Quantum Cryptography

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Overview

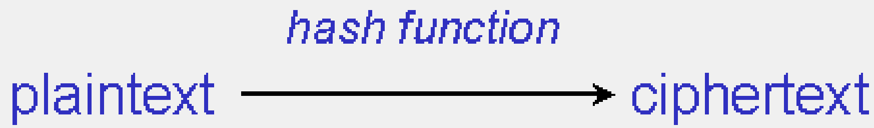
- Current Cryptography Methods
- Quantum Solutions
- Quantum Cryptography
- Commercial Implementation

Cryptography algorithms:

- *Symmetric* – encrypting and decrypting key are identical (Data Encryption Standard, Rivest Ciphers)
- *Asymmetric* – encrypting and decrypting keys differ (Elliptical Curve; Rivest, Shamir, Adleman)
- *Hash* – no decryption by design, meant to uniquely identify a message such as a password (Message Digest)

A) Secret key (symmetric) cryptography. SKC uses a single key for both encryption and decryption.

B) Public key (asymmetric) cryptography. PKC uses two keys, one for encryption and the other for decryption.



C) Hash function (one-way cryptography). Hash functions have no key since the plaintext is not recoverable from the ciphertext.

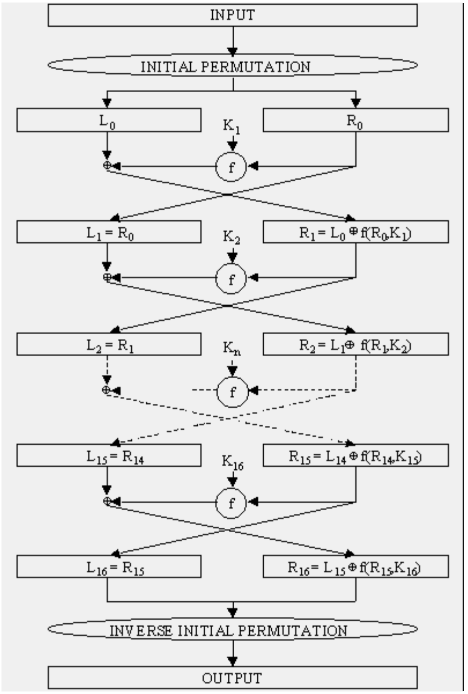
Gary Kessler 2007

Symmetric Key Distribution

- RC5 and others takes sufficiently long decrypt (72 bits with distributed computing ~1000 years for RC5)
- How do we securely distribute keys?
- Some methods work on simple binary addition:

$$s = m \oplus k, m = s \oplus k = m \oplus k \oplus k$$

- Others, such as DES, shuffle blocks of information



Asymmetric Key Distribution

- Rivest, Shamir, Adelman (RSA) use the property of factoring a large number in terms of primes is sufficiently complex with classical computers.
- Elliptical Curves make use of another sufficiently complex classical problem of calculating the discrete logarithm.
- Codes can be broken more readily than symmetric keys (72 bits sym ~ 2048 bits asym)

RSA Algorithm

- Pick two large prime numbers p and q and calculate the product $N = pq, \;\; \phi = (p-1)(q-1)$
- Choose a number that is co-prime with ϕ, c
- Find a number d to satisfy $cd = 1 \; mod \; \phi$, using a method such as Euclid's algorithm
- Using your plaintext, a, the ciphertext is encoded as $b = a^c \; mod \; N$
- To retrieve the plaintext, $a = b^d \; mod \; N$
- The numbers N and c are made public, so anyone can encrypt information, but only someone with d can retrieve the plaintext

Example

- Plaintext $a = 123$
- $p = 61$ and $q = 53$
- $N = pq = 3233$
- $\phi = (p - 1)(q - 1) = 3120$
- Pick a coprime of ϕ, $c=17$
- Find d such that $cd = 1 \mod \phi$, $d=2753$
- Encode with $a^c \mod N$, in this case $123^{17} \mod 3233 = 855$
- Decode message by evaluating $b^d \mod N$, in this case $855^{2753} \mod 3233 = 123$

855^2753 mod 3233

= 50432888958416068734422899127394466631453878360035509315554967564501
05562861208255997874424542811005438349865428933638493024645144150785
17209179665478263530709963803538732650089668607477182974582295034295
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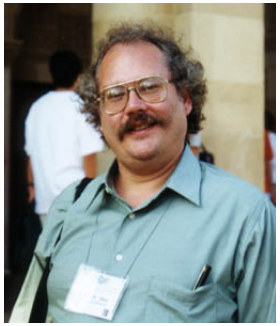
Enter Shor's Algorithm



