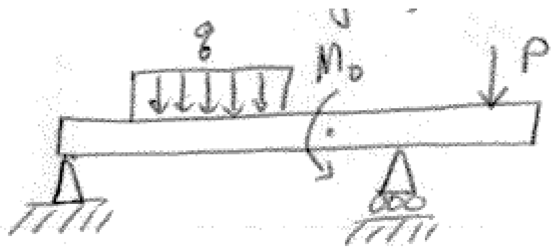
BIOEN 326 2014 LECTURE 8: LATERAL FORCES AND MOMENTS ON BEAMS

Also read Gere chapter 4.3 – 4.5

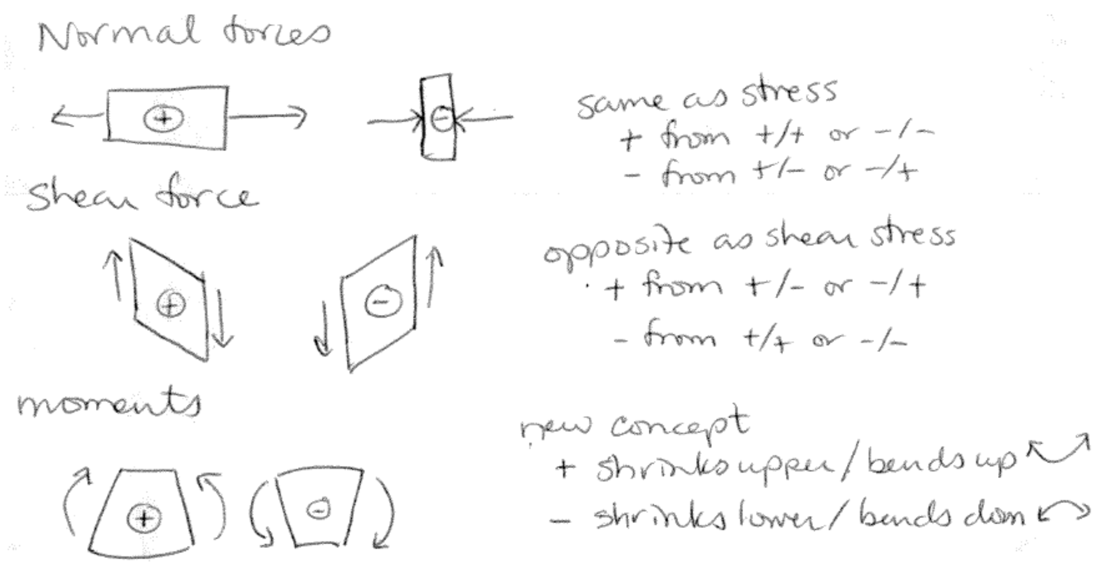
Here we consider bars that are subjected to **shear force**, which is force applied perpendicular to the axis of the beam (parallel to the face), or **couples**, which are moments that bend the beam. These take the form of distributed shear force (q in figure; units N/m), concentrated shear force (P in figure; units N), or a concentrated moment (Mo in figure, units Nm). All of these act to bend and shear the beam in the x-y plane, but do not apply a normal force or twisting moment.



As with axial and torsional loads, our first step is to identify the internal forces and moments at each position x along the beam. By convention, we use the following variables:

- **V(x) = internal shear force at position x**
- **M(x) = internal bending moment at position x**

Again, we use the **internal or deformation sign conventions** to indicate these forces and moments. (Recall that internal normal force was position for tension, which acts to elongate the bar). Just to be contrary, we consider internal shear force to be when it looks like it would deform a section of the beam like positive shear stress. More intuitively, we consider a bending moment to be positive when it acts to bend the beam upward. In summary, the deformation sign conventions for normal force, shear force and bending moments are:



How to calculate V(x) and M(x) from external loads using FBD

The following steps and instructions will give you the correct answer, although there are other variations that will solve the problem as well. The steps are provided here in one place for easy reference, but will make more sense after some examples.

1. **Support Reactions**. calculate the support reactions for all supports using a FBD as before (unless there is only one support, in which case you can skip this step and work from either end to the place of the support.)
2. **Cut at x**. Cut the bar at an arbitrary position x, and consider the beam on one side of the cut to be a solid support for the other side. Make sure you select which side of the cut to study, so that the support reactions are the only unknowns for that section of the beam.
3. **Draw V(x) and M(x)** as the two unknown support reactions (there is no x-direction forces), to complete the FBD for the beam section you chose. Draw them in the positive direction according to the internal sign conventions, so their sign will indicate the sign of V(x) and M(x) correctly.
4. **Equilibrium Equations**. Write the two equilibrium equations for forces in the y-direction and for Moments in the x-y plane. When you write these, look at the direction everything is pointing as assign the signs via the external sign conventions, just as you did for all previous free body diagrams.
5. **Solve** these simultaneously to obtain V(x) and M(x).
6. **Repeat** for any other segment of the beam where the cut would result in a different equation for forces or moments. This means you need to repeat every time you cross a concentrated shear force or a couple.

