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1. **(25 points)** A material is crystalline, and the ionic bonds in the material can be approximated with the bond energy function $U(r) = a(r - r_0)^2$, with $a = 200\ N/m$, and $r_0 = 2 * 10^{-10} m$. However, this energy function fails at $r = 2.04 * 10^{-10} m$, where the bond breaks.

   a. Do you expect this material to be viscous, viscoelastic, or elastic? Explain briefly.

2 → elastic, because crystalline materials are not
2: explain viscous/don't reform bonds, or no water.

   b. What is the approximate Young's modulus of this material? Explain or show your calculations.

2 for → $E \sim \frac{k}{r_0}$. here, $k = \frac{d^2U}{dr^2} @ r_0 = 2a = 400\ N/m$

3 for k: 2a

$r_0 = 2 \times 10^{-10} m$

1 for r_0

$E \sim \frac{400\ N/m}{2 \times 10^{-10} m} = 200 \times 10^{10}\ N/m^2$

2 for final #

$= 2000$ GPa. (oops - miscalculated >10x when writing question)

   c. Do you expect to see elastic or plastic deformation prior to material failure? Explain.

2: elastic
2: explain

elastic only; crystalline materials are brittle / bonds don't reform, etc.

   d. Do you expect this material to exhibit linear elasticity, strain hardening, or yielding? Explain.

2: linear
2: explain

linear - bonds break while the $U(r)$ still holds, & $U(r)$ has constant $\frac{d^2U}{dr^2}$

   e. At approximately what strain to you expect this material to fail? Explain or show your calculations.

2: very small or 0.02
2: explain
1: full calculation.

very small strains since crystalline are brittle

or break at strain $\varepsilon = \frac{\Delta r}{r} = \frac{0.04 \times 10^{-10} m}{2 \times 10^{-10} m}$

$= 0.02$.

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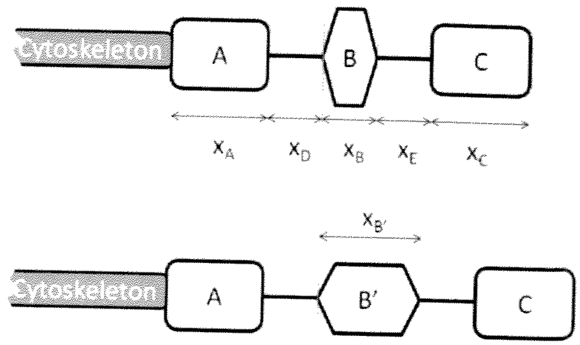
2. **(25)** As shown in the figure, an adaptor protein (white) connects through domain A to the cytoskeleton (gray). Domain B undergoes a conformational change between an inactive form B and active form B′. The various domains and peptide linkers between domains have lengths xA through xE as shown in the figure. Domain B changes length from x_B to x_B' upon activation; no other domain or linker length is affected by activation. If the cytoskeleton is not applying mechanical force to the adaptor protein, the free energy of domain B is G in the inactive conformation and G′ in the active conformation.



a. (12) For this adaptor protein to be a mechanotransducer, list the things that would need to be true. *(Consider how this protein must interact with other proteins or processes in the cell, and any quantitative information, such as what values must be positive or negative. Don't list things that were already stated above, but do indicate requirements about lengths regardless of whether this appears true in the figure.)*

$\boxed{2}$ for mech. $\boxed{2}$ for stretching B.

1) Connect to mechanical pathway in a way that stretches B. eg. domain C binds adhesive protein.

$\boxed{2}$ for biochem $\boxed{2}$ for conf dependent.

2) Initiates signaling pathway when B changes conformation (biochemical)

$\boxed{2}$ for G's in words or #'s $\boxed{2}$ for x_B's

3) Quantitative #'s right to be inactive w/o force ($G < G'$) & activatable by force / force favors ~~longer one of~~ active conf. ($x_B' > x_B$)

Note: or all opposite – so force inactives is ok too.

b. (13) If these things are all true, derive an equation (in terms of only the known values above) that indicates the fraction of time (i.e. probability) that the mechanosensor is active when the cytoskeleton applies a force F to the adaptor protein.