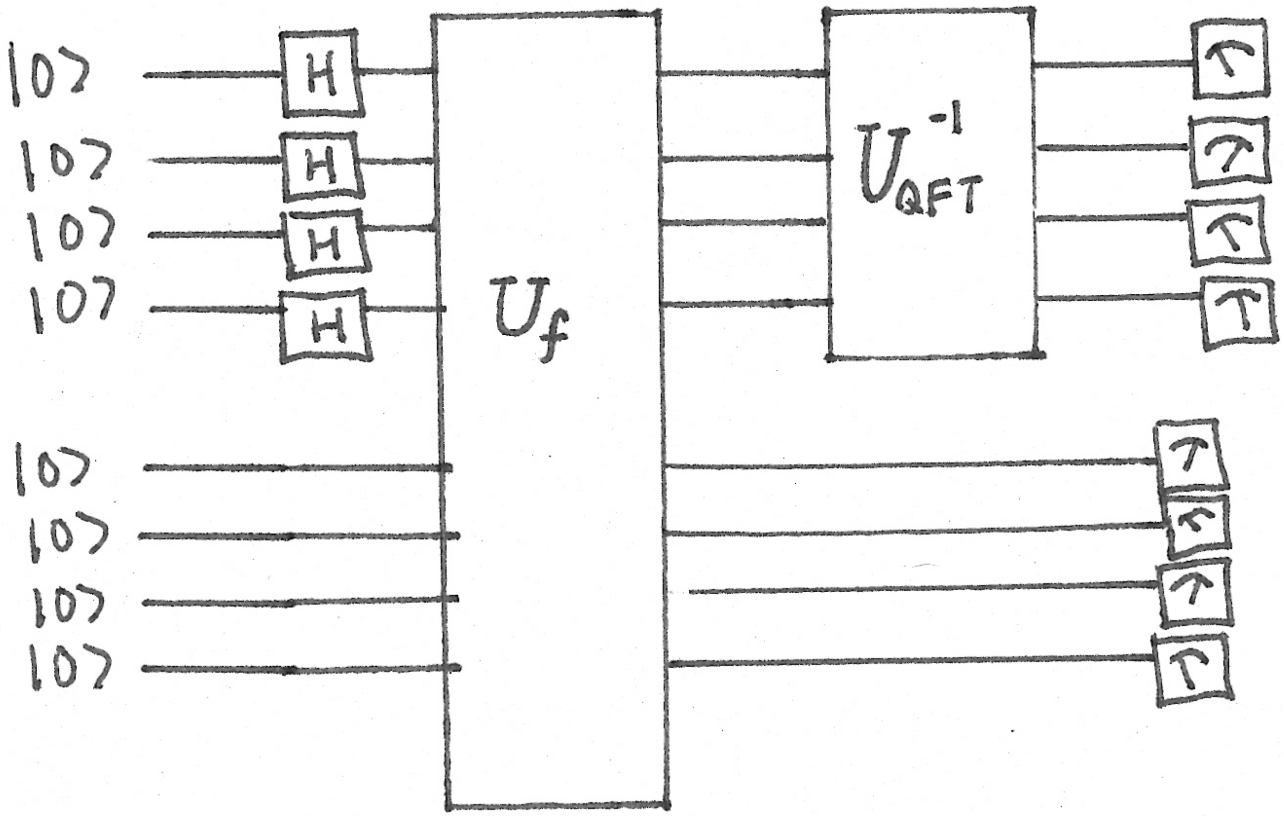
- Let $f(x) = b^x \mod N$, if we can find some r that $f(x) = f(x+r)$, then we can find a number d' such that $cd' = 1 \mod r$
- The value d' works like the decoding value we calculated from $cd = 1 \mod \phi$
- In addition, using different values for $b<N$, we can determine the prime components of N

What's the quantum algorithm?

- Initialize $log_2 N$ qubits an equal superposition state (input qubits)

$$|\psi\rangle = \frac{1}{2^{n/2}}\left(|000...000\rangle + |000...001\rangle + ... + |111...111\rangle\right)$$

- Using $log_2 N$ more qubits, enact $f(\psi)$ on them while retaining the state ψ in the input state (output qubits)
- Apply the quantum fourier transform on the ψ portion of the circuit
- Measure the input and output qubits $(y,f(x_0))$, with high probability you will measure an of $f(x_0 + (y/N)r)$ where y/N is close to an integer



