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Bioen 326 2012 FINAL EXAM

Rules: Closed Book Exam: Please put away all notes and electronic devices

Reminders:

- Wherever possible, show equations you use to answer questions.
- We give **partial credit** for the various steps needed to solve a problem, so include equations and steps. If you combine steps, but you get the right answer, you still get credit for all the steps.

Equations Provided on the Exam:

- $G = \frac{E}{2(1+\nu)}$
- $\sigma_{av} = \frac{\sigma_x+\sigma_y}{2}$
- $R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau^2}$
- $\sigma_{x\theta} = \sigma_{av} + \frac{\sigma_x-\sigma_y}{2}\cos(2\theta) + \tau\sin(2\theta)$
- $\tau_\theta = -\frac{\sigma_x-\sigma_y}{2}\sin(2\theta) + \tau\cos(2\theta)$
- $\sigma_{1,2} = \sigma_{av} \pm R$
- $\tau_{MAX} = R$
- $\epsilon_x = \frac{1}{E}\sigma_x - \frac{\nu}{E}\sigma_y - \frac{\nu}{E}\sigma_z$
- $\gamma_{xy} = \frac{1}{G}\tau_{xy} = \frac{2(1+\nu)}{E}\tau_{xy}$
- $\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}((1-\nu)\epsilon_x + \nu\epsilon_y + \nu\epsilon_z)$
- $\tau_{xy} = G\gamma_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy}$
- $e = \frac{\Delta V}{V_0} = \epsilon_x + \epsilon_y + \epsilon_z$
- $\delta = \int_0^L \frac{N(x)}{E(x)A(x)}dx$
- $k_a = \frac{EA}{L}$
- $k_t = \frac{GI_p}{L}$
- $k_c = \frac{3EI}{L^3}$
- $\int_A y^2 dA = I$
- $I = \frac{\pi}{4}r^4$
- $I = \frac{H^3W}{12}$
- $I = \frac{H^3W}{36}$, (neutral plane is at H/3 from flat edge).
- $\int_A r^2 dA = I_p$
- $I_p = \frac{\pi}{2}r^4$
- $\tau = \frac{Tr}{I_p}$
- $\kappa = M/EI$
- $\sigma_x(x,y) = -\frac{M(x)}{I}y$
- rectangle: $\tau(x,y) = \frac{V(x)}{2I}\left(\frac{H^2}{4} - y^2\right)$, max $= \frac{3V(x)}{2A}$
- circle: $\max(\tau(x)) = \frac{4V(x)}{3A}$
- hollow circle: $\max(\tau(x)) = \frac{4V(x)}{3A}\frac{(r_2^2+r_2r_1+r_1^2)}{r_2^2+r_1^2}$
- $\frac{d^2y}{dx^2} = \frac{M(x)}{EI}$
- $\sigma_L = \frac{pr}{2t}$
- $\sigma_C = \frac{pr}{t}$
- $\sigma = \frac{pr}{2t}$
- $\sigma_t = 0$
- $\sigma_C = \frac{pr}{t} = \frac{pr_0(1+\epsilon_C)}{t_0(1+\epsilon_T)}$
- $F_c = \frac{EI\pi^2}{L^2}$ (2 pins); $F_c = \frac{EI\pi^2}{4L^2}$ (solid support)
- $r = \sqrt{I/A}$; L/r is slenderness ratio
- $f(r) = \frac{k_BT}{L_p}\left(\frac{1}{4\left(1-\frac{r}{L_0}\right)^2} - \frac{1}{4} + \frac{r}{L_0}\right)$
- $f \approx \frac{3k_BT}{2L_p}\frac{r}{L_0}$
- $r_{rms} = \sqrt{2L_0L_p}$
- $E = \frac{U''(r_0)}{r_0} = \frac{k}{r_0}$ if linear
- $\frac{P_2}{P_1} = \exp\left(\frac{-\Delta G^0}{k_BT}\right)$
- $k_BT = 4e-21\,J$ at room & body temp.
- $\Delta G(f) = \Delta G^0 - f \cdot \Delta x(f)$
- $f_{eq}\Delta x^0 + f_{eq}^2\left(\frac{1}{k_2} - \frac{1}{k_1}\right) - \Delta G^0 = 0$
- $K_{eq}^0 = \frac{P_2}{P_1} = \exp\left(\frac{-\Delta G^0}{k_BT}\right)$
- $K_{eq}(f) = K_{eq}^0 \exp\left(\frac{f\Delta x^0}{k_BT}\right)$
- $P_2 = \frac{K_{eq}}{1+K_{eq}}$
- Voigt $E\epsilon + \eta\frac{d\epsilon}{dt} = \sigma$
- Maxwell $\sigma + \frac{\eta}{E}\frac{d\sigma}{dt} = \eta\frac{d\epsilon}{dt}$
- Kelvin $\sigma + \frac{\eta}{E_S}\frac{d\sigma}{dt} = E_P\epsilon + \eta\left(\frac{E_P+E_S}{E_S}\right)\frac{d\epsilon}{dt}$
- Transforms:
  - $\delta(t-\tau) = e^{-\tau s}$
  - $t \cdot \phi(t) = 1/s^2$
  - $(1-e^{-at})\phi(t) = \frac{a}{s(s+a)}$
  - $\sin(\omega t)\,\phi(t) = \frac{\omega}{s^2+\omega^2}$
  - $\cos(\omega t)\,\phi(t) = \frac{s}{s^2+\omega^2}$

