Bioen 326 2013 MIDTERM

Covers: Weeks 1-4, FBD, stress/strain, stress analysis, rods and beams (not deflections).

Rules: Closed Book Exam: Please put away all notes and electronic devices

Reminders:

- Show equations you use to answer questions. Even if your final answer is wrong, we give **partial credit** for the various steps needed to solve a problem, but we can't do this if you don't write the equations you used.
- We give **extra credit** if you realize your final answer is wrong and explain why, even if you do not have time to go back and find and fix the error. Since this can partially or fully make up for the mistake (depending on the type of mistake), I advise that you don't go back to fix until you finish the rest of the exam. Even then, don't erase what you have, but use the extra sheet and then cross out the first once you finish successfully, or you may run out of time and have erased your points.

Equations Provided on the Exam:

The following equations are provided on this cover sheet for the exam.

- $F_R = \int F_d(x)dx$ or $F_R = \int F_d(\vec{r})d\vec{r}$
- $x_R = \frac{\int xF_d(x)dx}{F_R}$, or $x_R = \frac{\int xF_d(\vec{r})d\vec{r}}{F_R}$
- $y_{COM} =$
- $\sigma_{av} = \frac{\sigma_x+\sigma_y}{2}$
- $R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau^2}$
- $\sigma_{x\theta} = \sigma_{av} + \frac{\sigma_x-\sigma_y}{2}\cos(2\theta) + \tau\sin(2\theta)$
- $\tau_\theta = -\frac{\sigma_x-\sigma_y}{2}\sin(2\theta) + \tau\cos(2\theta)$
- $\sigma_{1,2} = \sigma_{av} \pm R$
- $\tau_{MAX} = R$
- $\epsilon_x = \frac{1}{E}\sigma_x - \frac{\nu}{E}\sigma_y - \frac{\nu}{E}\sigma_z$
- $\gamma_{xy} = \frac{1}{G}\tau_{xy} = \frac{2(1+\nu)}{E}\tau_{xy}$
- $\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}((1-\nu)\epsilon_x + \nu\epsilon_y + \nu\epsilon_z)$
- $\tau_{xy} = G\gamma_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy}$
- $e = \frac{\Delta V}{V_0} = \epsilon_x + \epsilon_y + \epsilon_z$
- $e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)$
- $\delta = \int_0^L \frac{N(x)}{E(x)A(x)}dx$
- $k_a = \frac{EA}{L}$; $F = k_a\delta$
- $k_t = \frac{GI_p}{L}$; $T = k_t\phi$
- $\int_A y^2 dA = I$
- $I = \frac{\pi}{4}r^4$
- $I = \frac{H^3W}{12}$
- $I = \frac{H^3W}{36}$, (neutral plane is at H/3 from flat edge).
- $\int_A r^2 dA = I_p$
- $I_p = \frac{\pi}{2}r^4$
- $\kappa = M/EI$
- $\sigma_x = N/A$
- $\tau(r) = \frac{T}{I_p}(r)$
- $\sigma_x(x,y) = -\frac{M(x)}{I}y$
- $\tau_{xy}(x,y) = \frac{V(x)}{2I}\left(\frac{H^2}{4} - y^2\right)$, $\max\left(\tau_{xy}(x)\right) = \frac{3V(x)}{2A}$
- $\max\left(\tau_{xy}(x)\right) = \frac{4V(x)}{3A}$
- $\max\left(\tau_{xy}(x)\right) = \frac{4V(x)}{3A}\frac{(r_2^2+r_2r_1+r_1^2)}{r_2^2+r_1^2}$

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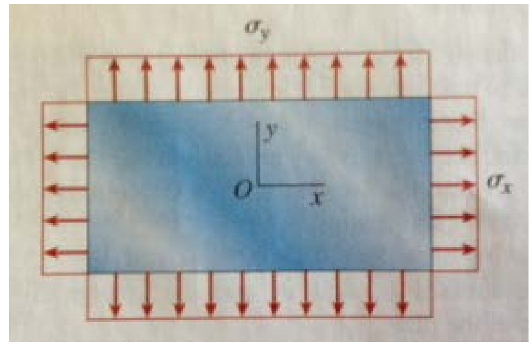

