Bioen 326 2013 FINAL EXAM

Rules: Closed Book Exam: Please put away all notes and electronic devices

Reminders: We give partial credit, so include equations and steps.

Equations Provided on the Exam:

- $F_R = \int F_d(x)dx$ or $F_R = \int F_d(\vec{r})d\vec{r}$
- $x_R = \frac{\int xF_d(x)dx}{F_R}$, or $x_R = \frac{\int xF_d(\vec{r})d\vec{r}}{F_R}$
- $\sigma_{av} = \frac{\sigma_x+\sigma_y}{2}$
- $R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau^2}$
- $\sigma_{x\theta} = \sigma_{av} + \frac{\sigma_x-\sigma_y}{2}\cos(2\theta) + \tau\sin(2\theta)$
- $\tau_\theta = -\frac{\sigma_x-\sigma_y}{2}\sin(2\theta) + \tau\cos(2\theta)$
- $\sigma_{1,2} = \sigma_{av} \pm R$
- $\tau_{MAX} = R$
- $\epsilon_x = \frac{1}{E}\sigma_x - \frac{v}{E}\sigma_y - \frac{v}{E}\sigma_z$
- $\gamma_{xy} = \frac{1}{G}\tau_{xy} = \frac{2(1+v)}{E}\tau_{xy}$
- $\sigma_x = \frac{E}{(1+v)(1-2v)}((1-v)\epsilon_x + v\epsilon_y + v\epsilon_z)$
- $\tau_{xy} = G\gamma_{xy} = \frac{E}{2(1+v)}\gamma_{xy}$
- $e = \frac{\Delta V}{V_0} = \epsilon_x + \epsilon_y + \epsilon_z = \frac{1-2v}{E}(\sigma_x + \sigma_y + \sigma_z)$
- $\delta = \int_0^L \frac{N(x)}{E(x)A(x)}dx$
- $k_a = \frac{EA}{L}$; $F = k_a\delta$
- $k_t = \frac{GI_p}{L}$; $T = k_t\phi$
- $\int_A y^2 dA = I$
- $I = \frac{\pi}{4}r^4$
- $I = \frac{H^3W}{12}$
- $I = \frac{H^3W}{36}$, (neutral plane is at H/3 from flat edge).
- $\int_A r^2 dA = I_p$
- $I_p = \frac{\pi}{2}r^4$
- $\kappa = M/EI$
- $\sigma_x = N/A$
- $\tau(r) = \frac{T}{I_p}(r)$
- $\sigma_x(x,y) = -\frac{M(x)}{I}y$
- $\tau_{xy}(x,y) = \frac{V(x)}{2I}\left(\frac{H^2}{4} - y^2\right)$, $\max\left(\tau_{xy}(x)\right) = \frac{3V(x)}{2A}$
- $\max\left(\tau_{xy}(x)\right) = \frac{4V(x)}{3A}$
- $\max\left(\tau_{xy}(x)\right) = \frac{4V(x)}{3A}\frac{(r_2^2+r_2r_1+r_1^2)}{r_2^2+r_1^2}$
- $\frac{d^2v}{dx^2} = \frac{M(x)}{EI}$
- $\sigma = \sigma_L = \frac{pr}{2t}$
- $\sigma_C = \frac{pr}{t}$
- $F_c = \frac{EI\pi^2}{L^2}$ (2 pins); $F_c = \frac{EI\pi^2}{4L^2}$ (solid support); $F_c \cong \frac{2EI\pi^2}{L^2}$ (pin and solid support); $F_c = \frac{4EI\pi^2}{L^2}$ (2 solid supports);
- $r = \sqrt{I/A}$; L/r is slenderness ratio
- Voigt $E\epsilon + \eta\frac{d\epsilon}{dt} = \sigma$
- Maxwell $\sigma + \frac{\eta}{E}\frac{d\sigma}{dt} = \eta\frac{d\epsilon}{dt}$
- Kelvin $\sigma + \frac{\eta}{E_S}\frac{d\sigma}{dt} = E_P\epsilon + \eta\left(\frac{E_P+E_S}{E_S}\right)\frac{d\epsilon}{dt}$
- Transforms:
  - $L[\delta(t)] = 1$
  - $L[\phi(t)] = 1/s$
  - $L[t\cdot\phi(t)] = 1/s^2$
  - $L[e^{-at}\,\phi(t)] = \frac{1}{s+a}$
  - $L[(1-e^{-at})\emptyset(t)] = \frac{a}{s(s+a)}$
  - $L[\sin(\omega t)\,\phi(t)] = \frac{\omega}{s^2+\omega^2}$
  - $L[\cos(\omega t)\,\phi(t)] = \frac{s}{s^2+\omega^2}$
- $k_BT = 4.1e-21\,J$ at room temp.
- $f(r) = \frac{k_BT}{L_p}\left(\frac{1}{4\left(1-\frac{r}{L_0}\right)^2} - \frac{1}{4} + \frac{r}{L_0}\right)$
- $f(r) \approx \frac{3k_BT}{2L_p}\frac{r}{L_0}$
- $r_{rms} = \sqrt{2L_0L_p}$
- $E = \frac{\frac{d^2U}{dr^2}(r_0)}{r_0}$
- $U_0[(r_0/r)^{12} - 2(r_0/r)^6]$
- $K_{eq} = \frac{P_2}{P_1} = \exp\left(\frac{-\Delta G^0}{k_BT}\right)$
- $P_2 = \frac{K_{eq}}{1+K_{eq}}$
- $\Delta G(f) = \Delta G^0 - f\cdot\Delta x(f)$
- $\Delta G(f_{eq}) = 0$
- $\Delta x(f) = x_2(f) - x_1(f)$
- $\Delta x_{1t}(f) = x_t(f) - x_1(f)$
- $K_{eq}(f) = K_{eq}^0 \exp\left(\frac{f\Delta x(f)}{k_BT}\right)$
- $k_{12}(f) = k_{12}^0 \exp\left(\frac{f\Delta x_{1t}^0(f)}{k_BT}\right)$