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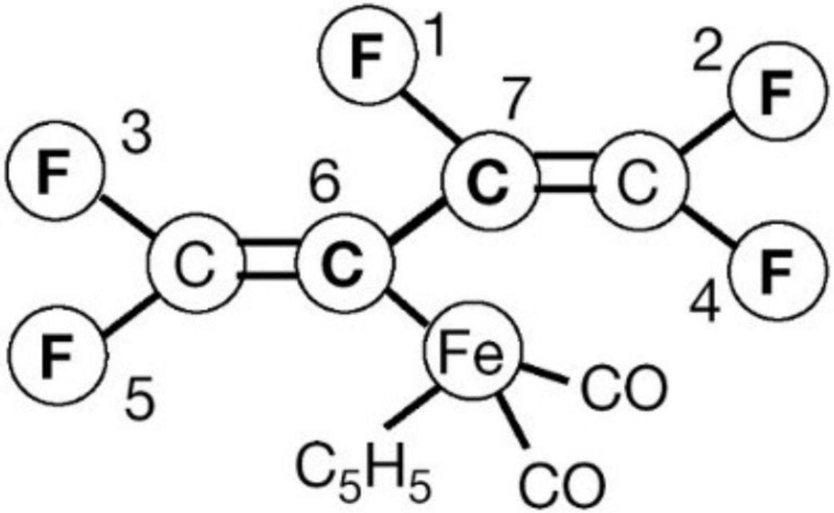
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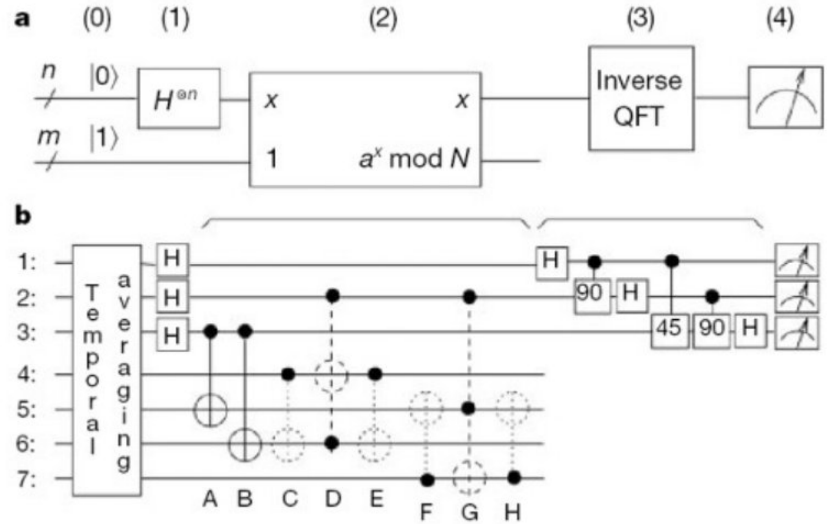
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$H^{\otimes n}$
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Inverse QFT
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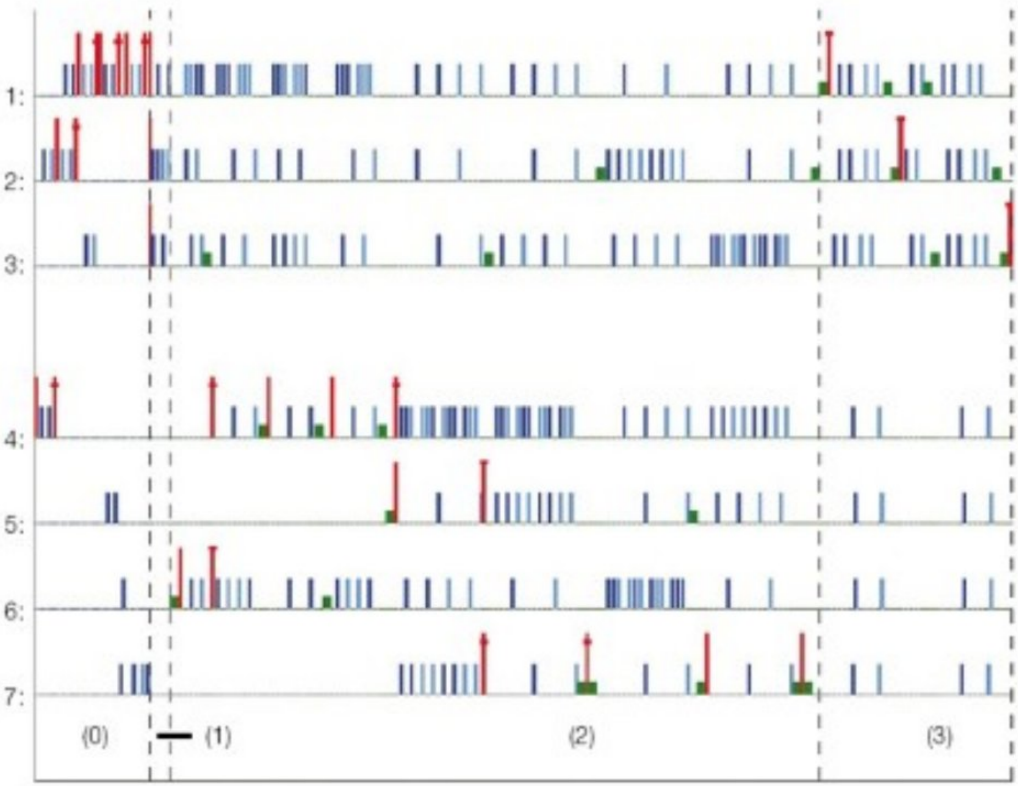
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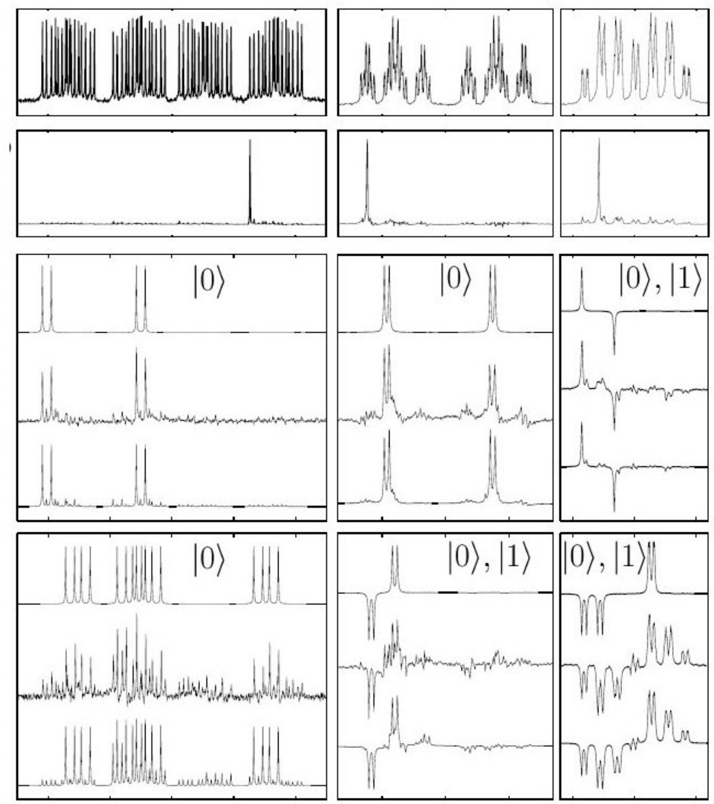
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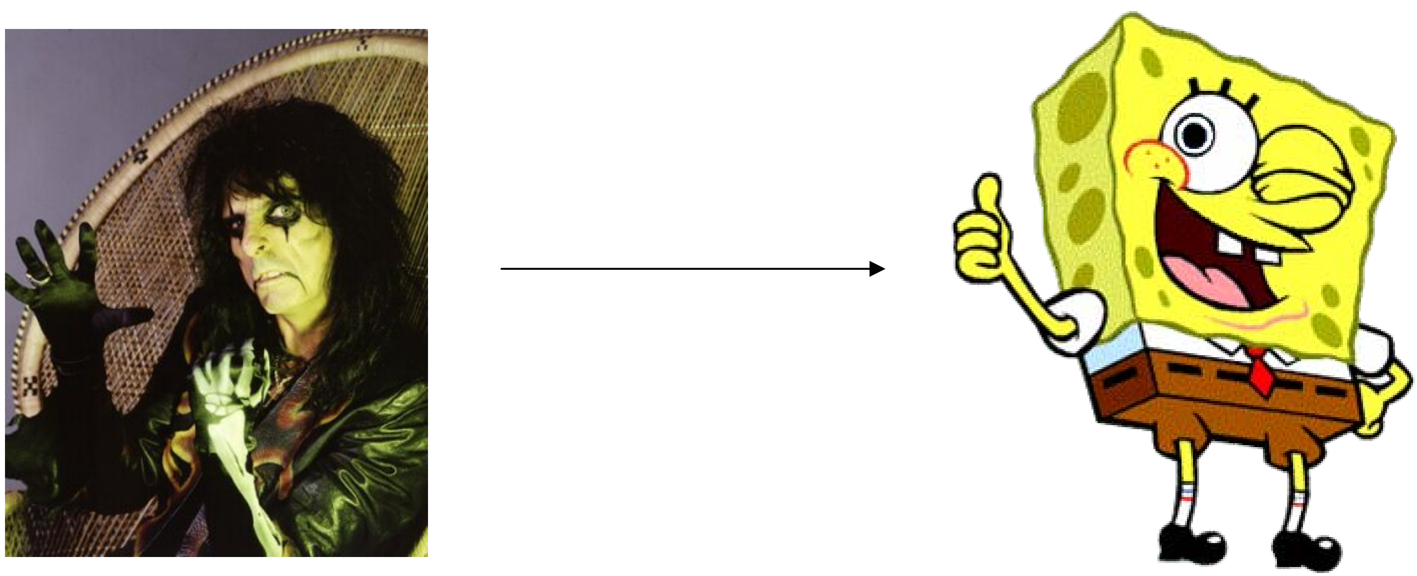
So we've broken it, now what?

