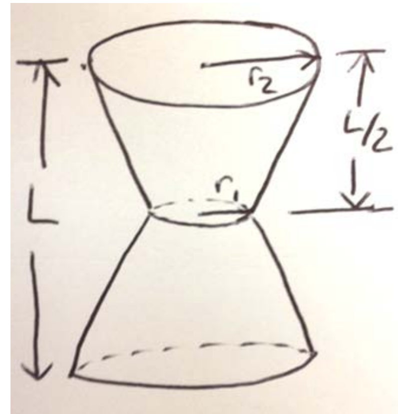
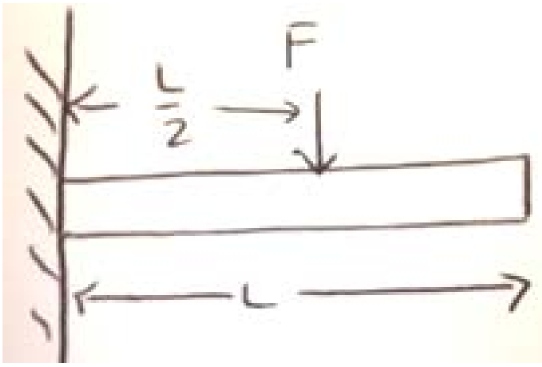
1. (25 points) If you have an object with the geometry indicated below, how much compressive force F (applied between top and bottom) would be required for the material to fail? Assume the following values.
    a) r1 = 10 mm
    b) r2 = 20 mm
    c) L = 40 mm
    d) E = 50 GPa
    e) $\nu = 0.3$
    f) USS = 10 MPa
    g) UTS = 40 MPa
    h) UCS = 30 MPa



2. A beam is attached to a solid support at x = 0, with external force applied at half its length (L/2), as shown below. The cross section of the beam is also shown below, and Young's modulus is E. You can assume that the deflection is small.

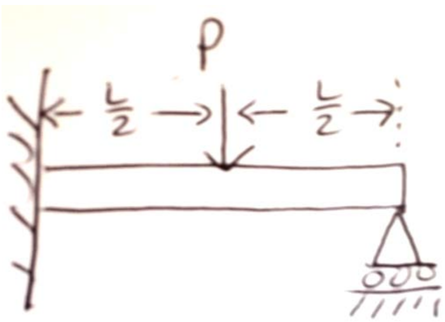


a) (15 points) What are the internal shear force V(x) and bending moment M(x) at a position x from the base?

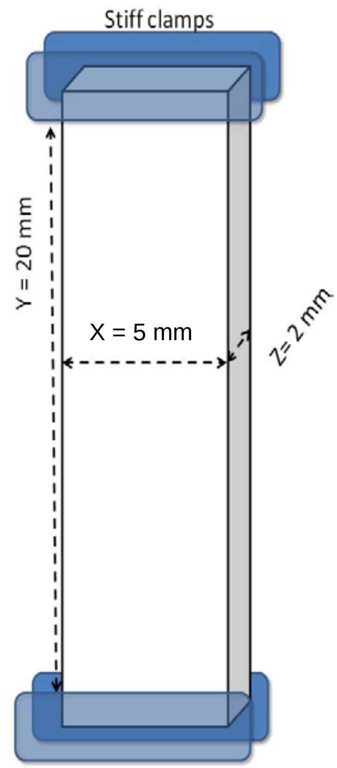
b) (10 points) What and where is the maximum tensile strain on the beam?

3. not for this year

4. (10 points) Draw the free body diagram and write the equations needed to solve for the support reactions for the beam below. You don't need to actually solve the equations.



3. You want to test the mechanical properties of a tissue engineered construct that is cut into strips that you clamp at opposite ends using sand-paper coated grips as shown in the figure on the right. Between the clamps, the construct is X = 5 mm wide, Y = 20 mm long and Z = 2 mm thick. You then stretch the material at 0.01 mm/sec by moving the clamps apart, and measure the length and tensile force. The force increases linearly to a force of 6 mN at 0.4 mm elongation. You note at this point that the width has decreased by 0.05 mm. As you continue to stretch the material, the force increases more rapidly until the material fails with 100 mN at 4 mm elongation. You can assume the material is isotropic.



a. What is the Young's modulus E, Elastic Modulus G, and Poisson ratio ν of the material?

b. What is the tensile strength of the material?