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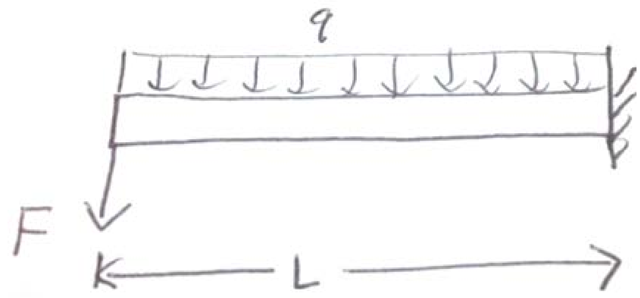
1. (**30 points**) A beam of length L is subjected to a force F as shown in the diagram, and to a uniform load $q = F/L$. The cross section of the beam is rectangular with height H and width W. The beam fails at USS = U, UCS = 2U, and UTS = 3U. What is the maximum force F that the beam can withstand before failure? Assume small deflections, and the beam is slender; that is, L >> H,W.



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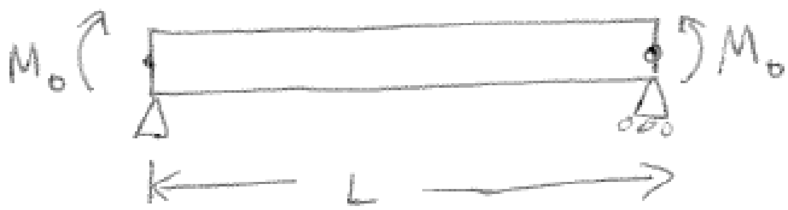
2. (30 points) A beam has length L, Young's modulus E, and a circular cross-section with radius r. What is the maximum deflection, v, of the beam in the situation below, and at what position x does this deflection occur? (you can assume small deflections)



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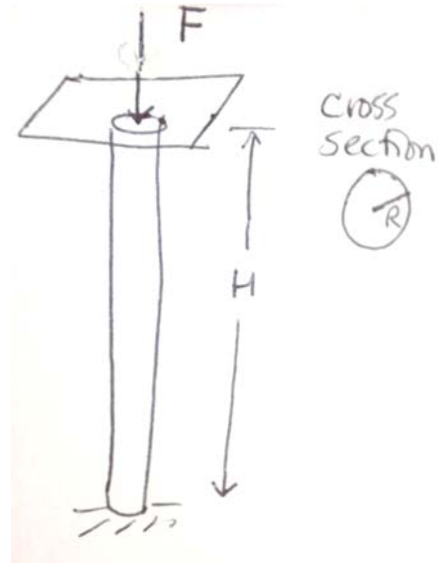
3. (**10 points**). A solid metal pole with Young's modulus E and a circular cross section of radius R is anchored upright into the ground. The top of the pole is at a height H above the ground and has a small platform. A weight causing a downward force, F, is placed on the platform directly above the pole. What is the highest allowable weight without buckling the pole?



