Constants and integrals you may need

\[ h = 6.63 \times 10^{-34} \text{ J s} \]
\[ \hbar = 1.05 \times 10^{-34} \text{ J s} \]
1 amu = 1.66 x 10^{-27} kg

Speed of light = 3.00 x 10^8 m s\(^{-1}\)

\[ \int_0^x x^n e^{-ax} dx = \frac{n!}{a^{n+1}} \]

\[ \int_0^a \sin \left( \frac{n\pi x}{a} \right) \sin \left( \frac{m\pi x}{a} \right) dx = \frac{a}{2} \delta_{nm} \]

\[ \int x \cos^2 x \, dx = \frac{x^2}{4} + \frac{1}{4} x \sin 2x + \frac{1}{8} \cos 2x \]

\[ \int \cos^2 x \, dx = \frac{1}{2} x + \frac{1}{4} \sin 2x \]

\[ \int x^2 \sin^2 x \, dx = \frac{x^3}{6} - \left( \frac{x^2}{4} - \frac{1}{8} \right) \sin 2x - \frac{x \cos 2x}{4} \]

\[ \int \sin^2 x \, dx = \frac{1}{2} x - \frac{1}{4} \sin 2x \]

\[ \int x^n e^{-ax} \, dx = \frac{n!}{a^{n+1}} \quad (a > 0, \, n \text{ positive integer}) \]

\[ \int x^{2n} e^{-ax^2} \, dx = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^{n+1}a^n} \sqrt{\frac{\pi}{a}} \quad (a > 0, \, n \text{ positive integer}) \]

\[ \int x^{2n+1} e^{-ax^2} \, dx = \frac{n!}{2 a^{n+1}} \quad (a > 0, \, n \text{ positive integer}) \]

\[ dV = r^2 \sin \theta \, dr \, d\theta \, d\phi \text{ in spherical coordinates} \]

NOTE: You must show your work to receive credit!
1) Calculate the average distance from the nucleus of an electron in the 2s orbital for the H atom. Use the normalized radial wavefunction

$$R_{2s}(r) = \frac{1}{2\sqrt{2}} \left( \frac{1}{a_0} \right)^{3/2} \left( 2 - \frac{r}{a_0} \right) e^{-r/2a_0}.$$
2) Calculate the probability that a particle in a one-dimensional box of length \( L \) \((0 < x < L)\) will be found between 0.49 \( L \) and 0.51 \( L \) when it is in its ground state. The normalized total energy eigenfunctions are given by \( \psi_n(x) = \sqrt{\frac{2}{L}} \sin \left( \frac{n\pi x}{L} \right) \). What would you expect for a classical particle? Why?
3) Identify the hydrogen atom wavefunctions shown in the plots below. Justify your answers.
4) Reply briefly to the questions or statements raised in the next four parts below.

a) Use the figure below to explain how an optical resonator leads to a laser linewidth that is less than the Doppler broadened linewidth.

![Resonator transmission diagram](image1)

b) Why are two separate laser pulses needed to achieve the isotope separation illustrated in the figure below?

![Isotope separation diagram](image2)
c) Using the figure below explain why the calculated probability for finding a particular value for the amplitude of a quantum mechanical harmonic oscillator will approach the classical result for large quantum numbers.

\[ x \]

\[ V(r) \]

\[ r_e \]

\[ D_e \]

\[ D_o \]

d) Why does the potential energy function \( V(r) \) for a diatomic molecule deviate from the harmonic potential \( V = \frac{1}{2} k r^2 \) at large \( r \)? Explain how this affects the spacing between adjacent energy levels for large values of the quantum number.
5a) Because the distance for the principle maximum follows the order 3d<3p<3s, one might expect the the n = 3 energy levels for multielectron atoms to follow the same order. Explain using the figure below why they follow the opposite order.

5b) Using the information about orbital energies in the table below, explain why atomic radii decrease in going from Be to F.

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<th>H</th>
<th>Li</th>
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<th>B</th>
<th>C</th>
<th>N</th>
<th>O</th>
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</table>
5c) Using the information about orbital energies in the table above, explain why the electronegativity increases in going from Be to F.

6) Calculate the average value of the linear momentum for the ground state of the harmonic oscillator. The normalized total energy eigenfunction is given by \( \psi_0(x) = \left( \frac{\alpha}{\pi} \right)^{1/4} e^{-\frac{1}{2}ax^2} \).