Chemistry 456 (10:30AM, Bagley 154)

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Problem Set 1 A KEY (due Friday, 5PM 1/6/12)

Problems in Text and Zumdahl, Ch 9.

Q1) A typical fire log weighs about 7 kilograms (about 15 pounds). How many fire logs would you need to heat the air (without worrying about heating the walls and roof) in a "typical" two bedroom house (3,000 square feet) assume the ceiling is 8'? Make appropriate assumptions about the composition of wood and the heat released and the heat capacity of air. Assume a typical winter day in Seattle, where the temperature outside is 40F and you want the house to be 70F.

A carbohydrate is worth 4 C/gram, you could look this up or read the label of a candy bar or any other carbohydrate based food. So one log produces

$\Delta U_{\log} = 4 \cdot 4.18 \cdot 10^3 \cdot 7 \cdot 10^3 J = 117 MJ / \log$

Now need the heat capacity of air. $C_{m,air} = \frac{5}{2} R = 20.5 \, {}^{J}\!/_{mole-K}$ and the Pressure is 1 Atm, the Vol is 24,000 ft3 (We assume the air stays in the house, so C is C at constant volume, but of course the extra air is driven out as the temperature goes up at one atm pressure.)

So the number of moles is: $n = \frac{PV}{RT} = \frac{1Atm * 24 \cdot 10^3 ft^3 / .035 \, {}^{ft^3}\!/_{liter}}{0.082 \cdot 300} = 28 \cdot 10^3 moles$

The molar heat capacity times the temperature change, $\Delta T = 17K$, is the heat needed to warm the air. $\Delta U_{Air} = n \cdot C_{m,air} \cdot \Delta T = 28 \cdot 10^3 \cdot 20.5 \cdot 17 = 9.8 MJ$. So it requires only a fraction of a single log. So why does it take a bunch of logs to heat a house?

Q1A) Follow-up: Heating a house requires heating some part of the walls too. Use the same house as above, and assume you need to heat the walls inside out to the insulation. So heat 5/8" Sheetrock from 40 to 70F. Sheet Rock is used to line the inside of the house; assume a single story, square footprint; and look up or estimate the heat capacity of sheet rock. Find the ratio of the heat needed for the sheet rock to the heat needed for the air inside the house. How important is heating the walls compared to heating the air? With the walls include the floor and the roof.

Looking on the web for wallboard or sheetrock I came upon the properties of gypsum, which has a density of about 800 kg/m^3, and a heat capacity around 1 k J/Kg/K. The volume of the sheet rock is

The floor and the ceiling each contribute 2 times 3,000 ft^2, and the 4 walls are about another 2,000 ft^2, so the volume of sheet rock is about 417ft^3 of sheet rock at a heat capacity of 800 kJ/m^3/K. So we can compare the total heat capacity of the air with the total heat capacity of the walls (including floor and ceiling).

$$C_{Sheetrock} = C_P \cdot V = 800 \, {}^{kJ}\!/\!_{K-m^3} \cdot \frac{417 \, ft^3}{0.035 \, {}^{ft^3}\!/\!_{liter} \cdot 10^3 \, {}^{l}\!/\!_{m^3}} = 9.5 MJ / K$$

Compare with the total heat capacity of the air:

$$C_{Air} = n \cdot C_{m,air} = 28 \cdot 10^3 \cdot 20.5 \, {}^{J}\!/\!_{K} = 575 \, {}^{kJ}\!/\!_{K}$$

The sheet rock then has a heat capacity that is about 20 times that of the air, so it would require several logs to heat the walls, not counting heat loss then to the outside air.

Q2) Food is a good example of a chemical reaction running a biological engine to do productive work (at constant pressure and temperature). A Hershey's chocolate bar has about 200 Cal. How many chocolate bars would it take for you to run (or walk) up the stairs of the Columbia Tower to the top? (You can assume 50% efficiency in use of the food). The Columbia tower is 788 feet of vertical climb (69 flights of stairs). You can do the big climb four days after our 456 final. To learn more for example see: http://www.llswa.org/site/PageNavigator/BC_homepage

Here we compare the work we do with the calories in food that we burn. The fundamental unit of carbohydrates is about 4 C/gram of food, and for typical fats it is about 9 C/gram. The C is a kilo calorie, so 1 Cal is worth 4.18 kJ of energy. The work is force through a distance in SI/MKS units (the easiest to use: $w = mgh = 75kg * 9.8 * 240m = 2.77 \cdot 10^5 J$, and a single candy bar is worth $200 * 4.18 \cdot 10^3 \cdot \frac{1}{2} = 4.18 \cdot 10^5 J$, so we end up with about ½ a candy bar is the energy used to get up the entire Columbia tower. There is a lot of energy in a candy bar.

