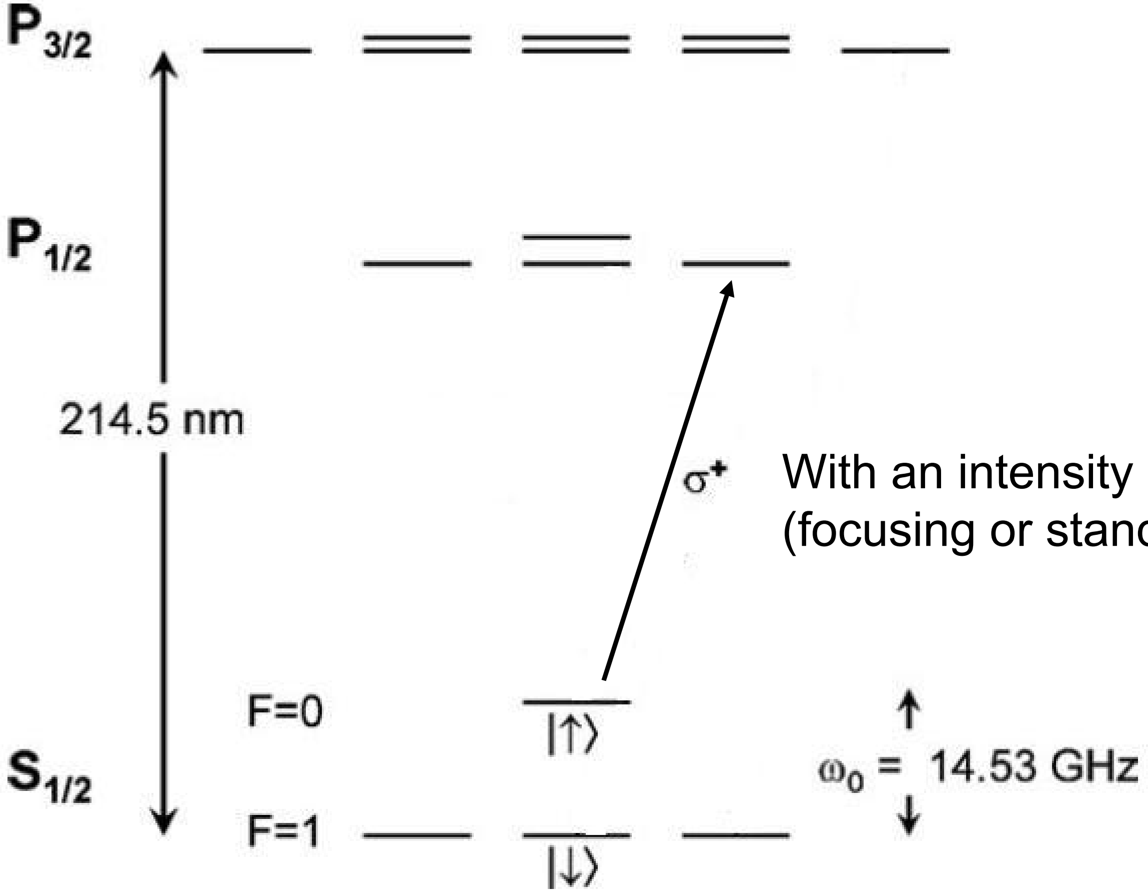
J-C Hamiltonian

$$H_{-1} = \eta \Omega_{\mathrm{SRT}} (\sigma^{+} a + \sigma^{-} a\dagger).$$

$$(\alpha|\downarrow\rangle + \beta|\uparrow\rangle)|0\rangle \Longrightarrow |\downarrow\rangle(\alpha|0\rangle + \beta|1\rangle)$$

Spin-dependent optical forces

- Laser beams' dipole force depends upon the state of the qubit |*S*> and certain excited state (atomic selection rules).
- Appropriate polarization of the light.
- Use intensity gradient of a laser beam or standing wave to control the motion of ion.



With an intensity gradient
(focusing or standing wave)