L →

$$P_a(f) = \frac{K_{eq}(f)}{1+K_{eq}(f)}, \text{ where } K_{eq}(f) = \frac{P_a(f)}{P_i(f)} = K_{eq}^0 \exp\left(\frac{f\,\Delta x^0}{k_BT}\right),$$

4 → where $\Delta x^0 = x_B' - x_B$

4 → and $K_{eq}^0 = \exp\left(\frac{-\Delta G^0}{k_BT}\right) = \exp\left(\frac{G'-G}{k_BT}\right)$

combine or box -

1 → so (they don't need to combine; can box all "where")

$$P_a(f) = \frac{\exp\left(-\frac{G'-G}{k_BT}\right)\exp\left(\frac{f(x_B'-x_B)}{k_BT}\right)}{1+\exp\left(-\frac{G'-G}{k_BT}\right)\exp\left(\frac{f(x_B'-x_B)}{k_BT}\right)}$$

can simplify also:

$$= \exp\left(\frac{G-G'+f(x_B}{k_BT}\right.$$

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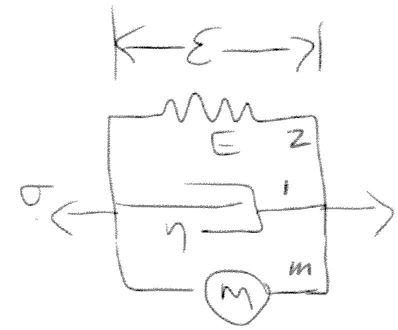
3. **(50)** Muscle fiber can be modeled as a stress-producing motor in parallel and series with viscous and elastic elements as shown in the figure. In the sign convention we use for this class, the contractive stress produced by the motor element is considered positive since it has the same effect as tensile stress applied to the element. This stress is independent of the strain or rate of strain of the element. That is, the element equation for the motor is $\sigma = \sigma_m(t)$, where $\sigma_m(t)$ is an input function that is controlled by the nerves. Since it does not depend on system behavior, think of it like a time-variable parameter.



a) (15 points) Build a mathematical model from this diagram, to derive an equation for the strain of the fiber, ϵ, vs the external stress applied to the fiber, σ.

4 $\sigma = \Sigma\sigma$ of the elements =

1 motor: $\sigma_m(t)$

4 dashpot $= \sigma_1(t) = \eta \frac{d\varepsilon(t)}{dt}$

4 elastic: $= \sigma_2(t) = E\,\varepsilon(t)$

2: combine $\sigma(t) = \sigma_m(t) + \sigma_1(t) + \sigma_2(t)$

$$\boxed{\sigma(t) = \sigma_m(t) + \eta\frac{d\varepsilon(t)}{dt} + E\varepsilon(t)}$$

a. (25 points) Using the differential equation you derived above, derive an algebraic equation for the strain, $\epsilon(t)$, of the muscle fiber, when the muscle starts to contract at t = 0 with a ~~force~~ stress of M, so $\sigma_m(t) = M\phi(t)$, and there is an external tensile stress on the fiber of $\sigma(t) = W\phi(t)$.

5 $\begin{cases} \mathcal{L}(\sigma_m(t)) = M/s \\ \mathcal{L}(\sigma(t)) = W/s \end{cases}$ $\quad \mathcal{L}(\varepsilon(t)) = X(s)$

I.C. are assume 0, so $\varepsilon(0) = 0$

10 for $\mathcal{L}$(ODE) $\mathcal{L}(\text{ODE}) = \frac{W}{s} = \frac{M}{s} + \eta s X(s) + E X(s)$

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5 to rearrange

$$\frac{W-M}{s} = (\eta s + E)\,X(s)$$

$$X(s) = \frac{W-M}{s(\eta s+E)}.$$ need form $\frac{a}{s(s+a)}$, with $a = E/\eta$

so $1/\eta$ on top & bottom.