- In general symmetric keys are harder to crack but tough to distribute, while asymmetric keys are easy to distribute but easier to crack
- Start thinking about using quantum systems to implement cryptography
- Restrictions on polarization bases measurements ($\uparrow \rightarrow \searrow \swarrow$ σ^{+} σ^{-})
- Restrictions on state duplication
- Very easy to create state perturbation

BB84 (Bennett and Brassard)

- We have two parties, Alice and Bob who want to securely distribute their symmetric key over a public channel



BB84

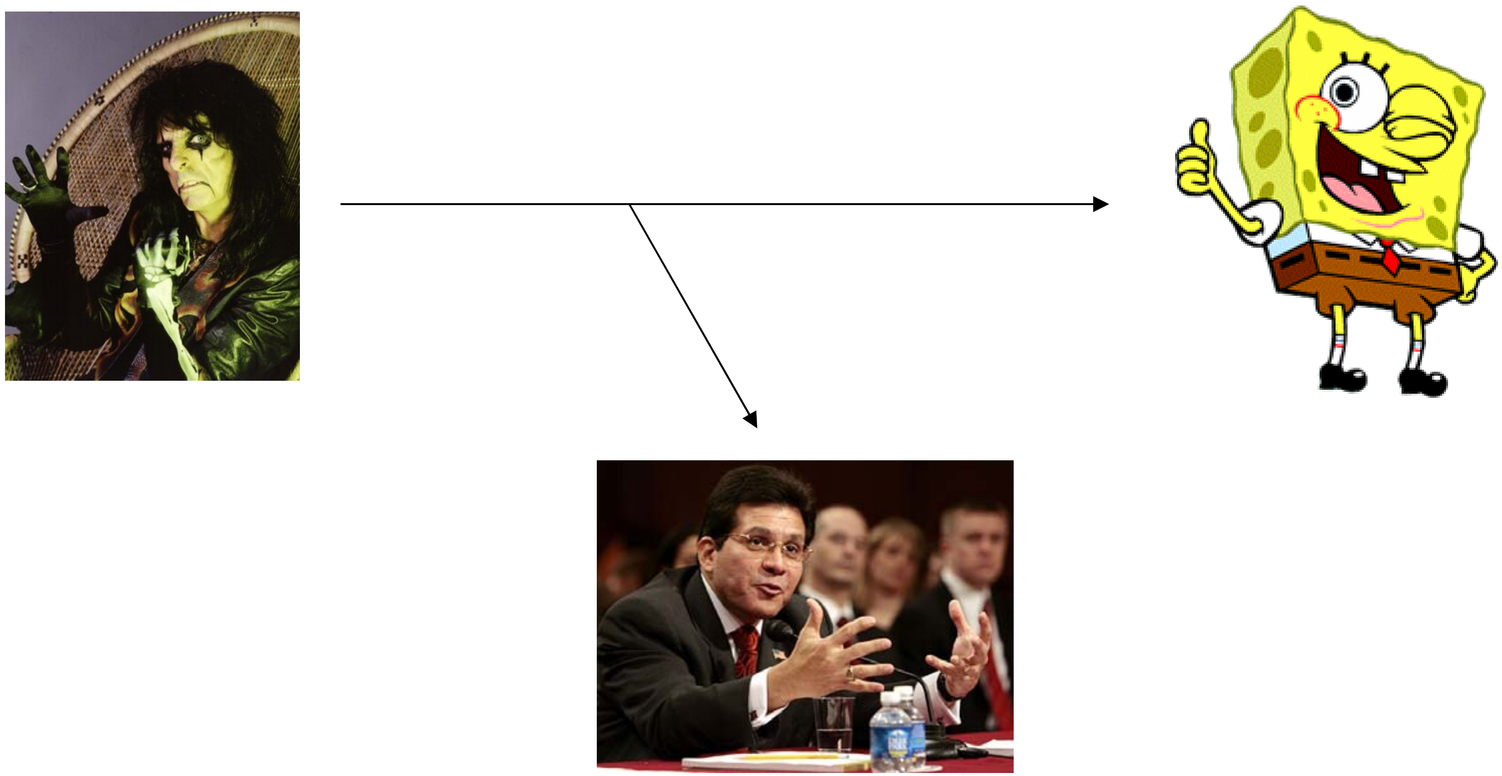
- Alice randomly chooses one of two orientations from two bases to measure in: (for spin ½ situation analogous to z-basis, and x-basis)
- Alice then assigns the value of 0 and 1 in each basis (up-z and up-x = 0, down-z and down-x =1)
- Alice sends a state from one of the four bases at random, and Bob selects (with his own random generator) a basis (x or z) to measure in
- If they choose the same basis, they will agree with 100% probability, if they choose a different basis they will have no way of correlating the results (error rate ~25%)

