# Superconducting Qubits

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# Outline

- How do we get macroscopic quantum behavior out of a many-electron system?
- The basic building block – the Josephson junction, how do we make it a two-level system?
- Current experiments
- Where is more research necessary – the DiVincenzo criteria

# Background superconductivity

- Discovered somewhat accidentally in 1911, theoretically explained in 1957
- Properties
  - Charge carriers are paired electrons (fundamental charge now 2e)
  - Zero electrical resistance
  - Expel magnetic fields inside (which leads to quantization of flux - useful)
  - All electrons condense to a single state described by one wavefunction, the superconducting order parameter

# The Theory you ask? Couldn't be simpler…

- Full microscopic theory in 1957 by Bardeen, Cooper and Schreiffer
- 2nd quantized Hamiltonian with 2-body interaction

$$\hat{\mathrm{H}} = \underbrace{\sum_{\mathbf{k},\sigma} \epsilon(\mathbf{k}) \hat{c}^{\dagger}_{\mathbf{k},\sigma} \hat{c}_{\mathbf{k},\sigma}}_{\text{Kinetic}} + \underbrace{\frac{1}{2} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q},\sigma,\sigma'} v(\mathbf{q}) \hat{c}^{\dagger}_{\mathbf{k}+\mathbf{q},\sigma} \hat{c}^{\dagger}_{\mathbf{k}'-\mathbf{q},\sigma'} \hat{c}_{\mathbf{k},\sigma} \hat{c}_{\mathbf{k}',\sigma'}}_{\text{Two-body interaction}}$$

- Can be solved variationally to give a ground state with paired electrons and an energy lower than the normal state by $\Delta \sim \hbar\omega_D$

$$|\psi\rangle = \prod_{|\mathbf{k}|<k_F} \frac{1 + \alpha_{\mathbf{k}} \hat{c}^{\dagger}_{\mathbf{k},\uparrow} \hat{c}^{\dagger}_{\mathbf{k},\downarrow}}{\sqrt{1+\alpha^2_{\mathbf{k}}}} |0\rangle$$

# Stepping back

- The problem can logically (and historically) be assaulted classically in the language of 2nd order phase transitions
- Ginzburg-Landau Free Energy

$$F[\psi] = F_N + \int d^3x \left( \alpha |\psi|^2 + \frac{\beta}{2} |\psi|^4 + \frac{1}{2m} \left| \left( \frac{\hbar}{i} \nabla - 2e\mathbf{A} \right) \psi \right|^2 + \frac{|H|^2}{2\mu_o} \right)$$

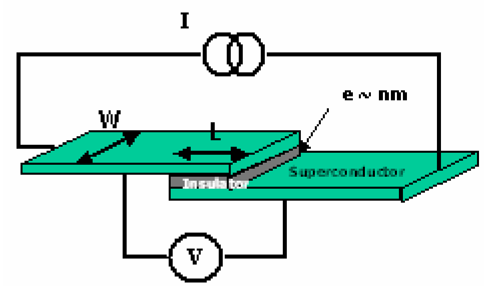
- Minimizing w.r.t. fluctuations in the order parameter we end up with a Schrödinger-like equation for $\psi$, which gives us a meaningful place to start talking about quantum computing

# The basic building block

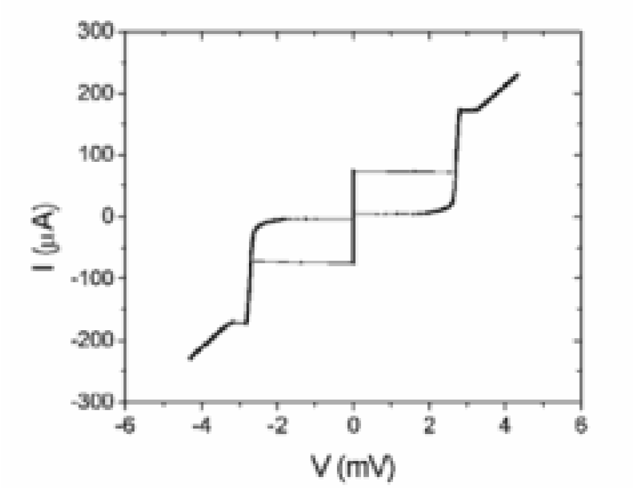
- Josephson tunnel junction
- With an applied bias, a tunneling current of Cooper pairs is observed

$$
\begin{aligned}
U(t) &= \frac{\hbar}{2e}\frac{\partial\theta}{\partial t} \\
I(t) &= I_c \sin(\theta)
\end{aligned}
$$