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Use this page if you need extra room for any questions. Just write “continued on empty page” or something so we know to look here.

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1. **(25 points)** A material is crystalline, and the ionic bonds in the material can be approximated with the bond energy function $U(r) = a(r - r_0)^2$, with $a = 200\ N/m$, and $r_0 = 2 * 10^{-10} m$. However, this energy function fails at $r = 2.04 * 10^{-10} m$, where the bond breaks.
   a. Do you expect this material to be viscous, viscoelastic, or elastic? Explain briefly.

   b. What is the approximate Young's modulus of this material? Explain or show your calculations.

   c. Do you expect to see elastic or plastic deformation prior to material failure? Explain.

   d. Do you expect this material to exhibit linear elasticity, strain hardening, or yielding? Explain.

   e. At approximately what strain to you expect this material to fail? Explain or show your calculations.

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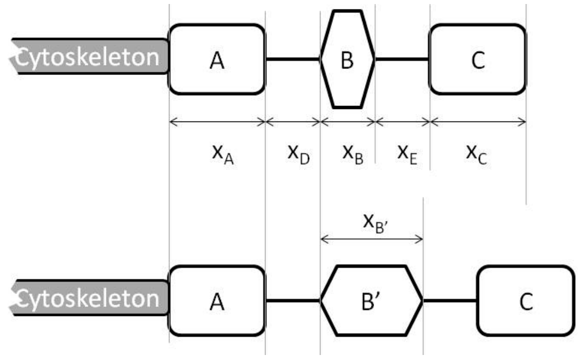
2. **(25)** As shown in the figure, an adaptor protein (white) connects through domain A to the cytoskeleton (gray). Domain B undergoes a conformational change between an inactive form B and active form B′. The various domains and peptide linkers between domains have lengths xA through xE as shown in the figure. Domain B changes length from x_B to x_B' upon activation; no other domain or linker length is affected by activation. If the cytoskeleton is not applying mechanical force to the adaptor protein, the free energy of domain B is G in the inactive conformation and G′ in the active conformation.



a. (12) For this adaptor protein to be a mechanotransducer, list the things that would need to be true. *(Consider how this protein must interact with other proteins or processes in the cell, and any quantitative information, such as what values must be positive or negative. Don't list things that were already stated above, but do indicate requirements about lengths regardless of whether this appears true in the figure.)*

b. (13) If these things are all true, derive an equation (in terms of only the known values above) that indicates the fraction of time (i.e. probability) that the mechanosensor is active when the cytoskeleton applies a force F to the adaptor protein.