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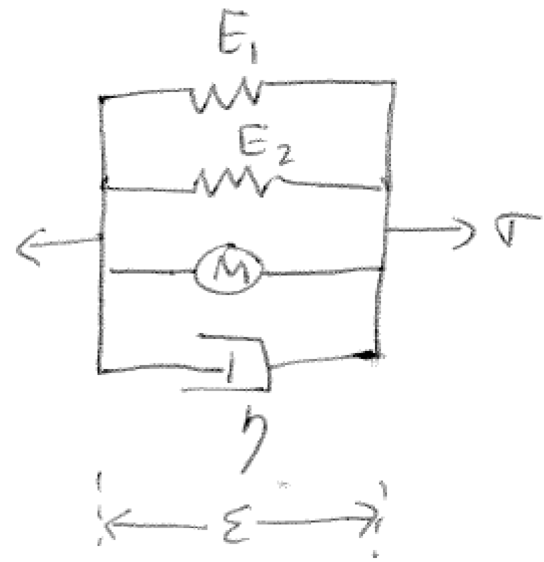
4. (**40 points**).
   a) (15 points) Derive a differential equation (ODE) relating stress, σ, to strain, ϵ, for the muscle model diagramed here. The motor element creates a tensile stress of $m(t)$.
   b) (25 points) If $m(t) = M\phi(t)$, and the material is at rest before the test starts, and there is no external stress on the material ($\sigma(t) = 0$), then what is the strain, $\epsilon(t)$?



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5. (**20 points; molecular biomechanics**). Several people have developed a fluorescent force sensing domain that can be genetically introduced into proteins to detect the amount of force applied to the proteins in vivo. The domain has two states (A and B) with different fluorescent spectra. You design such a domain in which state A and state B have the same length with no force ($x_A^0 = x_B^0$), but state A is stiffer ($\kappa_A > \kappa_B$). State A has energy $G_A^0 = 0$, and state B has energy $G_B^0 > 0$. **What is the equilibrium force, f_{eq}, at which the domain is equally likely to be in either state?**

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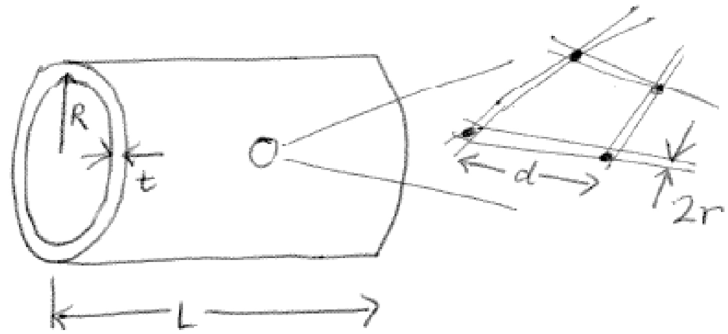
6. **(40 points)** You can cast a hydrogel scaffold of cross-linked collagen fibers, where the individual fibers have a circular cross-section of radius $r = 0.1\,\mu m$ and an average distance $d_{av} = 10\,\mu m$ between cross-links. The Young's modulus of collagen fibers is $E_C = 1\,GPa$. You cast this scaffold into a cylindrical tube that is L = 100 mm long, R = 10 mm in diameter, with t = 1 mm thick walls, clip the ends, and then pump liquid inside. How much pressure P should you use to pressurize the cylindrical tube if you want to cause an estimated 10% increase in radius of the cylinder? (*Hint: since we said 'estimated', you can make approximations if they are appropriate to this problem.*)



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7. (30 points) In the abstract of the del Rio paper (attached after this), the authors state that "**Application of physiologically relevant forces caused stretching of single talin rods that exposed cryptic binding sites for vinculin**." Analyze the appropriate level of certainty of this statement <u>if they had only performed the magnetic tweezers experiments (Figure 3 and the accompanying text)</u>.

   Note that we are saving time in your answer by not requiring the following: You do not need to discuss the atomic force microscopy experiment in Figure 4, nor the author's asserted level of certainty.

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