- Behavior is characterized by Stewart-McCumber parameter, based on the macroscopic Josephson equations, and resembles a non-linearized pendulum



http://www.lne.fr/en/r_and_d/electrical_metrology/josephson_effect_ej.shtml



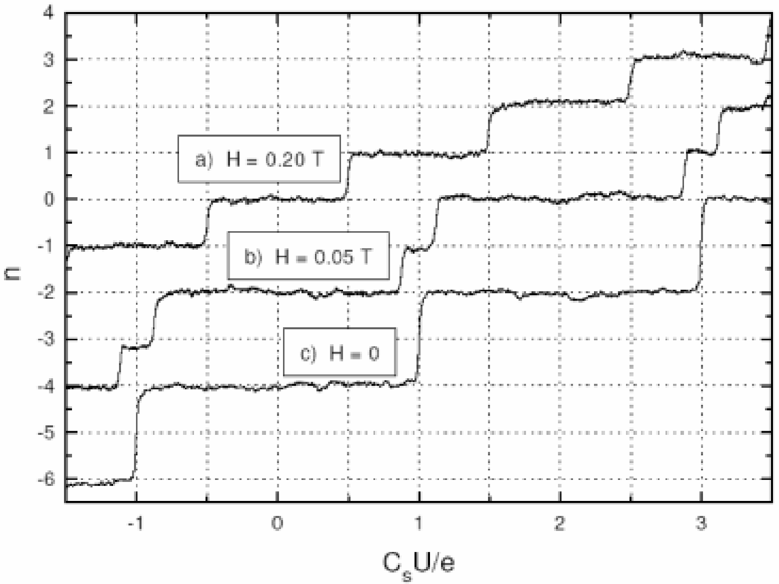
http://www.ifn.cnr.it/Groups/SQC/Research/JJ/jj.htm

# Quantum behavior

- If one thinks about a single Cooper pair tunneling across a capacitive barrier (changing energy density from $E=Q^2/2C$ to $E=(Q-2e)^2/2C$), the Coulomb “blockade” energy $E_C=(2e)^2/2C$ becomes apparent
- Comparing to thermal fluctation $k_BT$ and the uncertainty principle $\Delta E \Delta t>\hbar/2$, one arrives at size, temperature and resistance restrictions on the junction

# Quantum tunneling

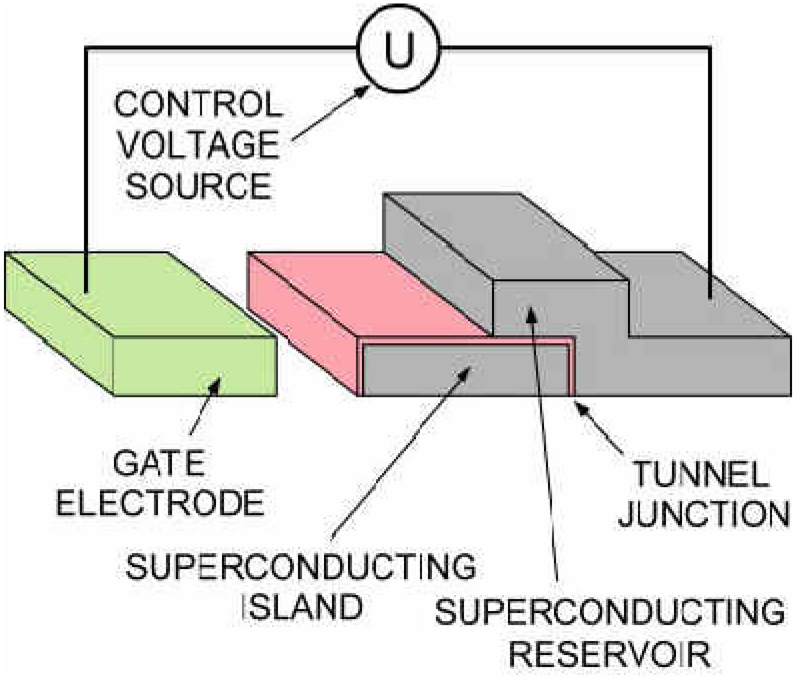
- The single Cooper pair tunneling is observable
- Conclusion – Cooper pair occupation number is a good quantum number to characterize the junction!



Bouchiat, et. al., *Physica Scripta* T76, 1998.

# The environment

- There is a problem – can’t simply attach leads and apply currents or voltages to JJ
- Büttiker solution (1987) – interact with JJ via a gate capacitance
- Result – “isolated” Cooper pair box (CPB)



http://www-drecam.cea.fr/drecam/spec/Pres/Quantro/Qsite/projects/qip/box.jpg

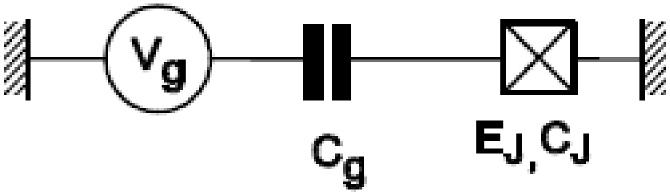
# The CPB Hamiltonian

$$\hat{H} = 4E_C \sum_p (\hat{N} - q)^2 |N\rangle \langle N| + \frac{E_J}{2} \sum_p (|N+1\rangle \langle N| + |N\rangle \langle N+1|)$$