& E on top & bottom

$$X(s) = \frac{W-M}{E} \cdot \frac{E/\eta}{s(s+E/\eta)}$$

5 to invert

$\mathcal{L}^{-1}\left(\frac{a}{s(s+a)}\right) = 1 - e^{-at}$, so

$$\boxed{\varepsilon(t) = \frac{W-M}{E}\left(1 - e^{-E/\eta\, t}\right)}$$

b. (10 points) interpret your solution for $\epsilon(t)$ to explain the requirements on the parameters $M, W, E,$ and η for the muscle fiber to be able to contract in this experiment. *If you didn't get a final solution, describe the requirement is for contraction, and explain how you would answer the question if you had a solution for $\epsilon(t)$.*

5 → contraction requires fiber to shorten, so need $\varepsilon(t) < 0$.

2 → Thus need to find values of W, M, E, η that make $\varepsilon(t) < 0$.

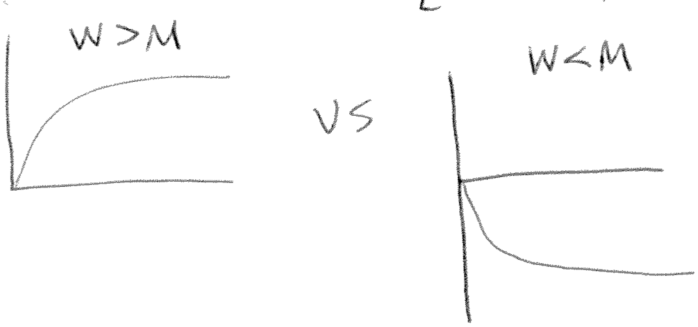
But E & η are always positive,

2 → so $e^{-E/\eta t}$ is < 1

so $1 - e^{-E/\eta t} > 0$

1 → final answer. so need $\frac{W-M}{E} < 0$, so need $\boxed{W < M.}$

or:



Thus, motor must outstress external stress.

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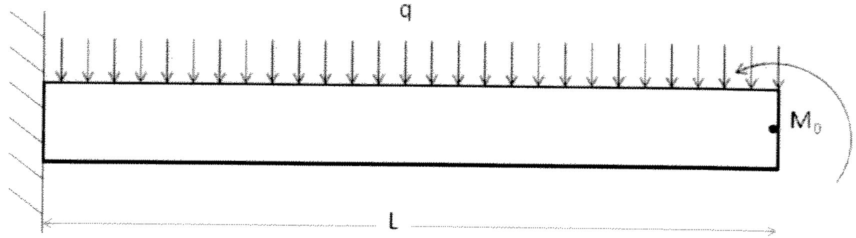
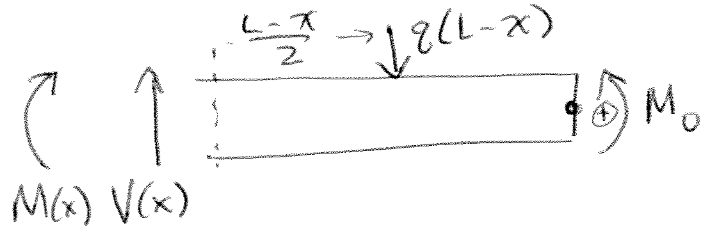
4. **(40)** A beam is attached to a solid support at $x = 0$, and is free at $x = L$. There is a uniform external load, q, across the beam. An external moment M_0 is applied at $x = L$, in the direction shown below. The cross section of the beam is a square with sides of length H, and the Young's modulus is E. You can assume that the deflection is small.



a) (15) Find the internal shear force, V(x) and internal bending moment, M(x) everywhere in the beam.



replace distributed load with $q(L-x)$ at centroid, $= \frac{L-x}{2}$ from cut & end.

$$\Sigma F_y = 0 : \boxed{V(x) = q(L-x)}$$

$$\Sigma M_z = 0 : -M(x) + \left(-\frac{L-x}{2}\right) q(L-x) + M_0 = 0$$

$$\boxed{M(x) = -\frac{q}{2}(L-x)^2 + M_0}$$

5pts for V(x) (-2pts for wrong sign)

5pts for q term in m(x) "

5pts for M_0 " "

