Chemistry 456 (10:30AM Bagley 154)

Winter 2012 Bruce H. Robinson

Problem Set 2 A (due Tuesday, 9 PM 1/10/12)

Q1) Consider stretching a spring by hanging a weight on the end of it. The spring obeys Hooke's Law, which is the force on the spring that causes an extension. $F_s = -k_s x$ where k_s is the spring force constant and the length beyond the rest length is x. We extend the spring by hanging a weight of mass, m, on the end. The spring will stretch and stop at equilibrium when the forces balance: $F_S + F_g = 0$, or $mg = k_s \ell$.

A) The energy stored in the spring at rest (as E=P.E. the potential energy) is $E = \frac{1}{2} k_s x^2$. Derive this relation from the Force Law for a spring, using Newton's definition of the force, $F = -\frac{d(P.E.)}{dx}$, where P.E. is the potential energy of the system. [Hint: Use the integral theorem of calculus here the same as you use in Q6 of HW 1A.]

B) Show that when a single weight is placed on the end of the spring, the energy stored in the spring (as potential energy) is less than the work done to stretch the spring. Decide on the best way to compare the work and stored energy. The energy stored in the spring (the system) is that given in A, and the work (in the environment) is that determined by the displacement of the mass in the gravitational field.

The stored energy is $\Delta E = \frac{1}{2} k_s \ell^2$, and the work $w = mgh$ is the work done in the environment.

C) Use the first law of thermodynamics to determine how much heat is given off or taken up. Explain whether the heat comes out or goes into the system. How does the sign of the heat help you determine the direction of heat flow (or transfer)?

Q2) Overview: In the following question, we will add a second mass (also of mass m) to the spring, and then compute the work done in this step and the energy stored in the spring. And then we will remove the second mass and return the spring to its original extension of length ℓ, and determine the work done by the spring and the change in stored energy in the spring. We will also determine the heat transferred from the system to the surroundings (or visa versa) in each step and then compute the net work and net heat for the two steps.

A) Determine the distance the string is stretched in terms of ℓ when the spring reaches its new equilibrium. (Remember the mass is 2m.)

B) How much work was done by the environment on the spring to make this move from position $x = \ell$ to its new position (don't do the total just do this one move)?

C) What is the increase in internal energy stored in the spring?

D) How does the increase in internal energy compare to the work done?

E) Using the first law determine the amount of heat transferred between the system and environment. From the sign of the heat transfer, explain whether the system releases or absorbs heat. (Exo or endo thermic??)

F) Now remove one weight, and repeat steps B thru E. as F thru I. Of importance is to notice the sign on the work and comment on who did what work.

J) Determine the net stored energy, net work and heat change for the sum of these two steps.

K) compare the next heat and work with what you would get if you just lowered a mass, m, distance ℓ, and removed the spring entirely. Why is this a valid comparison?

Q3) We don't often compare energy stored and used mechanically with that stored electrically. A standard 12V car battery can easily run a 60W bulb for 10 hours. (Actually most batteries are rated at 100Ah) which means it would run for 20 hours.
A) Compute the energy in a battery if it runs at 12V for 10 hours and lights a 60W bulb the entire time.
B) Now let's compare this with mechanical energy. How high would you have to lift a 1 Ton mass (assume this weighs 1000 kg) to contain the same energy as in the battery?

C) Suggest whether a battery or a mass can store energy more efficiently?

D) As a related idea, consider the Grand Coulee dam on the Columbia, which holds water up 550 feet (168 meters) and provides a total power of almost 7,000 MWatts. To sustain that type of output, how much water must spill over the dam and run the turbines to generate that much power? A watt is a Joule per second so you will want your answer in gallons per second. (A more convenient unit might be tons of water; where water has a density of 1 g/cc and 1000kg, a metric ton, is 1 nearly one US ton).

Q4) We are also looking at properties of functions that are (exact or inexact) differentials, or are (for exact differentials) indeed functions of two (or more) variables.

The first property we considered last time was Euler's test of whether a differential was exact. This test was based on the idea that if g is a function of x and y, then the cross derivatives of g should give the same answer regardless of order. Review the previous homework on this to refresh your memory. Now we consider another property of exact differentials, and that is that the integral from point A to point B is independent of path. This should be true because $\Delta g = \int dg = g(B) - g(A)$. Because of our differential form it is often hard to come up with the function to determine Δg directly from the function.

Here we are demonstrating that exact differentials yield the same value of the integral regardless of path, and those that are inexact differentials give results for the integrals that depend on path.

In the first example, consider the differential $dg = 6xy^3dx + 9x^2y^2dy$. You can use Euler's test to determine whether this is an exact differential. If it is, then the integral should be independent of path. To test this we will integrate over several different paths and see if we get the same answer.

A) In all cases below, integrate from point A being (x=0 , y=0) to point B where (x=1, y=1). Construct the function g of x and y and determine what Δg must be for this function between these two points.

B) Integrate Along the path where x=y, and see if you get the same answer as the first case.
C) Now integrate along the path where $x=y^2$.
D) Now integrate along the edge, first from A to C and then from C to B, where C is the point (x=0,y=1). [That is: Integrate along y from 0 to 1 holding x fixed at x=0, and then integrate from x=0 to x=1 holding y fixed at y=1.] Often integrating along an edge can be the most straightforward way to integrate along a path.

Q5) We repeat question 4 but now for the differential $dg = 4xydx + 3x^2ydy$ test whether this an exact or inexact differential. Integrate from A to B as specified in the previous problem (parts B,C and D) along the same paths as above and see if any of the paths yield the same answer. For part A you will have to explain why you could not construct the function for this case.