- Charge states would be degenerate
- Two energy terms compete
  - Coulomb blockage potential
  - Josephson energy
- The Josephson energy comes from the delocalization of electrons across the barrier

# Relevant parameters

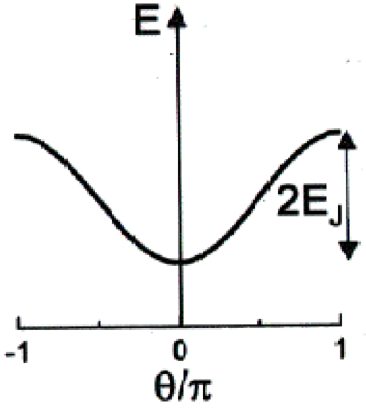
- Gate capacitance, $C_g$
- Bias, $U_g$
- Junction capacitance, $C_J$
- Junction characteristics
  - Competing Energy scales $\Delta$, $E_C$, $E_J$, $k_BT$



# Clearing things up

- With careful definition of the phase angle because of the integer (pos. or neg. of particle number), N and θ can be shown to be canonically conjugate
- H in this representation is diagonal

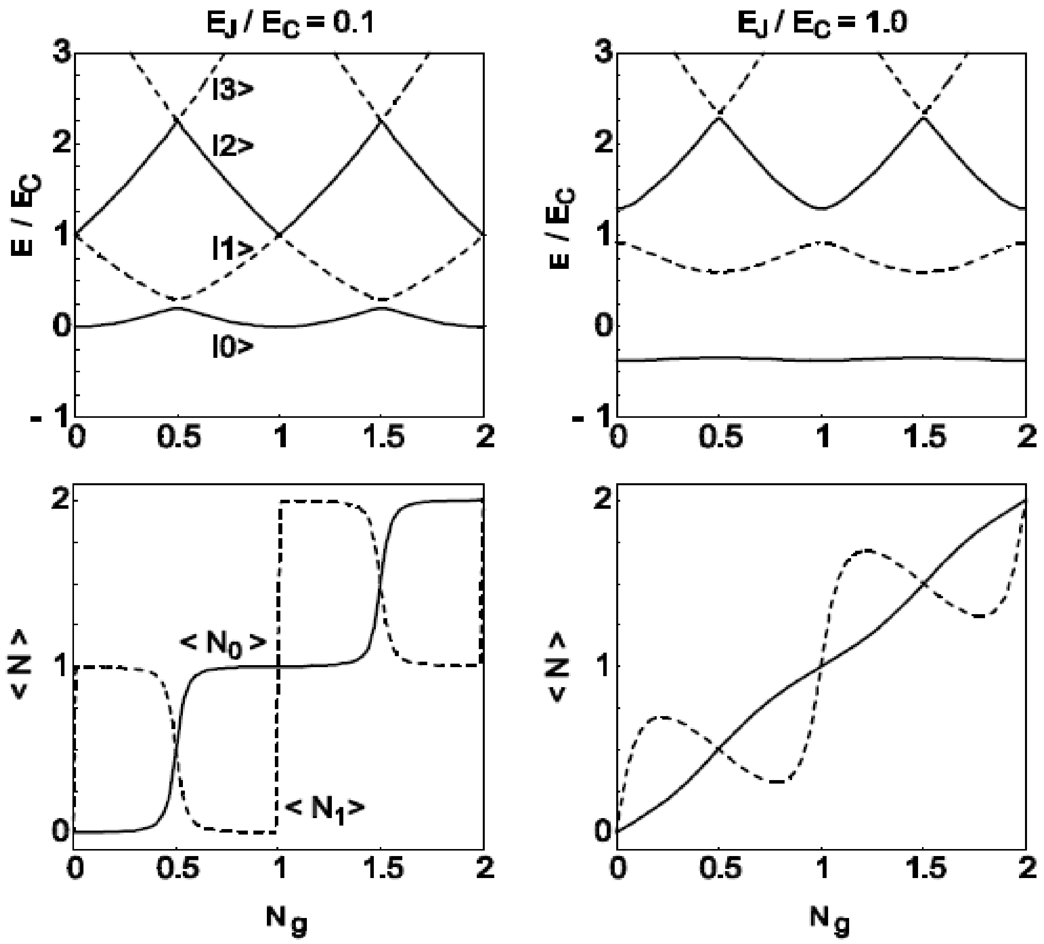
$$\hat{H} = 4E_C\left(\hat{N} - \frac{U_g C_g}{2e}\right)^2 - E_J \cos\hat{\theta}$$



Now, it is apparent that the ratio $E_C/E_J$ determines the dynamics

# Energy spectrum

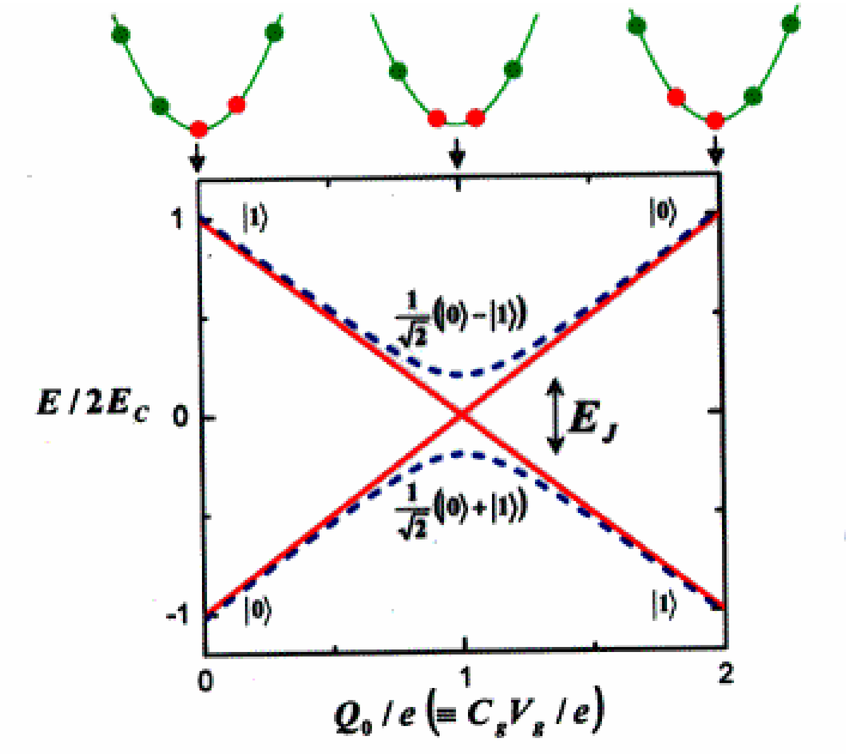
- At certain values of the offset charge, single Cooper pair states $|0\rangle$ and $|1\rangle$ are eigenstates of H
- More interestingly, at half integer multiples the eigenstates are $|0\rangle \pm |1\rangle$



Vion, D., *Josephson Quantum Bits based on a Cooper Pair Box*, 2004.

# Charge state mixing

- Operate near a gate bias which allows for only two states
- Energy separation at the microwave level



Nakamura, ITP conference on Nanoscience, 2001.

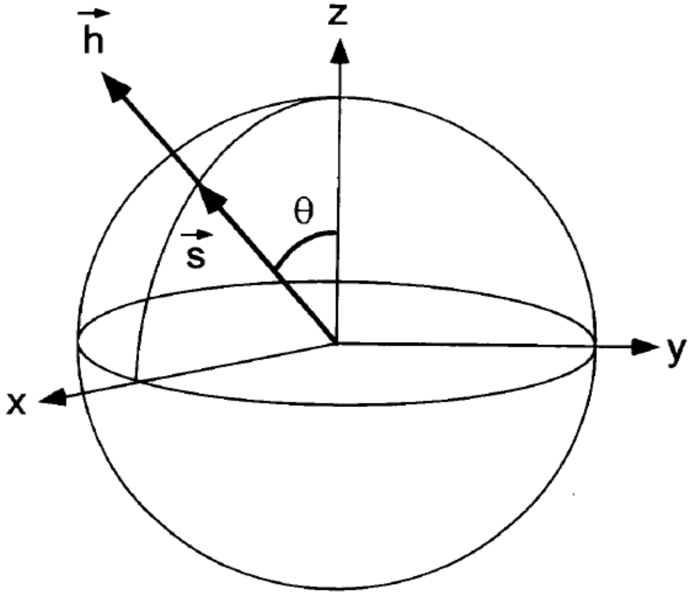
# Reduced Hilbert Space

- If the energies in the system restrict the box to the lowest two charge states, the Hamiltonian becomes

$$\hat{H} = 4E_C\left(n_g - 1\right)\sigma_z - \frac{E_J}{2}\sigma_x$$

$$\begin{aligned}\sigma_x &= \left|0\right\rangle\left\langle 1\right| + \left|1\right\rangle\left\langle 0\right| \\ \sigma_z &= \left|0\right\rangle\left\langle 0\right| - \left|1\right\rangle\left\langle 1\right|\end{aligned}$$