1. (20 points) A rectangular plate has isotropic and linear materials properties, with Young's modulus E = 1 MPa and Poisson ratio $\nu = 0.2$. If the unstressed thickness is $t_z = 1\,cm$. **What is the change in thickness, Δt_z,** under biaxial stress, with $\sigma_x = 2\,kPa$ and $\sigma_y = -3\,kPa$? (Use a positive sign in your answer if the material gets thicker, a negative for thinner).

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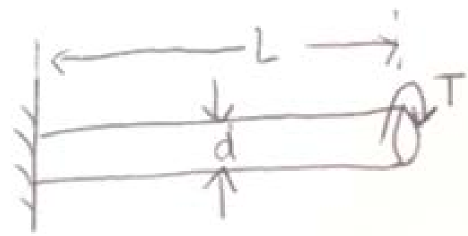
2. (20 points) An aluminum bar of solid circular cross section is twisted by an unknown torque T acting at the ends (see figure), causing it to twist by an angle of 0.4 radians. The rod is L = 1 m long, and has a diameter d = 2 cm, and the shear modulus is G. **What is the maximum value of the shear stress, $\max(\tau)$ and maximum shear strain $\max(\gamma)$?**



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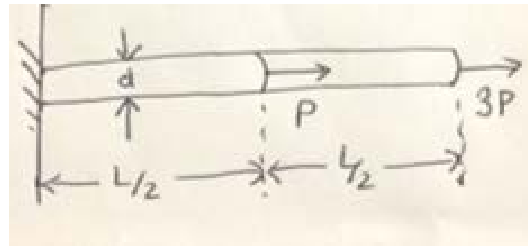

3. (30 points) The rod in the diagram has a circular cross-section with diameter d. It is exposed to a load P in the center and 3P at the tip as shown.

   a. **What is the maximum shear stress τ_{Max} in the rod?**
   b. **what locations or locations will this stress occur?**
   c. If the ultimate tensile stress, compressive stress, and shear stress are all identical, and equal to U, then **at what value of P will the rod fail?**

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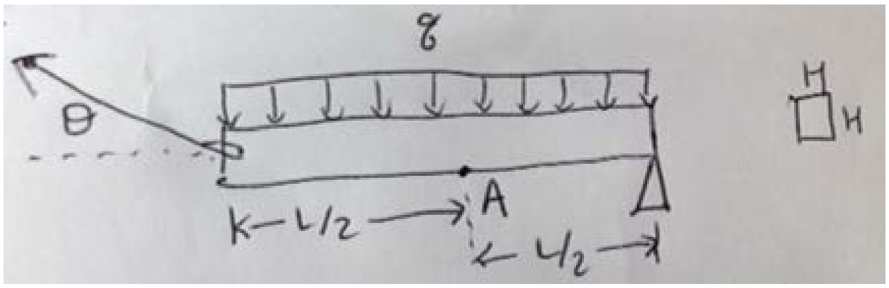
4. (30 points) In the diagram below, the cable held at an angle $\theta = \frac{\pi}{4} = 45°$ and the force on the cable is adjusted until the beam is parallel to the x-axis, as shown. The beam has a uniform distributed load with force density q over length L and the cross-section is an H by H square. Any deformations resulting from this are small. The Young's modulus is E, and the shear Modulus is G.

   What is the longitudinal normal stress, σ_x, at position A in the diagram, at the bottom of the beam at length L/2 from each end?