BB84

- In order to verify the transmitted information, Alice and Bob decide which bits can be kept and which bits will need to be retransmitted
- The correlated measurements will only be in compatible bases (both x or z)

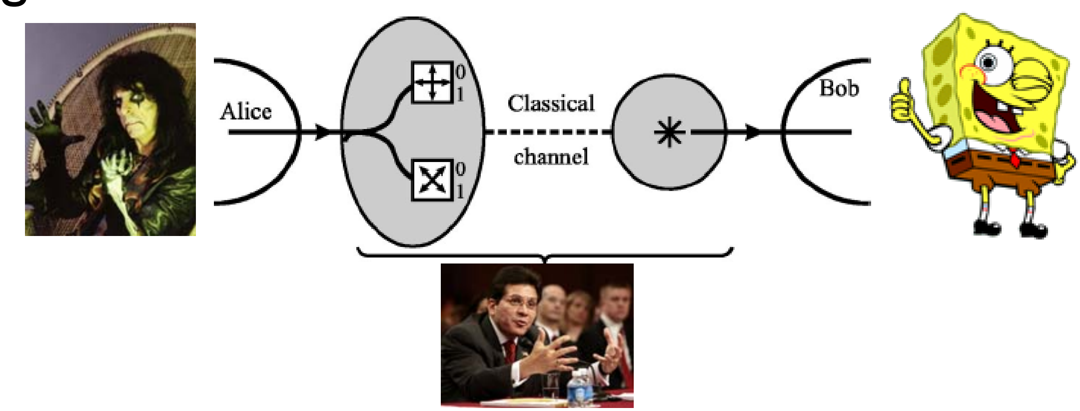
http://monet.mercersburg.edu/henle/bb84/demo.php