Comments:

- if you have a distributed load, you can do this in two ways, but either way, you only consider the part of the distributed load that still remains AFTER the cut:
  - you can integrate the load along the beam section for all values of ξ between the cut at x and the end of the beam, to obtain total force ($\int q\, d\xi$) and moment ($\int r \times q\, d\xi$, where r is vector from cut (x) to ξ).
  - replace the distributed load with a resultant force at the resultant location.

How to calculate V(x) and M(x) from external loads using integration

An alternative approach to solve these problems is to use integration, following a set of instructions that essentially do the same thing as drawing free body diagrams.

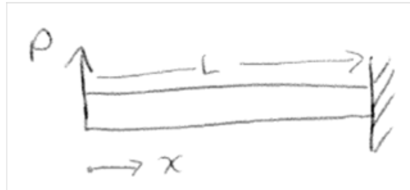
1. Integrate to obtain V(x): $\frac{dV(x)}{dx} = -q(x)$, and V(x) jumps by $-P$ at a concentrated load as you go in positive x direction.
2. Integrate to obtain M(x): $\frac{dM(x)}{dx} = V(x)$, and M(x) jumps by – M_0 at a couple.

In both cases, use the known values of V(x) and M(x) at one end to solve for the unknown constants of integration.

<u>Shear force and bending moment diagrams</u>.

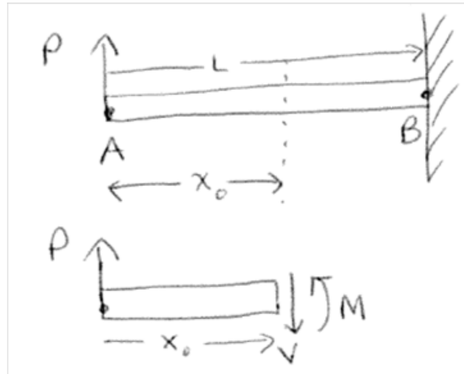
It is often helpful to plot V(x) vs x and M(x) vs x. This requires you to calculate their values at ends and at the places where two sections meet, as well as how the curve moves between. It allows you to quickly see where the maximum and minimum values occur, which is needed for later problem solving.

<u>Example 1: simple cantilever</u>



FBD method:

1. support reactions – we skip this step since only one support.
2. cut at x. I will select the left hand region, since the right hand has unknown support reactions at x = L, in addition to our unknown support reactions at the cut.
3. Draw V(x) and M(x). Note in the drawing here that V is in the positive direction because it is negative force on positive face, and M is positive because it acts to bend the beam upward.
4. Equilibrium Equations. Use static/external sign conventions. I will sum the moments around the x =0 spot, but it doesn't matter.
    a. $P - V(x) = 0; V(x) = P$
    b. $0P + M(x) - xV = 0; M(x) = xV(x) = xP$
5. Solve
    a. $V(x) = P$
    b. $M(x) = xV(x) = xP$
6. Repeat as needed
    a. These equations apply to all x, from 0 to L, so no other section needed.



Integration method:

$$\frac{dV}{dx} = 0; V(x) = C_1$$

$$\frac{dM}{dx} = V(x) = C_1; M(x) = C_1 x + C_2$$

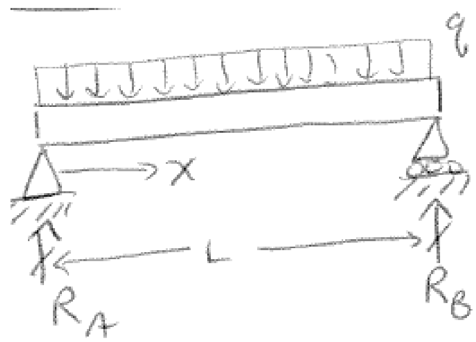
at x = 0, V(x) = P, since plus P on negative face. Use this to solve for $P = V(0) = C_1$. Thus,

$$V(x) = P$$

at x = 0, M(x) = 0, since no external couples or supports that can support a moment. Thus, $0 = M(0) = C_1 * 0 + C_2 = C_2$ so $C_2 = 0$ so

$$M(x) = Px$$

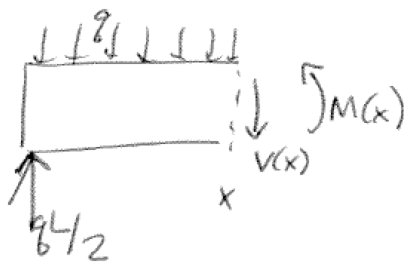
Example 2: beam supported at either end, with uniform distributed load.

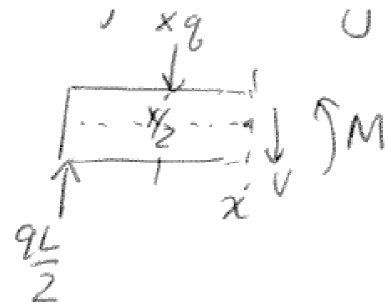


1. support reactions; total load is qL, and by symmetry, half this is applied by each support. Therefore, RA = RB = qL/2.

2. cut at x and consider left half only. Now resultant force from distributed load is qx, applied at x/2.