KEY

(25 points) If you have an object with the geometry indicated below, how much compressive force F (applied between top and bottom) would be required for the material to fail? Assume the following values.

a) r1 = 10 mm
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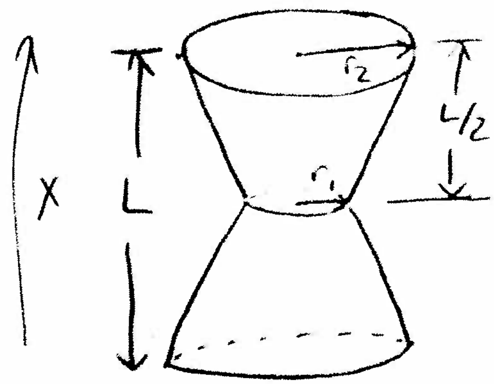


axial load, N = − F (compressive)

3 pts. $\sigma_x = \dfrac{-F}{A(x)}$. Need max, min σ_x. max = 0. ignore.

5 pts for where. $\min \sigma_x = -\dfrac{F}{\pi r_1^2}$ (where radius is smallest)

5 pts to do stress anal. $\max \tau = \left|\dfrac{\sigma_x}{2}\right| = +\dfrac{F}{2\pi r_1^2}$

5 pts [Fails if $UCS = -\min \sigma_x = +\dfrac{F}{\pi r_1^2}$
or $USS = \max \tau = \dfrac{F}{2\pi r_1^2}$

compare UCS, USS

5 to pick right one.

$F = UCS\ \pi r_1^2$

or $F = USS(2)\pi r_1^2$.

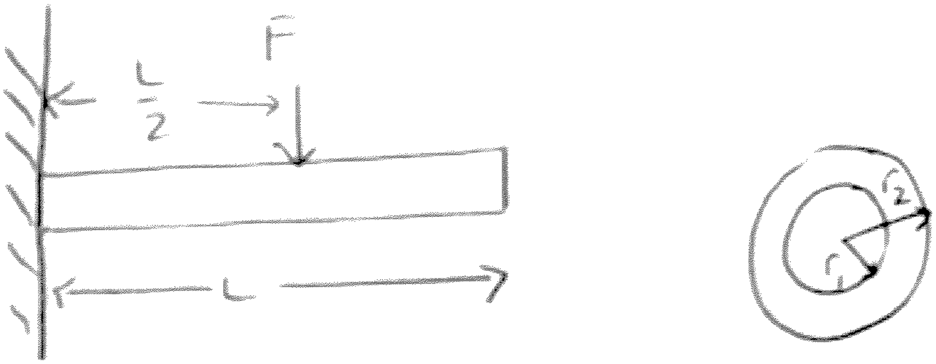
2USS < UCS, so ~~consider~~ will fail due to shear

$F = 10\ MPa\,(2)(3.14)(0.01)^2\ m^2$
$= 6.28\times10^7 \times 10^{-4} = \boxed{6.28\times10^3\ N}$

2 for right #'s. given rest.

KEY

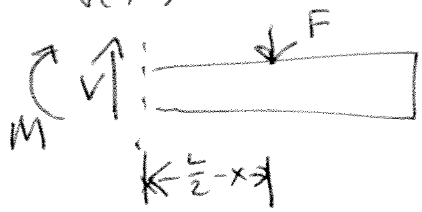
2. A beam is attached to a solid support at x = 0, with external force applied at half its length (L/2), as shown below. The cross section of the beam is also shown below, and Young's modulus is E. You can assume that the deflection is small.



a) (15 points) What are the internal shear force V(x) and bending moment M(x) at a position x from the base?

If $x > \frac{L}{2}$, $V(x) = 0$ (3) & $M(x) = 0$. (3)

If $x < \frac{L}{2}$



ΣF_y: $V(x) = F$ (3)

ΣM: $-M(x) - (\frac{L}{2} - x)F = 0$; $M(x) = -F(\frac{L}{2} - x)$

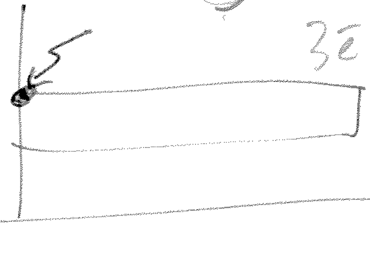
(−) (−)

(6) 3 for form, 3 for sign.

b) (10 points) What and where is the maximum tensile strain on the beam?

$\varepsilon_x = \frac{\sigma_x}{E} = -\frac{M(x)}{EI} y = \frac{F(\frac{L}{2} - x)}{EI} y$, (3) for eq. $I = \frac{\pi}{4}(r_2^4 - r_1^4)$

max @ $x = 0$, $y = r_2$: (3) (3)

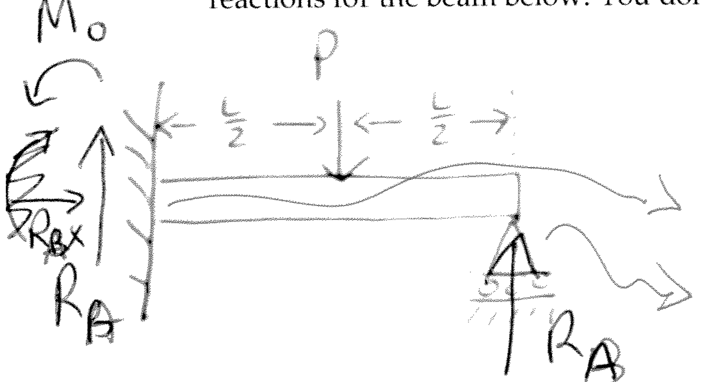


3e for finding correct x, y!

$\varepsilon_x = \frac{F(\frac{L}{2})}{EI} r_2 = \frac{FLr_2}{2EI}$,

(1) — bring together

4. (10 points) Draw the free body diagram and write the equations needed to solve for the support reactions for the beam below. You don't need to actually solve the equations.



$R_A + R_B - P = 0.$ (ΣF_y)

$M_0 - \frac{L}{2}P + L R_A = 0$ $(\Sigma M_{@B})$

$M_0 - L R_B + \frac{L}{2} P = 0$ $(\Sigma M_{@A})$

or

$(\Sigma M_{L/2})$

$R_{Bx} = 0$ (1) $(\Sigma F_x = 0)$

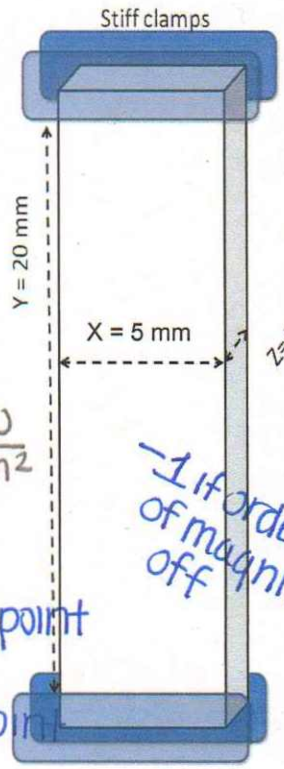
$\Sigma F_x = 0$ is optional, since obvious & we usually ignore it.

2pts for $\Sigma F_y = 0$

3pts for 1st moment Eqn

5pts for 2nd moment Eqn. (3 for correct, 2 for [illegible])

3. You want to test the mechanical properties of a tissue engineered construct that is cut into strips that you clamp at opposite ends using sand-paper coated grips as shown in the figure on the right. Between the clamps, the construct is X = 5 mm wide, Y = 20 mm long and Z = 2 mm thick. You then stretch the material at 0.01 mm/sec by moving the clamps apart, and measure the length and tensile force. The force increases linearly to a force of 6 mN at 0.4 mm elongation. You note at this point that the width has decreased by 0.05 mm. As you continue to stretch the material, the force increases more rapidly until the material fails with 100 mN at 4 mm elongation. You can assume the material is isotropic.



   a. What is the Young's modulus E, Elastic Modulus G, and Poisson ratio ν of the material? (15 points) 1 point

$E = \frac{\sigma}{\varepsilon}$ 2 points

$\sigma = F/A = \frac{6\times10^{-3}N}{(5\times10^{-3}m)(2\times10^{-3}m)} = 600\frac{N}{m^2}$

$\varepsilon = \frac{0.4mm}{20mm} = 0.02$ 1 point

$E = \frac{600 N/m^2}{0.02} = \boxed{3\times10^4 N/m^2 = E}$ 1 point

−1 if order of magnitude off

2 points

$\nu = -\frac{\varepsilon'}{\varepsilon} \Rightarrow \varepsilon' = \frac{-0.05\times10^{-3}m}{5\times10^{-3}m} = -0.01$ 1 point

$\nu = -\frac{-0.01}{0.02} = \boxed{1/2 = \nu}$ 1 point

1 point

$G = \frac{E}{2(1+\nu)} = \frac{3\times10^4 N/m^2}{2(\frac{3}{2})} = \boxed{1\times10^4 N/m^2 = G}$

2 points 2 points 1 point

−0 if E was wrong and used wrong E but had right equation

   b. What is the tensile strength of the material? (5 points)

σ_{max} = tensile strength $= \frac{F_{max}}{A} = \frac{100\times10^{-3}N}{(5\times10^{-3}m)(2\times10^{-3}m)}$ 1 point

2 points

1 point

$10\times10^{-6}m^2$

$\boxed{\sigma_{max} = 1\times10^4 N/m^2}$

1 points

$-3 - -6 = 3$

$10\times10^3 N/m^2$