Eavesdropping



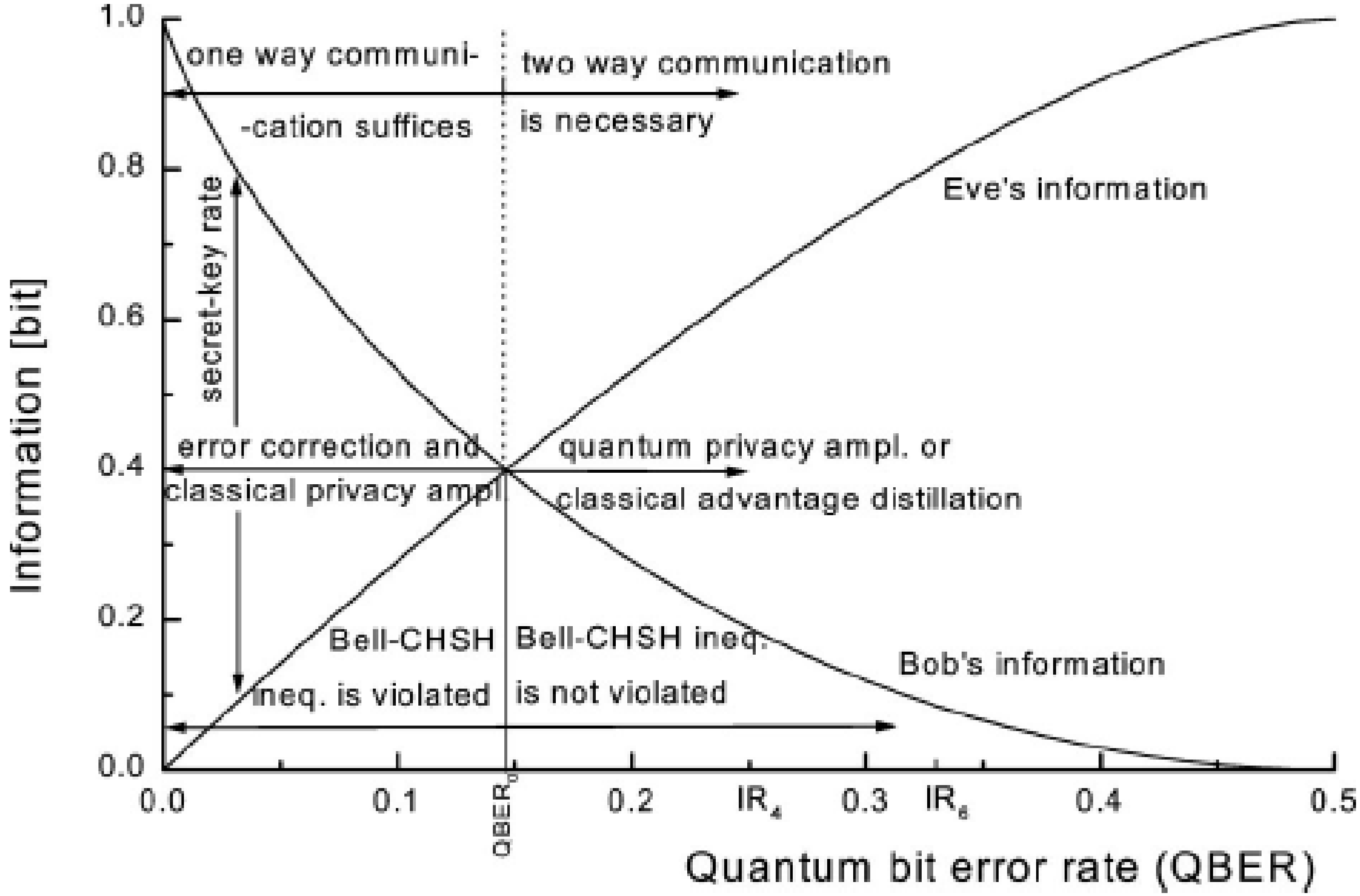
Multiple Photon Attack

- Eve can attack an optical channel by measuring multiple photon signals with a PBS and recreating the signals



2/3 of the time Eve can recreate the original state and send it to Bob, the rest of the time she introduces an error rate of 1/6

Eavesdropping Thresholds



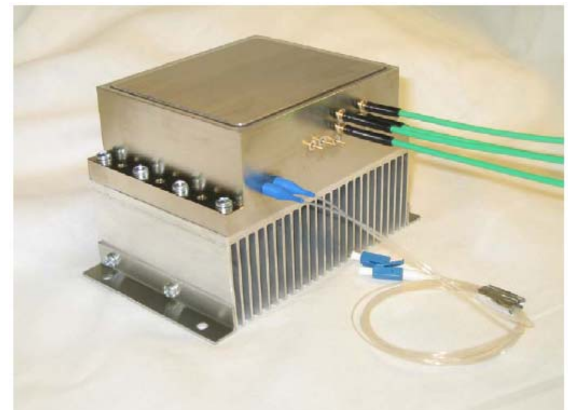
Commerical Quantum Crypto Systems

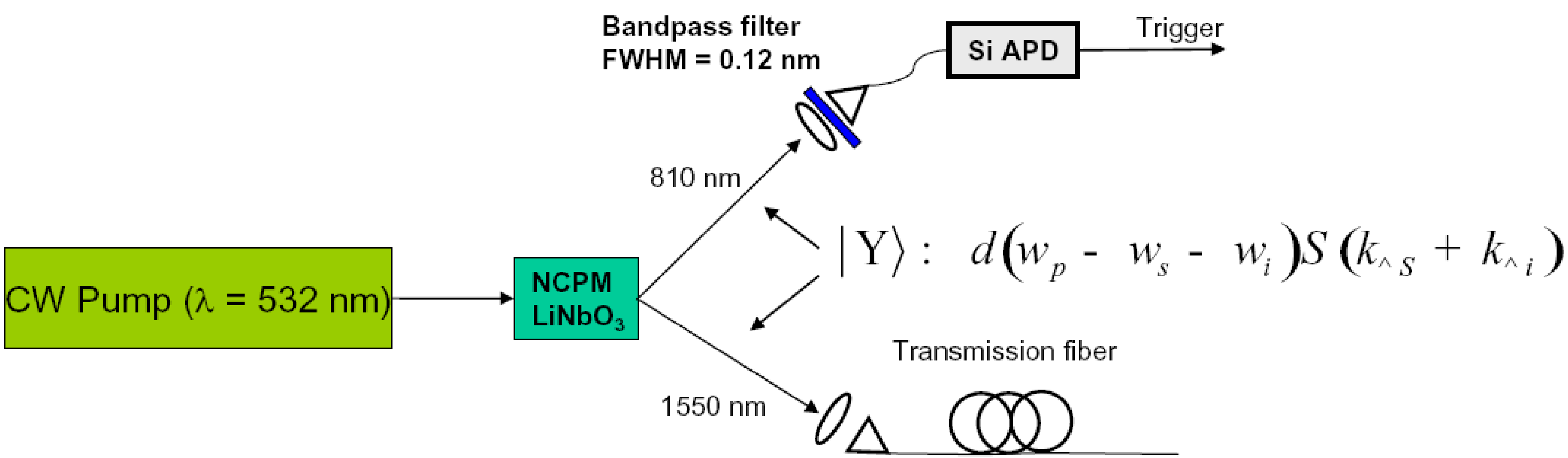
- Magiq: Currently have an implementation of a secure quantum network, the QPN 7505
- Works on a single photon source, and can transmit up to about 75 km with reasonable loss

