Problem Set 2B (due Day, 9PM Friday, 1/13/12)

Q2) We already showed that when a single weight is placed on the end of the spring the stored energy is less than the work done to stretch the spring. What is the factor between the work and energy?. Eg For a one step process, $\Delta E = \frac{1}{2} k_s \ell^2$, and the work $w = mgh$ where $h = \ell$, and the force balance requires that $mg = k\ell$. In this case then the work is twice the energy stored in the spring, $w = 2\Delta E$ (for a single step). In the next step we added a second weight and the spring was stretched an additional distance and the total energy stored in the spring is $\Delta E = 2k_s \ell^2$. The total work is the sum of the first and second steps. $w = mg\ell + 2mg\ell = 3mg\ell = 3k_s\ell^2$, the work is still larger than the energy but the fraction is less: $w = \frac{3}{2}\Delta E$ (for two steps). So the question is (to be done in the parts below): If we repeat the process N times, the work applied and the internal energy will be related by $w = f\Delta E$, so what is $f = f(N)$? And can we identify the reversible work this way?

Q1a) What is the total distance the spring will be stretched by adding N weights, each of mass m, onto the spring?

Q1b) Determine the amount of energy stored in the spring when N weights of mass m are loaded onto the spring.

Q1c) What is the total work done by adding N weights, each of mass m, sequentially on the spring as the spring is stretched? Write your answer in terms of N and $k_s \ell^2$.

Q1d) Now find the relation $w = f\Delta E$, and, thereby, f as a function of N.

Q1e) What is the heat that is generated by stretching the spring?

Q1f) From your results, show that the work that goes into the spring equals the energy stored in the spring for a large number of weights.

Q1g) We are interested in the reversible work, and the reversible work is that wherein the system (i.e. spring length) can be changed infinitesimally forward or backward at every step along the way with the addition (or removal) of an infinitesimally small amount of mass. So to do this we imagine that the individual masses are very small, $m \to dm$ and the length increase per step is also small: $\ell \to d\ell$, and the number of masses is such that the total length is $L = Nd\ell$. Compute the internal stored energy in the limit as N goes to infinit and dl goes to zero so that the product is a finite length. Compute the work done in this same limit (the reversible work); compare the reversible work with the internal energy stored.

Q1h) Compute the heat that is generated for the case of reversible work.

Q1i) Compare the equation of state, EoS, for the spring with the equation of state for an ideal gas, I.G.. Make as close a comparison as you can, finding corollaries in the spring EoS to all the variables in the I.G. EoS. Note similarities and differences, and comment on the role of temperature in the EoS of the spring.

Q2a) The lead bricks we used in class to compress the gas are about 14kg each. The pistons are each about 1cm across, and there are 2 of them. How much pressure does one brick generate on one (or each) cylinder?

Q2b) The piston is set to 40 mls before the lead brick is added. After the lead brick is added the piston reads about 25 mls. What pressure would produce that change? Assume the temperature of the gas remains constant in equilibrium with the room, and that the ideal gas EoS can be used. How does the pressure needed to compress the gas that far compare with the estimate of the pressure generated by the lead brick in part a?

Q2c) What is the work the brick did on the gas in part b? How much heat was transferred in this process and what direction did the heat flow. Use the relation of ΔU to temperature for the I.G.

Q2d) Remove the brick and let the gas re-expand back to it's original volume (40 mls). How much work did the gas do? Compare the amount of work the gas did under expansion to the amount of work the system did to compress the gas (part c).

Q2e) Under re-expansion how much heat flows (at constant temperature) and in what direction?