- Control is achieved by manipulating the Josephson energy

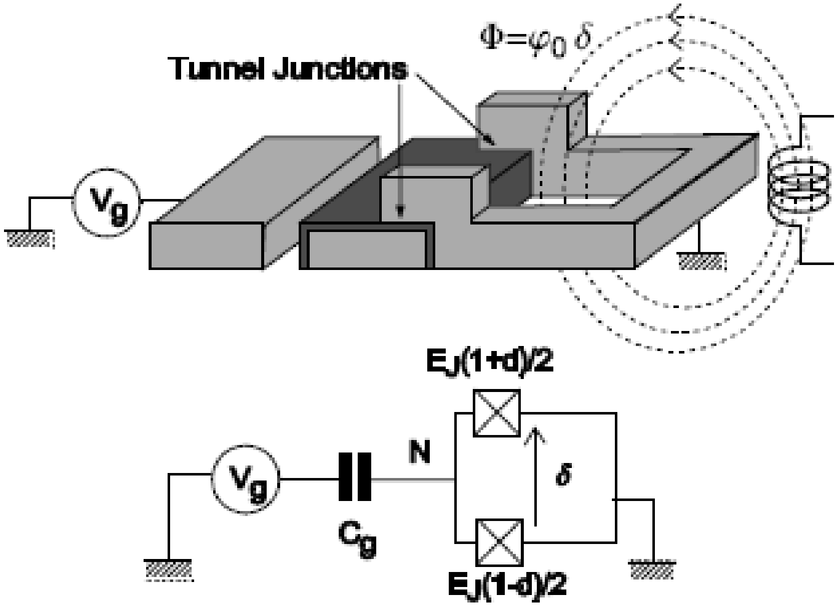


Bouchiat, et. al., Physics Scripta T76, 1998.

# Splitting the box – the real scheme

- Have to gain some degree of control over $E_J$
- Solution – thou shalt split the box in twain Clark, Proc. IEEE 77, 1208 (1989)
- Now, the flux determines Josephson Energy

$$E_J(\Phi_{ext}) = E_J \cos(\pi\Phi_{ext}/\Phi_o)$$

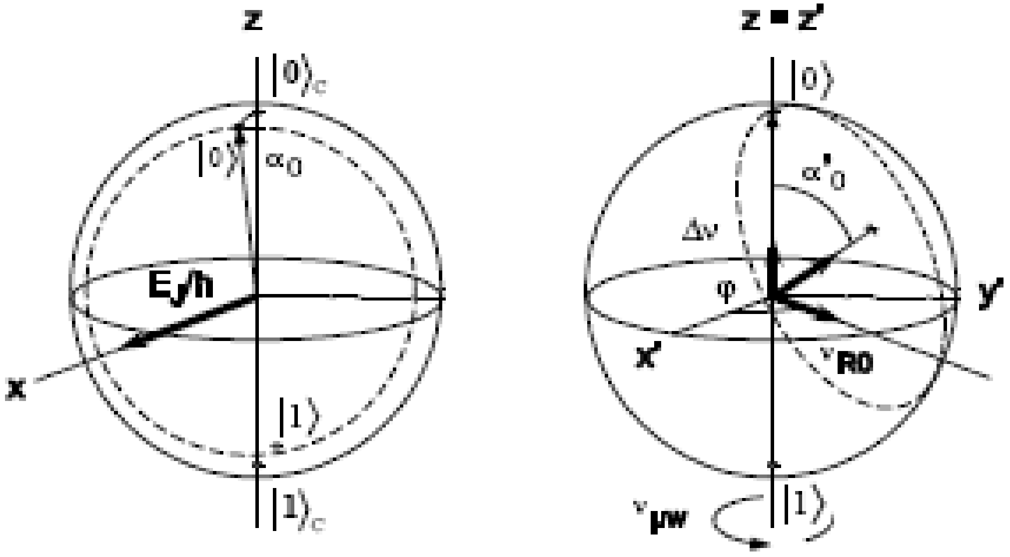


Vion, D., *Josephson Quantum Bits based on a Cooper Pair Box*, 2004.

# Where are we?

- We have a macroscopic state with quantized excitations
- By manipulating Coulomb and Josephson energy scales, we can operate in an effective two-state regime
- By adding a second junction, we can manipulate the coupling between the states
- Sounds like the mid-90’s