Q3) Angel Falls, in Venezuela, has a height of 979 m (3,212 ft, 0.6 mi).
Assume that all of the heat stays in the water (i.e. the water falls adiabatically, which actually is not a bad assumption, given the massive amount of water, were it not for the spray). How much hotter is the water at the bottom of the falls than the top. Explain any assumptions you feel you have to make to get an answer.

The tradeoff is the potential energy to heat, which goes through kinetic energy. Due to conservation of energy the KE of the water at the bottom of the falls is equal to the chane in PE as a result of falling. Then the KE is abruptly halted by hitting the bottom and then is converted into heat and that goes to temperature through the heat capacity of water.
The heat capacity of water

$$mgh = C\Delta T$$

The heat capacity of water is nominally 1 cal per gram/K, or 4.18 J/gram/K. This is how the calorie was standardized in the first place. For convince then set m=1 gram

$$\Delta T = \frac{mgh}{C} = \frac{1 \cdot 10^{-3} \cdot 9.8 \cdot 979}{1 \cdot 4.18} K = 2.3K$$

Q4) What is the mass of a single weight (assume one step work) you need to put on a bike pump to take the air in the pump from 1 atmosphere to 9 atmospheres? Assume the diameter of the piston of the bike pump is 3 cm (about an inch). (The SI unit of pressure is the Pascal, and is generally the most convenient unit of pressure when determining energy.) How much force does this mass generate? You can assume that the heat during compression leaves the gas and the temperature stays constant.

We add a mass that produces a pressure of 8 Atm over an area of 3 cm diameter bike pump.

$$P = 8Atm = 8 \cdot 1.01 \cdot 10^5\, Pa = F / Area$$

$$Area = \pi r^2 = 7 \cdot 10^{-4} m^2$$

$$F = P * Area = mg$$

$$m = \frac{P * Area}{g} = \frac{8.08 \cdot 10^5 \cdot 7 \cdot 10^{-4}}{9.8} = 57kg$$

This is getting close to what a person weighs. No wonder it is hard to pump up a bike tire. The really high pressure ones go to about this pressure.

Q5) We have talked about a function of two (independent) variables. Consider this function:

$$g = g(x, y) = 6x^2 y^3$$

A) Is it a function of x and y?

Yes, it is clearly a function of x and y.

B) Now examine the same function in the difference form:

$$dg = M(x, y)\,dx + N(x, y)\,dy$$

And find M and N.

$$dg = M(x, y)\,dx + N(x, y)\,dy$$

$$M(x, y) = \left(\frac{\partial g}{\partial x}\right)_y = 12xy^3$$

$$N(x, y) = \left(\frac{\partial g}{\partial y}\right)_x = 18x^2 y^2$$

C) Verify that the cross derivative of g are the same in either order (Euler's test for exactness), i.e. show that:

$$\left(\frac{\partial \left(\frac{\partial g}{\partial x}\right)_y}{\partial y}\right)_x = \left(\frac{\partial \left(\frac{\partial g}{\partial y}\right)_x}{\partial x}\right)_y$$

For this function g, by explicitly evaluating both sides independently.

$$\left(\frac{\partial\left(\frac{\partial g}{\partial x}\right)_y}{\partial y}\right)_x = \left(\frac{\partial M(x,y)}{\partial y}\right)_x = \left(\frac{\partial 12xy^3}{\partial y}\right)_x = 36xy^2$$

$$\left(\frac{\partial\left(\frac{\partial g}{\partial y}\right)_x}{\partial x}\right)_y = \left(\frac{\partial N(x,y)}{\partial x}\right)_y = \left(\frac{\partial 18x^2y^2}{\partial x}\right)_y = 36xy^2$$