3. Draw V(x) and M(x). Note that they are drawn in the positive direction by deformation conventions.





4. Equilibrium Equations, at point x (for moments)

$$\frac{qL}{2} - qx - V(x) = 0$$

$$M(x) + \frac{x^2 q}{2} - \frac{xqL}{2} = 0$$

5. Solve

$$V(x) = \frac{qL}{2} - qx = q\left(\frac{L}{2} - x\right)$$

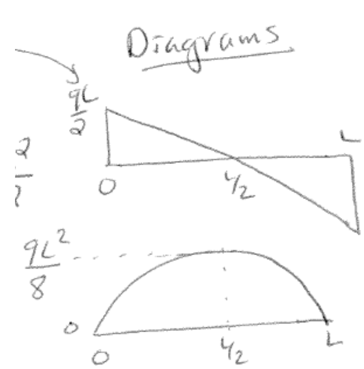
$$M(x) = -\frac{x^2 q}{2} + \frac{xqL}{2} = \frac{qx(L-x)}{2}$$

This is positive, consistent with beam bending into a smile.

For integration method:

$dV/dx = -q$, so $V(x) = -qx + C$. To find C, remember that $V(0) = R_A = \frac{qL}{2}$ from the diagram, and $V(0) = -q * 0 + C = C$, so $V(x) = \frac{qL}{2} - qx$. This is same as before.

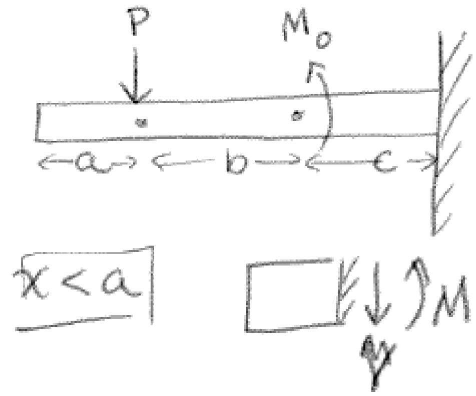
$\frac{dM}{dx} = V(x) = \frac{qL}{2} - qx$. Thus, $M(x) = C + \frac{qL}{2}x - \frac{q}{2}x^2$. To find this C, remember that $M(0) = 0$, so $C = 0$, so $M(x) = \frac{qL}{2}x - \frac{q}{2}x^2$, again same as before.



To make the diagram, calculate $V(0) = \frac{qL}{2}, V(L) = -\frac{qL}{2}$; $M(0) = M(L) = 0$; $V\left(\frac{L}{2}\right) = 0$; $M\left(\frac{L}{2}\right) = \frac{qL^2}{8}$.

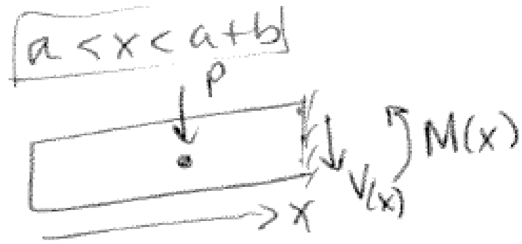
V(x) is linear, and M(x) is quadratic, so curved.

Example 3: beam with concentrated load and couple.



1. if I start from left, I don't need to calculate the support reactions at the start
2. cut at x within segment of length a:
3. Draw V(x) and M(x).
4. write equilibrium equations: $V(x) = 0$ and $M(x) = 0$.
5. already solved.

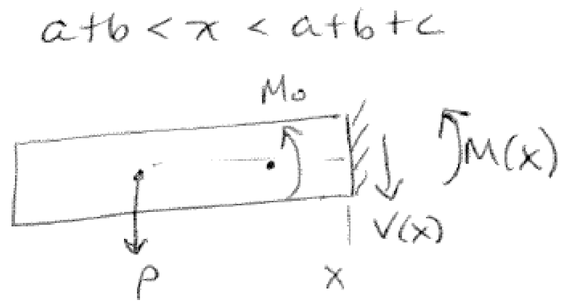
repeat within b:



4. Force and moment equations:
$$-V(x) - P = 0$$
$$M(x) + P(x - a) = 0$$
5. solve:
$$V(x) = -P$$

$M(x) = -P(x - a)$

repeat within c:



4. Force and moment equations:
$$-V(x) - P = 0$$
$$M(x) + P(x - a) + M_0 = 0$$
5. solve:
$$V(x) = -P$$
$$M(x) = -P(x - a) - M_0$$

Now do this by integration:

Between 0 and a, V(x) = 0 and M(x) = 0, obviously.

At x = a , V(x) jumps by –P, so $V(x) = -P$ from a to a+b. $\frac{dM}{dx} = V(x) = -P$ so $M(x) = -Px + C$. Use $M(a) = 0$ to find C; $M(a) = -Pa + C = 0$, so $C = Pa$. Thus $M(x) = -P(x - a)$.

At x =b, V(x) stays the same, so $V(x) = -P$ and M(x) jumps by $-M_0$, so $M(x) = -P(x - a) - M_0$

Draw the shear force and bending moment diagrams: