Chemistry 455B
First Midterm
October 21, 1994

Constants and integrals you may need

$h = 6.63 \times 10^{-34}$ J s

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int \sin^2 x \, dx = \frac{1}{2}x - \frac{1}{4}\sin 2x$$

$$\int_0^L \sin\frac{n\pi x}{L} \cos\frac{m\pi x}{L} dx = 0$$

1. (30 pts)

a) Using the complex representation of sin and cos, show that

$$\sin\left(\theta + \frac{3\pi}{2}\right) = -\cos\theta$$

b) Determine whether $y^2 - \frac{2}{3}$ is an eigenfunction of the operator

$$2y\frac{d}{dy} - (2-y^2)\frac{d^2}{dy^2}$$

c) Plot the function $y(x,t) = a \sin(kx - vt)$ as a function of x for a constant $t > 0$. Take v to be positive. What is the value of y when $x = 0$? For what positive value of x does y go through its first zero?

2. (40 pts) Respond to the following statements or questions. Try to stay within the allocated space.

a) An eigenfunction is an acceptable wave function, but a wave function is not necessarily an eigenfunction of a given quantum mechanical operator.

b) You are told to determine the momentum of a particle in a box. Repeatedly making the measurement N times on the same box will give you the same set of answers as if you had carried out one measurement each on N identically prepared boxes.

c) Explain how the de Broglie relation can be used to determine whether quantum or classical mechanics is appropriate for a given problem.

d) What experimental observation concerning the photoelectric effect could you use to support the particle view of light?

3. (30 pts) The wave function describing a particle in a one dimensional box with the potential

$$V(x) = 0 \quad 0 < x < L$$
$$= \infty \quad \text{otherwise}$$
$$\text{is } \psi(x) = A \sin\frac{2\pi x}{L} + B \sin\frac{4\pi x}{L}$$

You need not normalize this function.

a) Is this function an eigenfunction of the kinetic energy operator? Explain your answer.

b) With the system in the state described by $\psi(x)$, what values of the kinetic energy will you determine in an individual measurement?

c) What is the expectation value of the kinetic energy for the system described by $\psi(x)$? Leave your answer in term of A, B and fundamental constants.