# DC vs. microwave control

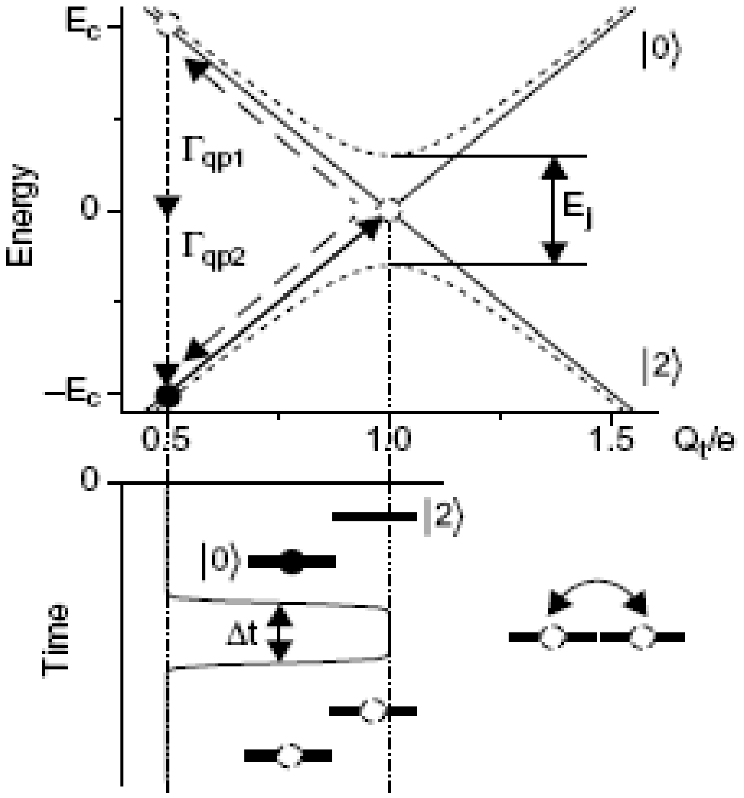


Vion, D., *Josephson Quantum Bits based on a Cooper Pair Box*, 2004.

- At the left, an approach shown in 1999 to be successful by Nakamura, and at the right, the approach developed by Devoret with “Quantronium”

# First coherent control

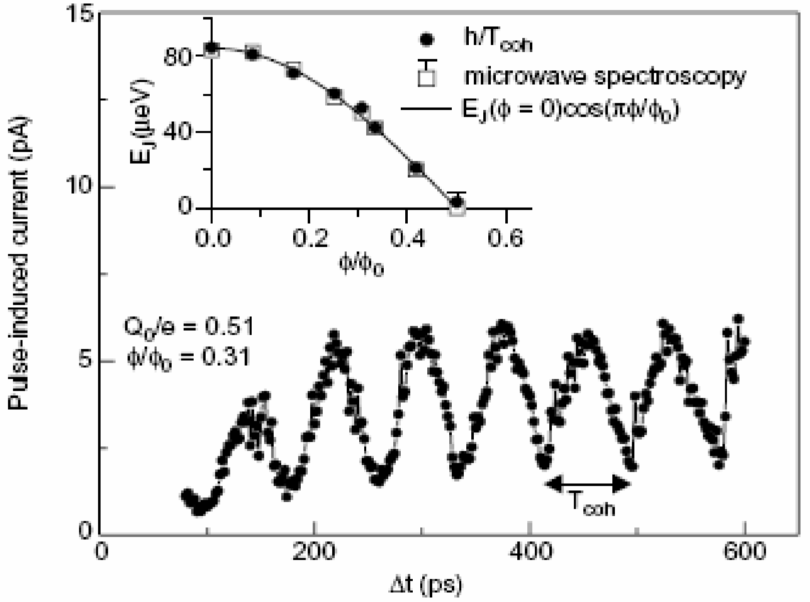
- Published in Nature, 1999 by Nakamura at NEC labs, Japan
- The circuit starts out in the ground state, with the control parameter far to left. A DC pulse brings the two states into resonance for a time Δt. Afterwards, the system is allowed to decay.



Nakamura, Y., et. al., *Nature* **398**, 768.

# Rabi oscillations

- Junction can be tuned with an external flux
- The magnitude of the Josephson energy determines in what regime the circuit operates
- Large source of decoherence is quantum fluctuations in the offset charge



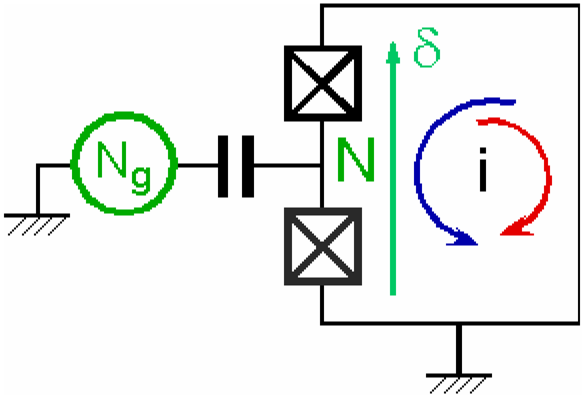
Nakamura, Y., et. al., *Nature* **398**, 768.

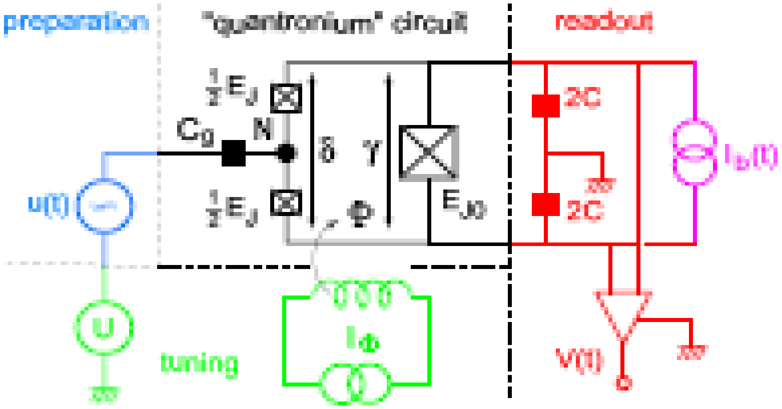
# Quantronium (2001)

