Problem Set 1 A (due Friday, 5PM 1/6/12)

Problems in Text and Zumdahl, Ch 9.

Q1) A typical fire log weighs about 7 kilograms (about 15 pounds). How many fire logs would you need to heat the air (without worrying about heating the walls and roof) in a "typical" two bedroom house (3,000 square feet) assume the ceiling is 8'? Make appropriate assumptions about the composition of wood and the heat released and the heat capacity of air. Assume a typical winter day in Seattle, where the temperature outside is 40F and you want the house to be 70F.

Q1A) Follow-up: Heating a house requires heating some part of the walls too. Use the same house as above, and assume you need to heat the walls inside out to the insulation. So heat 5/8" Sheetrock from 40 to 70F. Sheet Rock is used to line the inside of the house; assume a single story, square footprint; and look up or estimate the heat capacity of sheet rock. Include in your estimate of the amount of sheetrock what is on the ceiling and an equivalent material on the floor. Find the ratio of the heat needed for the sheet rock to the heat needed for the air inside the house. How important is heating the walls compared to heating the air?

Q2) Food is a good example of a chemical reaction running a biological engine to do productive work (at constant pressure and temperature). A Hershey's chocolate bar has about 200 Cal. How many chocolate bars would it take for you to run (or walk) up the stairs of the Columbia Tower to the top? (You can assume 50% efficiency in use of the food). The Columbia tower is 788 feet of vertical climb (69 flights of stairs). You can do the big climb 13 days after our 456 final. To learn more for example see:
http://www.llswa.org/site/PageNavigator/BC_homepage

Q3) Angel Falls, in Venezuela, has a height of 979 m (3,212 ft, 0.6 mi).
Assume that all of the heat stays in the water (i.e. the water falls adiabatically, which actually is not a bad assumption, given the massive amount of water, were it not for the spray). How much hotter is the water at the bottom of the falls than the top. Explain any assumptions you feel you have to make to get an answer.

Q4) What is the mass of a single weight (assume one step work) you need to put on a bike pump to take the air in the pump from 1 atmosphere to 9 atmospheres? Assume the diameter of the piston of the bike pump is 3 cm (about an inch). (The SI unit of pressure is the Pascal, and is generally the most convenient unit of pressure when determining energy.) How much force does this mass generate? You can assume that the heat during compression leaves the gas and the temperature stays constant.

Q5) We have talked about a function of two (independent) variables. Consider this function:

$$g = g(x, y) = 6x^2 y^3$$

A) Is it a function of x and y?

B) Now examine the same function in the difference form:

$$dg = M(x,y)\,dx + N(x,y)\,dy$$

And find M and N.

C) Verify that the cross derivative of g are the same in either order (Euler's test for exactness), i.e. show that:

$$\left(\frac{\partial\left(\frac{\partial g}{\partial x}\right)_y}{\partial y}\right)_x = \left(\frac{\partial\left(\frac{\partial g}{\partial y}\right)_x}{\partial x}\right)_y$$

For this function g, by explicitly evaluating both sides independently.

D) Using Euler's test for exactness show whether it is possible to write g as a function of x, and y for the case where

$$dg = 4xy\cdot dx + 3x^2 y\cdot dy$$

And construct the original function $g = g(x,y)$.

E) Using Euler's test for exactness show whether it is possible to write g as a function of x, and y for the case where

$$dg = 16xy^3\cdot dx + 24x^2y^2\cdot dy$$

And construct the original function $g = g(x,y)$.

Q6) When you take your foot off the gas of your car it will roll to a stop eventually due to air and other sources of friction. Assume that you noticed that the speedometer showed exponential time decay: $V = V_o e^{-rt}$ and the rate of slowing constant $r = 0.1 \,/\, min$.

A) Assume you are traveling at 60 mph, how far will you coast after you take your foot off the gas? It is rather interesting to realize that it takes an infinite amount of time to travel a finite distance.

B) The slowing rate assumes it takes about 10 minutes to reduce your velocity by about half. Can you find the exact half-life for slowing? What would you estimate from your experience on the highway?

C) Explain how this problem is connected to the central tenet of calculus that:

$$\Delta z = \int dz = \int \left(\frac{\partial z}{\partial x}\right)_y dx$$