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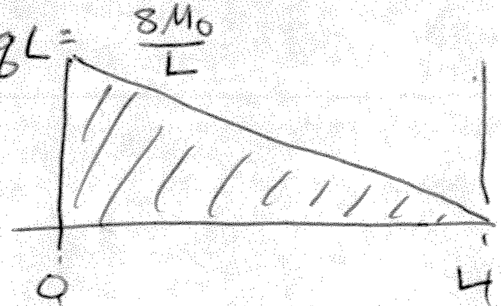
b) (15) In the special case where $q = \frac{8M_0}{L^2}$, draw the shear force and bending moment diagrams

$V(0) = q(L) = \frac{8M_0}{L^2}$

$V(L) = q(L-L) = 0$



2 pts for qL or $\frac{8M}{L}$ @ $x=0$

2 pt for 0 @ $x=L$

1 pt for strait line/correct plot

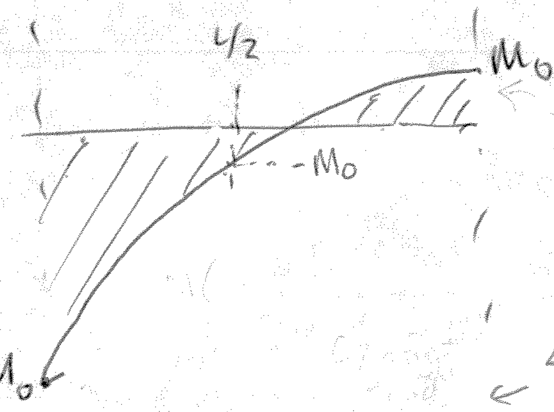
$M(x) = -\frac{8M_0}{L^2}(L-x)^2 + M_0$

$M(0) = -8M_0\frac{L^2}{L^2} + M_0 = -7M_0$

$M(L) = -8M_0(0) + M_0 = M_0$

$M(x)$ has extreme @ $x=L$ (max, since form $-ax^2$)

or $M(\frac{L}{2}) = -\frac{8M_0L^2}{L^2 \cdot 4} + M_0 = -2M_0 + M_0 = -M_0$



3 pts for x(0)

3 pts for x(L)

4 pts for decent curve w/ No no other max/min

c) (10) in the same special case, identify the values of x and y that you would need to test for failure.

5 Need to consider x where $V(x)$ is max for shear τ stress, which is @ $y=0$. so, at $(x=0, y=0)$

5 Need to consider where $M(x)$ is max and min. Actually, just $|M(x)|$ is greatest, since rod is symmetrical, so $x=0$, and y = extremes, $\pm H/2$.

$(0, H/2)$ & $(0, -H/2)$ consider σ_x

OK to say both $x=0$ & $x=L$ – always OK to test too many.

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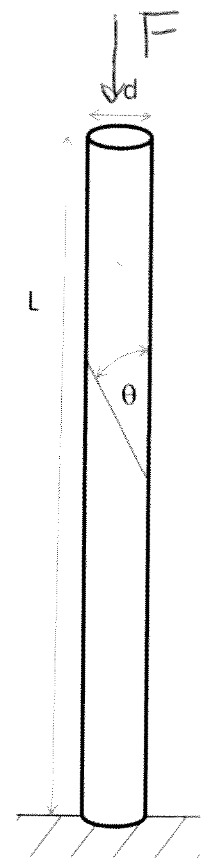
5. **(30)** A column has a weld as shown in the figure. What is the maximum force, F, that the column can withstand? Express your answer in terms of the following values:
θ = angle of the weld, from the vertical
L = length
d = diameter
E = the Young's modulus
USS_w = ultimate shear stress allowed parallel to the weld
UTS_w = ultimate tensile stress allowed normal to the weld



5 pts (check for buckling or assume none)

3 pts for right F_c

2 pts for I & combine

Buckling: $F_c = \frac{E I \pi^2}{L^2}$ since solid/free ends

$I = \frac{\pi}{4} R^4 = \frac{\pi d^4}{64}$

$$\boxed{F_c = \frac{E\pi^3 d^4}{256 L^2}}$$

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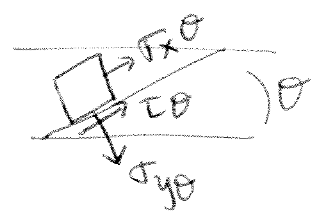
or Failure at Weld:

1 for Eqn, 2 for right values ($-F$, $R = d/2$) → $\sigma_x = -F/A = \frac{-F}{\pi R^2} = \frac{-4F}{\pi d^2}$ (<0, since compressive)

What is τ_W & σ_W, shear & tension on weld?