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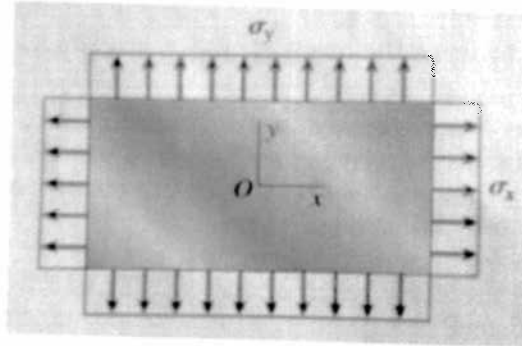
1. (20 points) A rectangular plate has isotropic and linear materials properties, with Young's modulus E = 1 MPa and Poisson ratio $\nu = 0.2$. If the unstressed thickness is $t_z = 1\,cm$. **What is the change in thickness, Δt_z,** under biaxial stress, with $\sigma_x = 2\,kPa$ and $\sigma_y = -3\,kPa$? (Use a positive sign in your answer if the material gets thicker, a negative for thinner).



To find Δt_z, need to find ε_z. ($\Delta t_z = \varepsilon_z \cdot t_z$)

ε_z can be found from 3D Hooke's Law:

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu\sigma_x - \nu\sigma_y) =$$

$$\sigma_z = 0, \text{ so } \varepsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y)$$

Now fill in #'s:

$$\varepsilon_z = -\frac{0.2}{10^6\,Pa}(2-3)10^3\,Pa = +\frac{0.2}{10^3} = 0.2\times10^{-3}$$

$$\Delta t_z = 0.2\times10^{-3}\cdot 1\,cm$$
$$= 2\times10^{-3}\,mm$$
$$= 2\,\mu m$$
$$= 2\times10^{-6}\,m$$

any of these are OK; (positive sign, or "gets thicker")

Assigned Points:

8 pts for correct Hooke's Law. (3pts if in wrong direction)
(5 for realizing 3D Hooke's needed, 3 for right direction)

5 pts for definition of strain: $\Delta t_z = t_z \cdot \varepsilon_z$

7 pts for right answer after all plug ins.

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$\tau(r) = \frac{T}{I_p} r$. max is when $r = \frac{d}{2}$, radius.

$\max(\tau) = \frac{T}{I_p} \cdot \frac{d}{2}$

$I_p = \frac{\pi}{2} r^4 = \frac{\pi}{2}\left(\frac{d}{2}\right)^4$. so I_p is known.

But T is not known, so need Eqn for it:

$T = k_t \phi$, ϕ is known

$k_t = \frac{G I_p}{L}$, L is known, G is known (but not given)

combine:

$\max(\tau) = \frac{k_t \phi}{I_p}\left(\frac{d}{2}\right) = \frac{G I_p \phi}{L I_p}\left(\frac{d}{2}\right) = \frac{G\phi}{L}\left(\frac{d}{2}\right)$

This is pure shear, so $\tau_{max} = \max(\tau)$.

$\max(\gamma) = \tau/G = \frac{\phi}{L}\left(\frac{d}{2}\right)$.

Plug in values:

$\max(\gamma) = \frac{0.4}{1m}\left(\frac{2cm}{2}\right) = \frac{0.4\,(1\,cm)}{100\,cm} = \boxed{0.004.}$

$\max(\tau) = \boxed{0.004\,G.}$

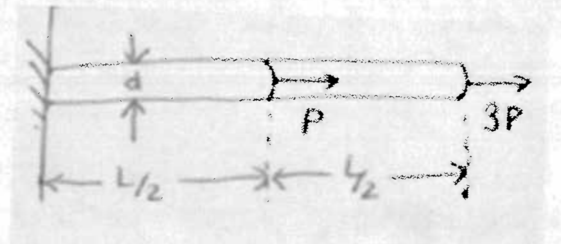
Points assigned:

2 pts for each Eqn (2×7 = 14)

3 pts for plugging in to get each correct answer (2×3=6)

If you need more room, please use one of the extra pages and indicate here: continued on page ______

3. (30 points) The rod in the diagram has a circular cross-section with diameter d. It is exposed to a load P in the center and 3P at the tip as shown.