- Tries to escape the charge fluctuation by not using it for readout
- Rather relies on supercurrent

$$\hat{I} = \frac{2e}{\hbar}\frac{\partial \mathbf{H}}{\partial \theta}$$

which has different sign in each of the charge states

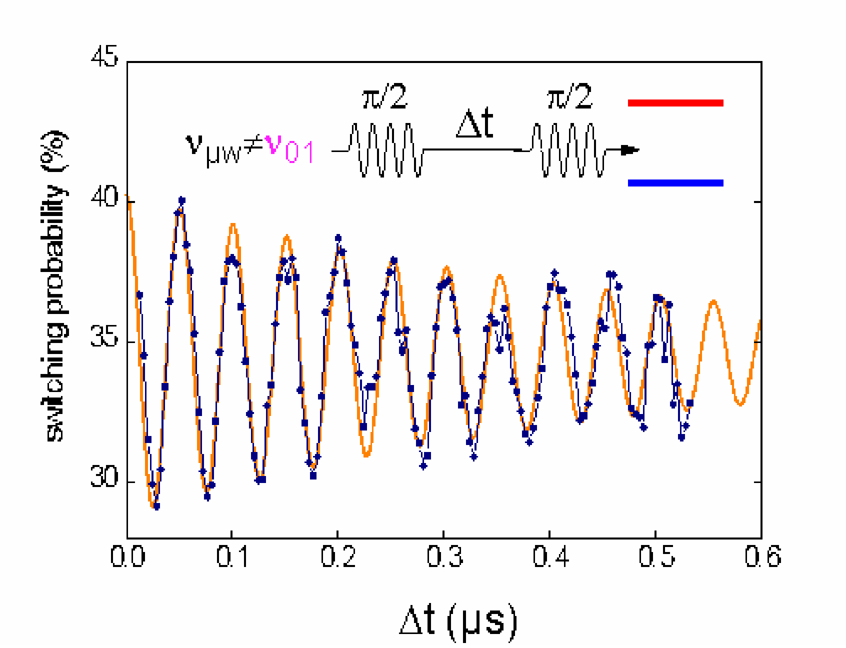




Benenti, G. and G. Casati, *Europhysics News* (2005) Vol. 36 No. 1

# Decoherence time from Ramsey fringe measurement

- Fits to a decaying exponential oscillation give a decoherence time of ~500 ns
- With energy separations on the typical 1-10GHz scale, this corresponds to several thousand bit flips before dephasing (has since improved)



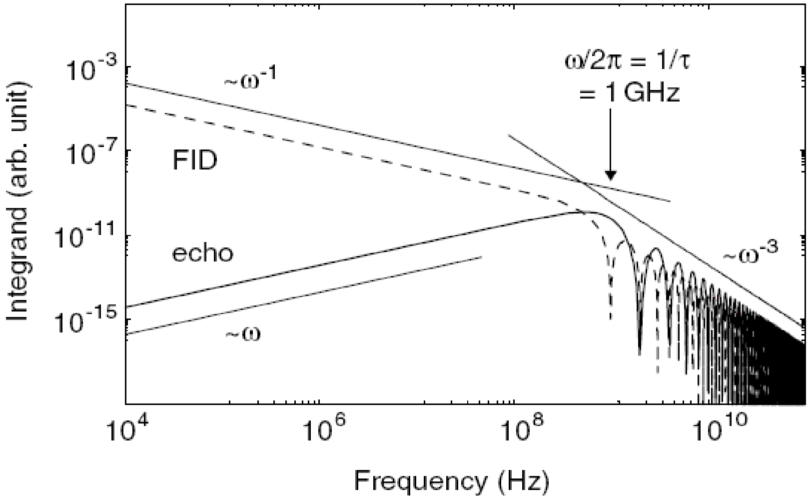
http://www-drecam.cea.fr/drecam/spec/Pres/Quantro/Qsite/projects/qip/ramsey.gif