D) Using Euler's test for exactness show whether it is possible to write g as a function of x, and y for the case where

$$dg = 4xy\cdot dx + 3x^2y\cdot dy$$

First test if this is a function of x and y:

$$\left(\frac{\partial\left(\frac{\partial g}{\partial x}\right)_y}{\partial y}\right)_x = \left(\frac{\partial\left(\frac{\partial g}{\partial y}\right)_x}{\partial x}\right)_y$$

$$\left(\frac{\partial(4xy)}{\partial y}\right)_x \stackrel{?}{=} \left(\frac{\partial(3x^2y)}{\partial x}\right)_y$$

$$4x \stackrel{?}{=} 6xy$$

This g is not a function of x and y. So one cannot construct a function.

And construct the original function $g = g(x,y)$.

E) Using Euler's test for exactness show whether it is possible to write g as a function of x, and y for the case where

$$dg = 16xy^3\cdot dx + 24x^2y^2\cdot dy$$

And construct the original function $g = g(x,y)$.

$$\left(\frac{\partial\left(\frac{\partial g}{\partial x}\right)_y}{\partial y}\right)_x = \left(\frac{\partial\left(\frac{\partial g}{\partial y}\right)_x}{\partial x}\right)_y$$

$$\left(\frac{\partial 16xy^3}{\partial y}\right)_x \stackrel{?}{=} \left(\frac{\partial 24x^2y^2}{\partial x}\right)_y$$

$$48xy^2 \stackrel{?}{=} 48xy^2$$

Yes, it is a function of x, y so by inspection $g = 8x^2y^3 + C$. The constant, C, must be independent of x and y.

Q6) When you take your foot off the gas of your car it will roll to a stop eventually due to air and other sources of friction. Assume that you noticed that the speedometer showed exponential time decay: $V = V_o e^{-rt}$ and the rate of slowing constant $r = 0.1 \,/\!_{min}$.

A) Assume you are traveling at 60 mph, how far will you coast after you take your foot off the gas? It is rather interesting to realize that it takes an infinite amount of time to travel a finite distance.

We usually try to use Newton's laws in the differential form, but the integral from is much easier for a problem like this.

$$v = \frac{dx}{dt}$$

The force on the object can be compute from $F = ma = m\frac{dv}{dt} = -mrv$, notice the minus sign suggest the force is in the opposite direction from the motion (it opposes the motion). But this does not get us the position. The most straight forward way is to use the definition of velocity and integrate both sides:

$$\int_0^\infty v\,dt = \int_0^\infty \frac{dx}{dt}dt = \int_o^x dx = \Delta x$$

$$\Delta x = \int_0^\infty v\,dt = \int_0^\infty V_o e^{-rt}dt = \frac{V_o}{r} = \frac{60mph}{.1/\min}\cdot\frac{1}{60\min/hr} = 10miles$$

B) The slowing rate assumes it takes about 10 minutes to reduce your velocity by about half. Can you find the exact half-life for slowing? What would you estimate from your experience on the highway?

$$\frac{1}{2} = e^{-r\tau}$$

$$\ln 2 = r\tau$$

$$\tau = 7\min$$

The half life, is the time to drop to ½ the initial velocity, and that is 7 minutes. Hard to know if this is about how long it takes in a car: at low speeds it seems like I could coast forever; on the highway one can't afford to just try coasting, but I would be surprised if it took more than a minute to drop to 30mph. But then at high speeds Stoke's law is replaced by Rayleigh's law.

C) Explain how this problem is connected to the central tenet of calculus that:

$$\Delta z = \int dz = \int \left(\frac{\partial z}{\partial x} \right)_y dx$$

Z is the position, and the velocity is dx/dt so the chain rule lets us write the problem in exactly this form.

A comment about the velocity expression:
Viscous drag was quantified by Newton, even though it does not often occur in introductory physics courses. The effect of viscous drag is to produce a force on the car proportional to the car's velocity (Stoke's Law). This gives us a differential equation for the velocity: Newton's Law;

$$F = ma = m\frac{dv}{dt}$$

$$F = -\eta v = m\frac{dv}{dt}$$

$$v = v_o e^{-rt}$$

$$-\eta v = -rmv$$

$$r = \frac{\eta}{m}$$