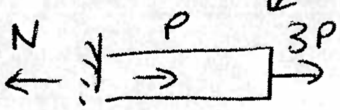


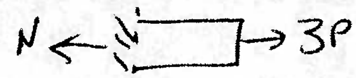
   a. **What is the maximum shear stress in the rod?**
   b. **what locations or locations will this stress occur?**
   c. If the ultimate tensile stress, compressive stress, and shear stress are all identical, and equal to U, then **at what value of P will the rod fail?**

Uniaxial stress, so $\sigma_x = N(x)/A(x)$

$A(x) = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \frac{\pi}{4}d^2$ for all x.

$N(x) = +4P$ from 0 to L/2, $+3P$ from L/2 to L





$-N+3P=0$
$N=3P$

$-N+P+3P=0$
$N=4P$

$N=4P$ is greater, so

$\sigma(x) = \frac{4P}{\frac{\pi}{4}d^2} = \frac{16P}{\pi d^2}$; (a) $\tau_{max} = \sqrt{\left(\frac{\sigma_x - 0}{2}\right)^2 + 0^2} = \frac{\sigma_x}{2} = \boxed{\frac{8P}{\pi d^2}}$

(b) This occurs at every point from $\boxed{0 \text{ to } L/2}$ since diagram to get 4P is for any x there.

(c) UTS = UCS = USS = U; will fail at lowest P that causes one to fail: $\frac{16P}{\pi d^2} = U$ or $\frac{8P}{\pi d^2} = U$; $\boxed{P = \frac{U\pi d^2}{8}}$ or $\frac{U\pi d^2}{16}$

Points assigned:

(a) 15 pts
  (3 for correct N(x)
  2 for " A(x)
  5 for σ_x from N, A
  5 for τ_{max} from σ_x

(b) 5 pts for correct location

(c) 10 pts: 5 for choosing whether to use UTS & σ_x or UCS & τ_{max}.
  5 for calculating P from that.

the logic here is right; select lowest P, but the math is wrong; we should have circled the other one since 1/16 is less than 1/8. Since logic is right, this answer should have gotten 3 or4 of the 5 points. If you cicled wrong one with no logic, no points. I fyou circled right one with no logic, 5 points.

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Name Key

4. (30 points) In the diagram below, the cable held at an angle $\theta = \frac{\pi}{4} = 45°$ and the force on the cable is adjusted until the beam is parallel to the x-axis, as shown. The beam has a uniform distributed load with force density q over length L and the cross-section is an H by H square. Any deformations resulting from this are small. The Young's modulus is E, and the shear Modulus is G.

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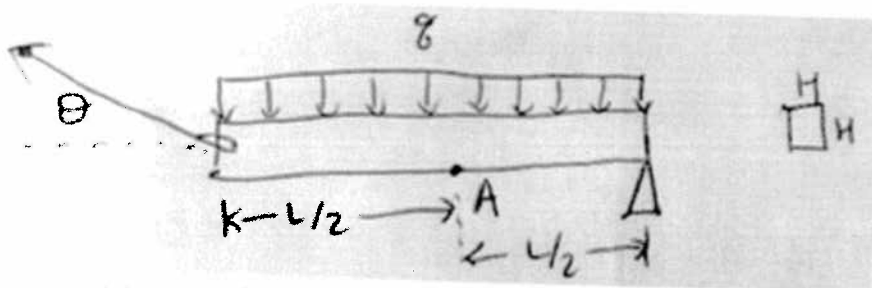
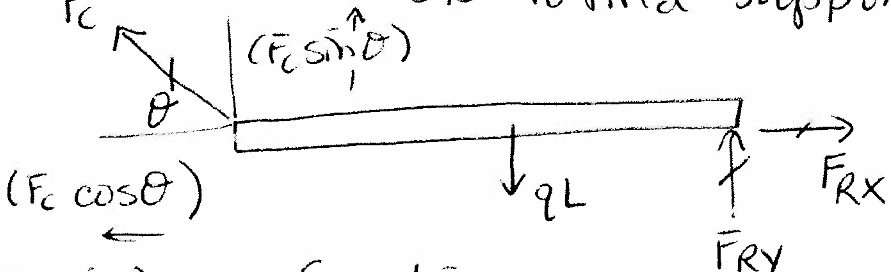


