

## LECTURE 0: ESSENTIAL PRIOR KNOWLEDGE FOR BIOEN 326:

Note that this is not today's lecture notes; Lecture 1 and Homework 1 are posted on the web page, on today's date on the schedule page. See the course homepage: <http://courses.washington.edu/bioen326/index.htm>. You should have a link to this through myUW.

### Prerequisite Knowledge:

I assume you have taken classes that are prerequisites for the prerequisites. This includes knowledge from:

- CHEM 142-162 (chemical reactions, thermodynamics, molecular bonds)
- PHYS 121 (friction, force, moments, free body diagrams, newton's laws)
- MATH 126, (introduction to multivariable differential calculus)
- MATH 307, solutions to linear differential equations
- MATH 308, especially matrix algebra
- AMATH 301: numerical solutions to ODEs and matrix algebra.
- BIOEN 315: protein structure and energy, thermodynamics, kinetics.

### Ordinary differential equations (ODEs)

For this class you should be able to solve linear ODEs. When you have a simultaneous system of more than one ODE, you need to know how to convert to one or solve two simultaneously. You should know how to solve these using Laplace transforms. Recall the following:

For first order ODEs, you need one ODE and one initial condition for each dependent variable. For second order ODEs, you need the initial condition for both the function and its derivative. (We are not addressing boundary value problems here, although we will touch on these briefly when we discuss buckling of columns).

The Complementary solution is when the initial Condition holds, but there is no pushing function. The Particular solution is when there is a Pushing function, but the initial conditions are zero. Since the equations are linear, the overall solution is the sum of the response to the Initial conditions and the pushing function, or the complementary plus particular solution.

To use Laplace transforms

- you can define  $Y(s) = L(y(t)) = \int_0^{\infty} y(t)e^{-st} dt$ . We often abbreviate  $Y = L(y)$ .
- Then,  $L\left(\frac{dy}{dt}\right) = sY(s) - y(0)$ , where  $y(0)$  is the initial condition.
- Some common transform pairs are:
  - $L(\delta(t)) = 1$  (impulse function)
  - $L(\emptyset(t)) = 1/s$  (step function; 0 below  $t=0$ , and 1 above  $t=0$ )
  - $L(t\emptyset(t)) = 1/s^2$
  - $L(e^{-at}\emptyset(t)) = \frac{1}{a+s}$
  - $L(\sin(at)) = \frac{a}{a^2+s^2}$
  - $L(\emptyset(t)(1 - e^{-at})) = \frac{a}{s(s+a)}$
  - $L(b_1y_1 + b_2y_2) = b_1L(y_1) + b_2L(y_2)$  (linear property of Laplace transforms)
- Method to apply Laplace transforms:
  - transform both sides of the ODE. You may want to do this for the Particular solution only, so you can set all initial conditions to zero.
  - manipulate to get  $Y = G(s)$ , where  $G$  is a polynomial in  $s$ , or a sum of such polynomials, one for particular and one for complementary.
  - Use partial fraction decomposition ([http://en.wikibooks.org/wiki/Complex\\_Analysis/Residue\\_Theory/Partial\\_Fractions](http://en.wikibooks.org/wiki/Complex_Analysis/Residue_Theory/Partial_Fractions)) or method of residues (also called Heaviside cover up method)

([http://www.math.vt.edu/people/dlr/m2k\\_opm\\_lappf2.pdf](http://www.math.vt.edu/people/dlr/m2k_opm_lappf2.pdf) or

[http://en.wikipedia.org/wiki/Heaviside\\_cover-up\\_method](http://en.wikipedia.org/wiki/Heaviside_cover-up_method)) to manipulate  $G(s)$  to be a sum of terms that are known transform pairs (eg on the right side of the transform pair list above).

- If you kept the nonzero initial conditions, you should have separate terms for the particular (has the input/forcing function) and the complementary solution (has initial conditions). Take the inverse transform of all terms.
- Example: find  $y(t)$  if  $\frac{dy}{dt} = -ky$  and  $y(0) = C$ 
  - first we let  $Y(s) = L(y(t))$ .
  - Then  $L\left(\frac{dy}{dt}\right) = sY(s) - y(0)$ .
  - Now use these to transform the equation:  $sY(s) - C = -kY(s)$
  - Now rearrange to solve for  $Y(s)$ :  $sY(s) + kY(s) = C$ , so  $Y(s) = \frac{C}{s+k}$
  - Now go to the transform tables, and see that  $L(e^{-at}) = \frac{1}{s+a}$ , and our answer is just a constant times that, with  $a = k$ . Transforms are linear, so the constant factor just stays. Thus,  $y(t) = Ce^{-kt}$ , which you probably already knew.

### Partial Derivatives

Recall that a partial derivative is indicated by the del operator, as in:  $\frac{\partial u}{\partial x}$  and refers to the derivative of the variable in the numerator with respect to that in the denominator while holding all other variables constant. For example, if  $u = y^2 + z \sin(x)$ , then  $\frac{\partial u}{\partial y} = 2y$ ,  $\frac{\partial u}{\partial x} = z \cos(x)$ ,  $\frac{\partial u}{\partial z} = \sin(x)$ . You will use these a lot in 325, but we will touch on them here.