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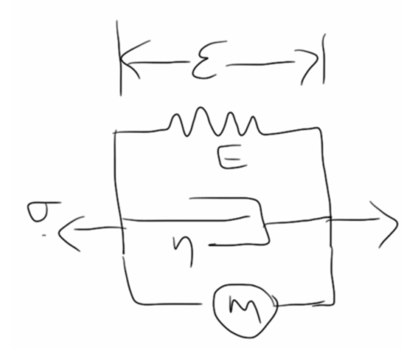
3. **(50)**Muscle fiber can be modeled as a stress-producing motor in parallel and series with viscous and elastic elements as shown in the figure. In the sign convention we use for this class, the contractive stress produced by the motor element is considered positive since it has the same effect as tensile stress applied to the element. This stress is independent of the strain or rate of strain of the element. That is, the element equation for the motor is $\sigma = \sigma_m(t)$, where $\sigma_m(t)$ is an input function that is controlled by the nerves. Since it does not depend on system behavior, think of it like a time-variable parameter.



a) (15 points) Build a mathematical model from this diagram, to derive an equation for the strain of the fiber, ϵ, vs the external stress applied to the fiber, σ.

a. (25 points) Using the differential equation you derived above, derive an algebraic equation for the strain, $\epsilon(t)$, of the muscle fiber, when the muscle starts to contract at t = 0 with a force of M, so $\sigma_m(t) = M\phi(t)$, and there is an external tensile stress on the fiber of $\sigma(t) = W\phi(t)$.

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b. (10 points) interpret your solution for $\epsilon(t)$ to explain the requirements on the parameters $M, W, E, and\ \eta$ for the muscle fiber to be able to contract in this experiment. *If you didn't get a final solution, describe the requirement is for contraction, and explain how you would answer the question if you had a solution for* $\epsilon(t)$.

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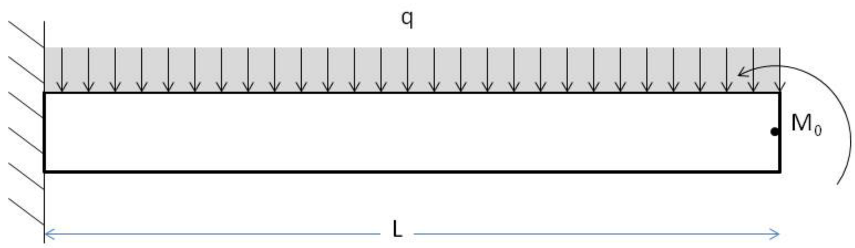
4. **(40)** A beam is attached to a solid support at $x = 0$, and is free at $x = L$. There is a uniform external load, q, across the beam. An external moment M_0 is applied at $x = L$, in the direction shown below. The cross section of the beam is a square with sides of length H, and the Young's modulus is E. You can assume that the deflection is small.



a) (15) Find the internal shear force, V(x) and internal bending moment, M(x) everywhere in the beam.

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b) (15) In the special case where $q = \frac{8M_0}{L^2}$, draw the shear force and bending moment diagrams

c) (10) in the same special case, identify the values of x and y that you would need to test for failure.

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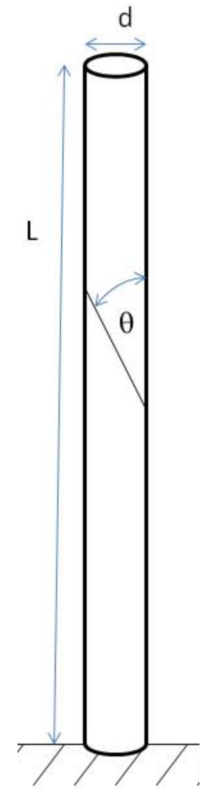
5. **(30)** A column has a weld as shown in the figure. What is the maximum force, F, that the column can withstand? Express your answer in terms of the following values:
   θ = angle of the weld, from the vertical
   L = length
   d = diameter
   E = the Young's modulus
   USS_w = ultimate shear stress allowed parallel to the weld
   UTS_w = ultimate tensile stress allowed normal to the weld



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6. **(30)** A thin-walled cylindrical pressure vessel of radius r and thickness t is subjected simultaneously to an internal pressure p and a compressive force F at the ends, as shown in the figure. The material has linear elasticity with Young's modulus E and Poison ratio v, and you can assume small deformations. How much force F, do you need to apply in order to produce zero longitudinal strain in the cylinder walls?