Use FBD to find support Rxns:



[5pts for FBD]

Equilibrium Equations:

$F_c \cos\theta = F_{RX}$

$F_c \sin\theta + F_{RY} - qL = 0$

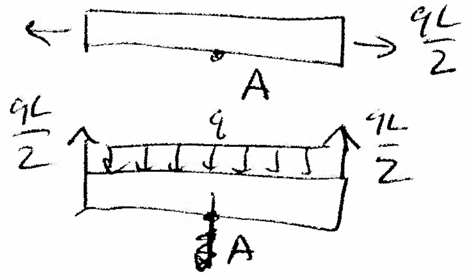
(moment around L/2) $F_c \sin\theta \left(\frac{L}{2}\right) = F_{RY}\left(\frac{L}{2}\right)$; $F_c \sin\theta = F_{RY}$

so $2F_{RY} = qL$; $\boxed{F_{RY} = qL/2}$ $\boxed{F_c \sin\theta = \frac{qL}{2}}$

$\sin\theta = \cos\theta$ for $\theta = \frac{\pi}{4}$, so $\boxed{F_{RX} = F_c \cos\theta = \frac{qL}{2}}$

[5pts for support rxns]

This is a combined load, so solve two problems: [5pts for combined]



Find σ_{xA} @A due to axial forces

Find σ_{xL} @ A due to lateral

$\sigma_x = \sigma_{xA} + \sigma_{xL}$

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[4 pts to find σ_{xA}]

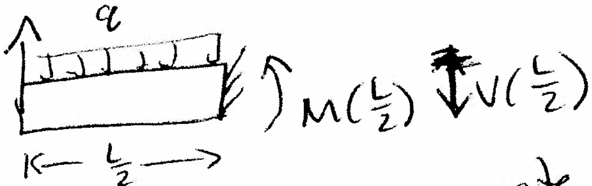
$$N(x) = \frac{qL}{2}$$

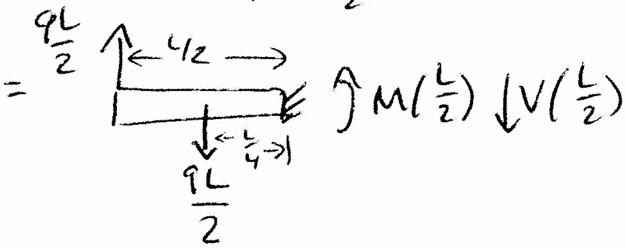
$$A(x) = H^2$$

$$\sigma_{xA} = \frac{N(x)}{A(x)} = \frac{qL}{2H^2}$$

[6 pts to find $M(\frac{L}{2})$, bending moment @ A]

Cut @ $\frac{L}{2}$





write $\Sigma M_z = 0$ @ $\frac{L}{2}$:

$$M(\tfrac{L}{2}) - \tfrac{L}{2}\left(\tfrac{qL}{2}\right) + \tfrac{L}{4}\left(\tfrac{qL}{2}\right) = 0$$

$$M(\tfrac{L}{2}) = \frac{qL^2}{4} - \frac{qL^2}{8} = \boxed{\frac{qL^2}{8}}$$

[5 pts to find σ_{xL} at A from $M(\frac{L}{2})$:]

$\sigma_{xL} = -\frac{M(x)}{I}y$ for bending. $I = \frac{H^3W}{12}$, but $W = H$) $\frac{H^4}{12}$

$y = -H/2$

So $\sigma_{xL} = -\frac{qL^2}{8} \cdot \frac{12}{H^4}\left(-\frac{H}{2}\right) = \frac{12qL^2}{16H^3} = \boxed{\frac{3qL^2}{4H^3}}$

Now, wrap up combined loading:

$$\sigma_x = \sigma_{xA} + \sigma_{xL} = \boxed{\frac{qL}{2H^2} + \frac{3qL^2}{4H^3}}$$

both are units $\frac{q}{m} = \frac{N/m}{m} = N/m^2 = Pa$ ✓

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