Price? $97000

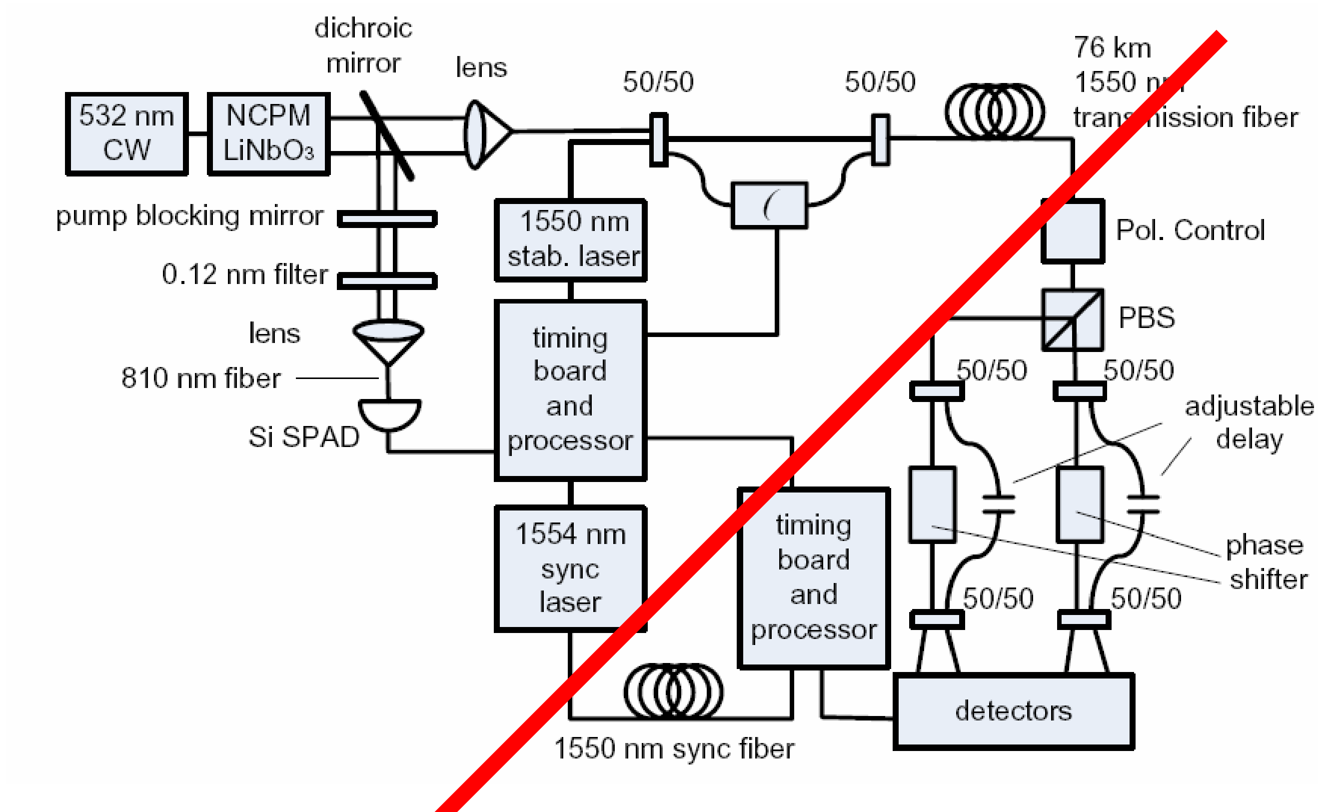
Single Photon Manipulation

Entanglement occurs in the time-frequency domain, there is a high probability that a single photon is produced, and low probability of multiple photons



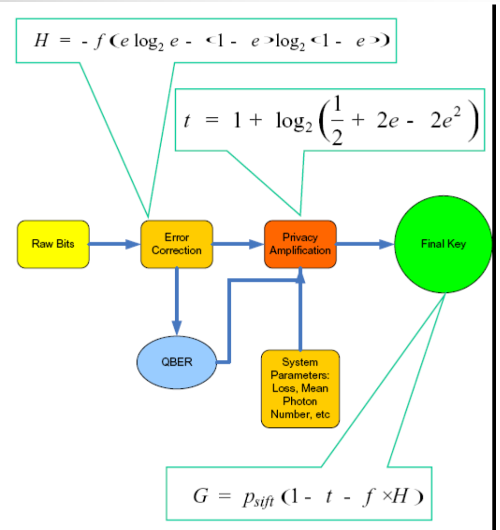


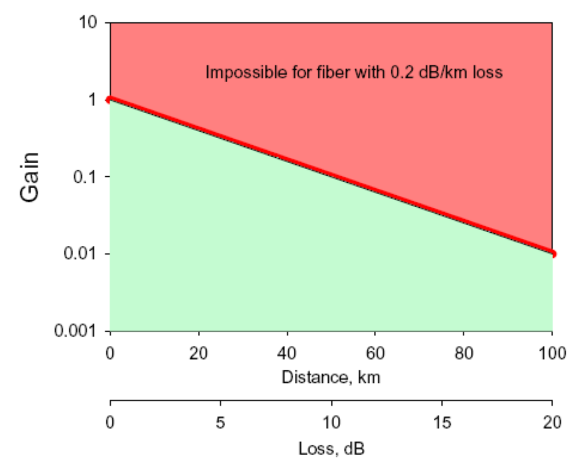
Single Photon setup



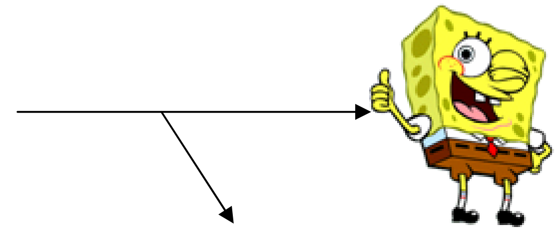
Fidelity Considerations

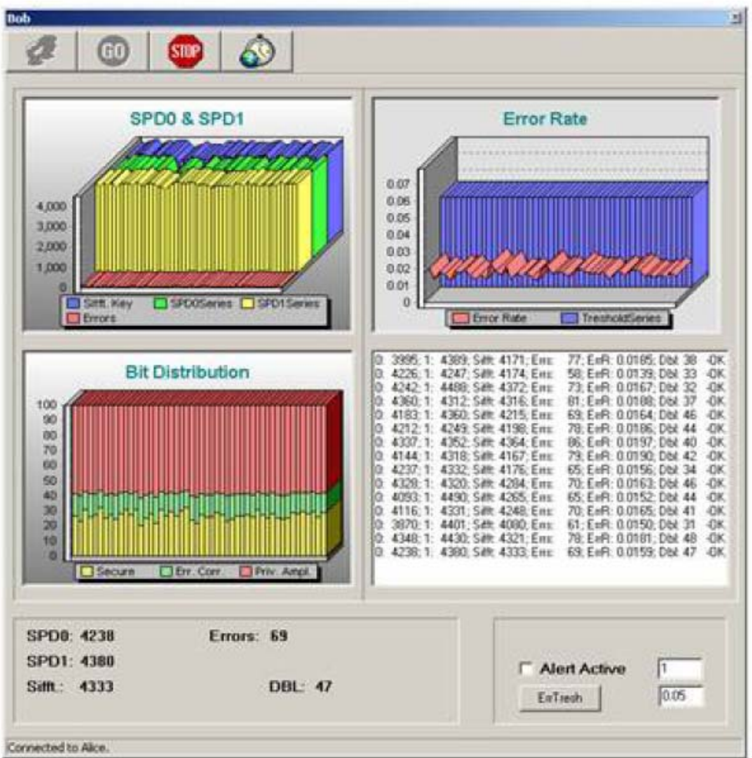
- The system tries to maximize *G*, the probability of transmitting a secure bit with a single initial pulse
- This attenuates about 10dB for every 50 km of transmission

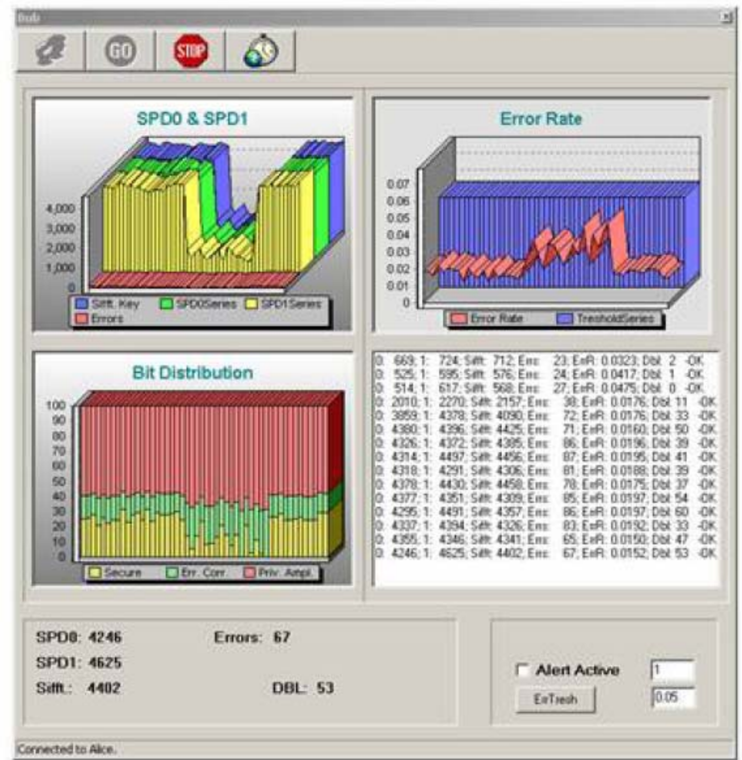




Eavesdropping!





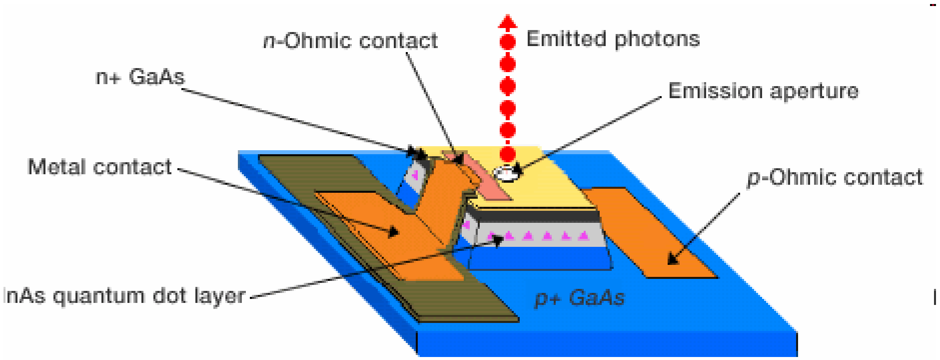


Other systems

- Id quantique:

  Vectis – Their crypto system, uses typical QKD and AES (Advanced Encryption Standard)

  Quantis – Random number generator, based on standard 50/50 polarization probabilities (4 Mbit/s number generation) PCI hardware
- Toshiba Research – Cambridge, QKD and single photon emission with quantum dots



Sources

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- "*Secure Communication with single photons*"; A. Trifonov; 2005
- "*An Overview of Cryptography*"; Gary Kessler; 2007
- "*Applications of Quantum Cryptography in Government Classified, Business, and Financial Communications*"; Audrius Berzanskis; 2005
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