1 for Eqn → $\tau_W = \tau_\theta = \frac{\sigma_x - \sigma_y}{2} \sin(2\theta) + \tau \cos(2\theta)$



4 for right σ_x, σ_y, τ, & simplify: $= \frac{\sigma_x}{2} \sin(2\theta) = \frac{-4F}{2\pi d^2} \sin 2\theta$

5 pts for this & simplify → fails when $\tau_\omega = USS_w$; $\boxed{F = \frac{USS_w \cdot \pi d^2}{2\sin(2\theta)}}$

(sign of τ doesn't matter, so removed (−) to keep F compressive, since F>0 defined as compress.

5 pts for any logic excluding this

Also fails when $\sigma_W = UTS$, but:

$\sigma_W = \sigma_{y\theta} = \sigma_{x(\theta+90)} = \frac{\sigma_x}{2} - \frac{\sigma_x}{2}\cos(2\theta + 180) = \underbrace{\frac{\sigma_x}{2}}_{<0}\underbrace{(1-\cos)}_{>0} < 0$ so compress.

(or can just note that $\sigma_{av} = \frac{\sigma_x}{2} < 0$, $R = |\frac{\sigma_x}{2}|$, so never $\sigma_{x\theta} > 0$, or just that never get tension in pure uniaxial compression.)

2 points for "min of"

$$\boxed{\text{Thus, fails at } F = \min\left(\frac{E\pi^3 d^4}{256 L^2}, \frac{USS_w \pi d^2}{2\sin(2\theta)}\right)}$$

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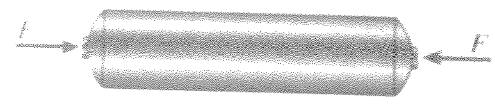
6. **(30)** A thin-walled cylindrical pressure vessel of radius r and thickness t is subjected simultaneously to an internal pressure p and a compressive force F at the ends, as shown in the figure. The material has linear elasticity with Young's modulus E and Poison ratio v, and you can assume small deformations. How much force F, do you need to apply in order to produce zero longitudinal strain in the cylinder walls?



15 pts — To get zero long. strain, need σ_L such that Hooke's law in 3D will result in $\varepsilon_L = 0$.

12 for Hooke's, 3 for $\varepsilon_L = 0$.

$$\varepsilon_L = \frac{1}{E}\sigma_L - \frac{\nu}{E}\sigma_C = 0$$

This is the main point of this "A" problem, so worth most points.

So $\sigma_L = \nu\sigma_C$.

10 → Now need to find σ_L & σ_C:

3 ⟶ $\sigma_C = \frac{pr}{t}$ regardless of F.

3 for $\frac{pr}{2t}$ — $\sigma_L = \frac{pr}{2t} + \left(-\frac{F}{A}\right)$ (stress due to compression is <0, but F is positive parameter)

↑ pressure ↑ compression

4 for $-\frac{F}{2\pi r t}$ (-2 for wrong sign)

$A = 2\pi r t$

5 to combine all to solve for F.

So $\left(\sigma_L =\right) \frac{pr}{2t} - \frac{F}{2\pi r t} = \nu\frac{pr}{t} \left(= \nu\sigma_C\right)$

Solve for F:

$$\frac{F}{2\pi r t} = \frac{pr}{2t} - \nu\frac{pr}{t}$$

$$F = \frac{2\pi r t\left(\frac{1}{2}-\nu\right)pr}{t} = 2\pi r^2\left(\tfrac{1}{2}-\nu\right)p$$

$$\boxed{F = \pi r^2(1-2\nu)p}$$