# Decoherence sources

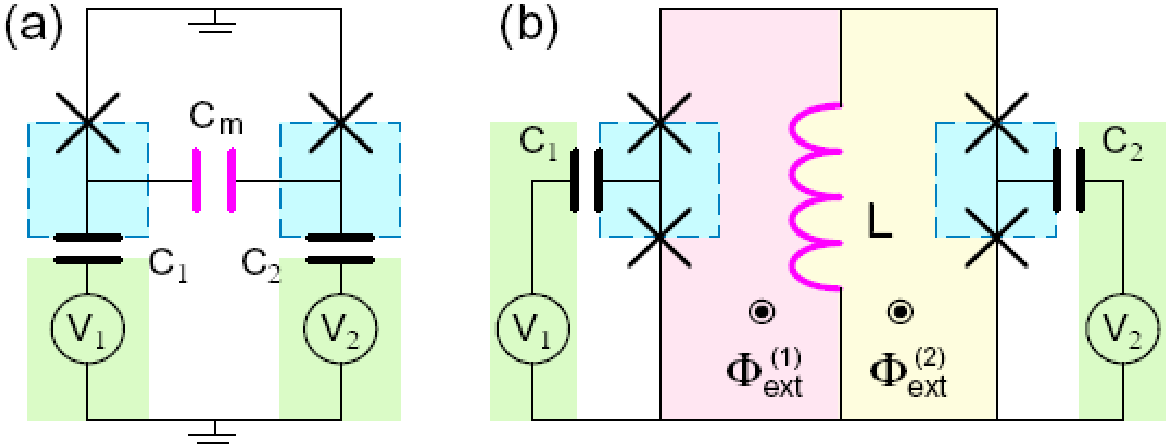
- 1/f noise due to offset charge fluctuations that arise from biasing to operate at the degeneracy point
- Voltage fluctuations from gate impedance
- Magnetic flux noise
- Readout back action
- Internal noise in the tunneling gate due to imperfections

# Decoherence continued

- Times are approaching microseconds
- Decoherence quality factor ($Q=2\pi T_{dec}\omega_{01}$) approaching $10^5$
- Either increase speed (smaller junctions) or decrease noise (cleaner junctions, quieter electronics, ?)



# The last ingredient – two qubit gates



You, J. Q. and F. Nori, Physics Today, 2005.

- Couple capacitively (a) or inductively (b)
- Both have their own difficulties, the first arising from the difficulty of controlling capactiance, the second from stray flux

# C-NOT gate

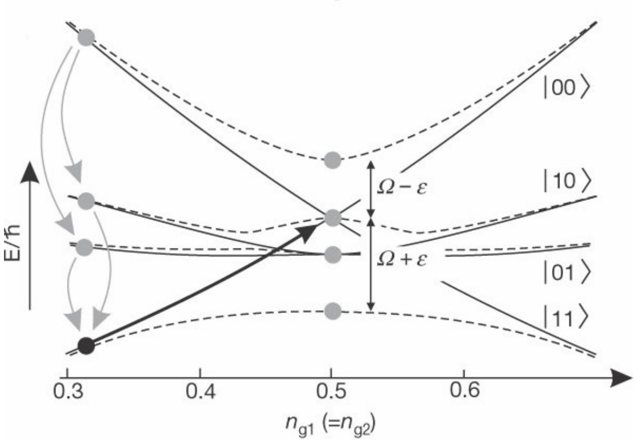
- Operate both qubits at the degeneracy point (gate offset charge of ½)
- Manipulations are made by adjusting fluxes in the interbit term of the Hamiltonian

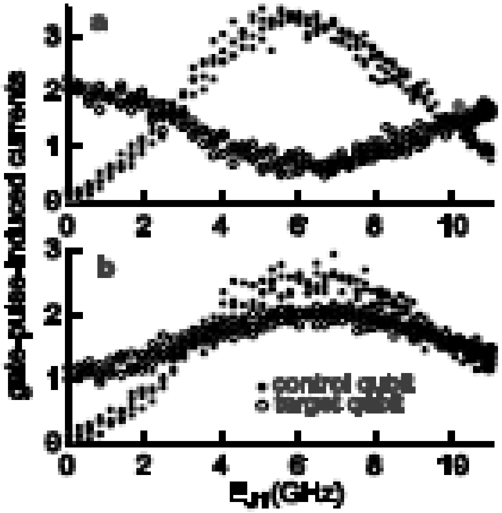
$$H = -E^*_{J1}\,\sigma_x^{(1)} - E^*_{J2}\,\sigma_x^{(2)} + \chi\,\sigma_x^{(1)}\,\sigma_x^{(2)}$$

- The four eigenvalues change with the changing coupling, but the states do not → viable two-bit gate operated by microwave pulses

# Experimental verification

- Qubit 1 is prepared in a pure state far from the resonance point
- Proper biasing brings all four states to the degeneracy point, where the superposition of all four states evolves for a certain time
- On decay, the probe current will determine the coefficients of the superposition





Yamamoto, T. et. al., *Nature* **425** (2003).

# Now where are we?

1. Viable qubit – Charge states
2. Initialization – success rates of >90%
3. Long decoherence time – mediocre ($Q \sim 10^5$)
4. Universal set of gates – only recently and with low fidelity
5. Readout – charge states or critical current
6. Covert to flying qubits X
7. Transmit flying qubits X

# Current research

- Nakamura – NEC Japan – CPB
- Devoret – Yale – Quantronium
- Esteve, Bouciat – Saclay, France – Quantronics
- Kouwenhoven – Delft – CPB with persistent current readout
- Schoelkopf – Yale – CPB
- Simmonds – NIST Boulder – CPB
- Nori – Michigan – Theory
- Schon – Karlsruhe – Theory
- Bruder – Basel, CH – Theory