Problem 17.6:

Evaluate the commutator $[x, p_x^2]$ by applying the operators on an arbitrary test function, $f(x)$. Use the theorem of problem 1 where $x = \hat{A}$, and $B = \hat{C} = p_x$, and see if the derivation is easier or harder? Compare your result with problem 4, which gives $[x^2, p_x] = i2\hbar x$, and comment on the parallel of the results.

Problem 17.15

Apply the Heisenberg uncertainty principle (HUP) to estimate the zero point energy for the particle in the box. You might take a moment to see what the exact answer is for the particle in the ground state in the box, and that the HUP is just satisfied. In this problem the bounds on $x$ and $p$ are a little bit larger so that the estimates are above the uncertainty. The problem is nice because you can estimate values for the uncertainties just based on plausibility arguments and come pretty close to the exact answers. You can use forms 17.8 and 17.9 to do the problem. See text for details.

Problem 13.29 and 13.30, now applied to spin.

You already did these problems but now, knowing about spin, you can go back and rework the problem.

I recast 13.29 in the following form. Let’s define two matrices

$$S_z = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{and} \quad S_x = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

and given the four vectors: $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

Show whether each vector is an eigen vector the two operators $S_x$ or $S_z$, and where it is an eigenvector, give the eigen value. (Later we will put Planck’s constant into the definitions of these operators.)

Now the point is to realize that two of the vectors are eigenvectors of $S_x$ and two go with $S_z$. And more importantly you will find that the set of (two) eigenvalues for $S_x$ are the same as those for $S_z$. What does that mean? How does that relate to how the Stern-Gerlach apparatus worked when each magnet set up (the operator) split the beam into two distinct answers?

Now show that $S_x$ and $S_z$ do not commute, and that they make a new operator defined by: $[S_z, S_x] = iS_y$

Find the operator (matrix) $S_y$: 
Now show that $[S_x, S_y] = iS_z$.

By the symmetry of what you have what would you guess for the commutator: $[S_y, S_z]$?