**Mechanics** (most of phys 121 or equivalent).

#### Definitions

- a force,  $\vec{F}$  is a vector applied at a point.
- a moment results when you consider the effect of this force on a second point O:  $\vec{M} = \vec{r} \times \vec{F}$ , where  $\vec{M}$  is the moment at point O, and  $\vec{r}$  is the vector from O to where the force is applied. Recall that the cross product is perpendicular to the plane of the two vectors involved, so comes out of the paper in this example. Specifically,  $\vec{r} \times \vec{F} = rF \sin\theta \cdot \vec{n}$ , where  $\vec{n}$  is the unit vector with direction determined by the right hand rule, and  $r$  is the distance between O and where force is applied, while  $F$  is the magnitude of the force. When  $\vec{r}$  and  $\vec{F}$  are in-line, then  $\sin\theta = 0$ , so there is no moment. The moment is maximum when the two are perpendicular. Finally, the moment increases with the distance  $r$  and force  $F$ .

Free body diagrams. Recall that an object at equilibrium is either at rest or has a constant linear and angular velocity. This occurs when all forces and moments on the object are balanced, but in phys 121, you focused on problem where there were no moments so we just review this. To use a free body diagram to calculate forces:

- Draw the object, indicating distances and angles of interest, whether known or unknown.
- Indicate external forces acting on the object. These may be known or unknown:
- Write down the knowns and unknowns to clarify which are which.
- Write the equilibrium equations. At equilibrium, for every point on the object, the forces in each direction must sum to zero. In 2D,  $\sum F_x = 0$ ,  $\sum F_y = 0$ .
- Solve for the unknowns. You should have one independent equation for every unknown, or the problem is not solvable.

**Other:**

Linear algebra: You should understand what eigenvector and eigenvalues are.

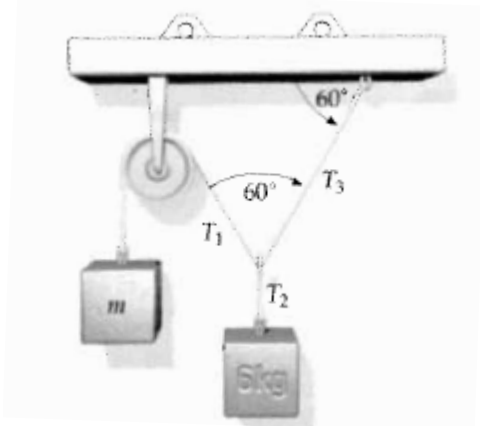
Chemical thermodynamics: you should know the Boltzmann distribution, which states that the probability of a molecule (or similar system) being in state 1 is:  $p_1 \propto \exp\left(-\frac{e_1}{k_B T}\right)$ , where  $e_1$  is the energy of the element in that state. Alternatively, you may be familiar with the formula:  $P_1 \propto \exp\left(-\frac{E_1}{RT}\right)$ , where  $P_1$  is the fraction of molecules in state 1, and  $E_1$  is the energy of a mole of molecules in that state. However, we will go over this again in class, so this prior knowledge is not essential.

Mathematical Programming: you should be able to use MATLAB or a similar mathematical programming software to solve linear algebra and ODE problems. You should be able to:

- write and run an m-file script
- write m-file functions to be called
- define variables of interest
- use matrix algebra to multiply arrays, and solve for eigenvalues and eigenvectors.
- use ODE solvers such as ode23s to solve ODE problems.
- make figures to plot results

### Test Yourself! Can you solve these Review Problems?

1. Laplace Transforms to solve ODE: given the equations  $\frac{dy}{dt} = -ay + b$ , and  $y(0) = c$ ,
  - a. Use Laplace Transforms to solve for  $y(t)$ .
  - b. verify your result by plugging the solution back into the original equations.
2. partial derivatives:
  - a. given the function,  $u = \frac{x}{y}$ , what are  $\frac{\partial u}{\partial x}$  and  $\frac{\partial u}{\partial y}$ ?
3. mechanics:
  - a. draw the free body diagram for the following problem (from Tipler problem 4-52c): For the system in the following figure to be in equilibrium, find the unknown tensions  $T_1$ ,  $T_2$ , and  $T_3$  and mass  $m$ . (The weight is 5 kg in case you can't read it; the bar is horizontal.)



4. linear algebra
  - a. what is  $\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} e \\ f \end{bmatrix}$ ? what is  $\begin{bmatrix} e \\ f \end{bmatrix} * \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ ?
  - b. If  $A$  is a matrix, then what is the mathematical definition for an eigenvector and eigenvalue of  $A$ ?
  - c. What is the relationship between diagonalization of a matrix and its eigenvectors and eigenvalues?
5. chemical thermodynamics (optional; you will learn this in class, this is just to jog your memory)
  - a. What is the relationship of the Boltzmann constant,  $k_B$ , to the gas constant,  $R$ ?
  - b. How are these used to calculate the probability of a molecules being in a state?

- c. If a molecule can have two states, one with energy  $E_1$  and the other with  $E_2$ , what is the probability of being in each state?
6. mathematical programming: use a mathematical program such as MATLAB to find the numerical solution to the following, and verify that this is the same as your analytic solution obtained above:
  - a. problem 1d, with  $a = 2$ ,  $b = 5$  and  $c = 0$  (remember how to use ODE solvers?)
  - b. problem 4a, with  $a = b = 1$ ,  $c = 2$ ,  $d = 0.5$ ,  $e = 1$ ,  $f = 1$ .