Sep 30 Lecture – Bioen 485

Introduction to Course Policies
Introduction to Modeling
Compartmental Models
Checking Solutions
Block Diagrams
Higher Order ODE's.

Paperless Course – see one & only handout for course websites

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Assignments & Grading:

40%: weekly labs (~8, or ~5% each) due Thurs
Read before lab section, start if possible, get help in lab, finish on own later, due Thurs. answer questions, show work, be legible.

10% Weekly Reading Discussions due Tues
"Read" article & write ideas for discussion
we just check some (1 vs 0 pts) & grade others (0.0 – 4.0)

25% Project 1, due Nov 9 – posted
Reproduce & Evaluate model paper
Read rubric carefully! Note that you apply skills from Reading & labs both.

25% Project 2, due Exam week.
Team project
Design model to solve a problem

Helping vs Cheating –
Do discuss, tutor, point out type of error
Don't give answers, provide code, ~~plg~~ plagerize
see web site

Intro to Modeling

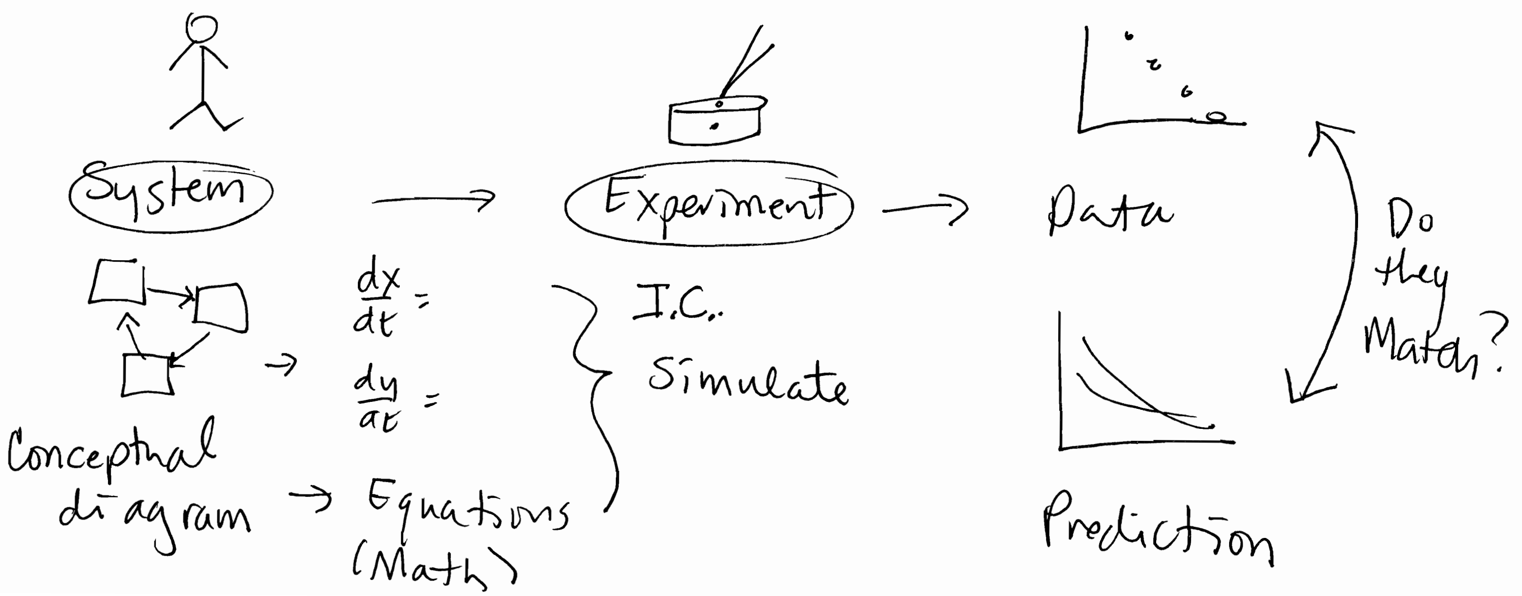
This course covers Engineering models,
Not: image analysis, bioinformatics, data analysis
Target Audience -
1) Interested in doing comp work, but mostly:
2) Experimentalist who wants engineering tools to interpret work, aid in quantitative hypothesis testing, or design.

☆ Engineering tools we cover include
- mostly linear & nonlinear ODE's (control systems)
- stochastic differential Equations
- Partial differential Equations (PDE's) } a little
  = transport, mechanics

☆ Types of Models

Black Box	Gray box
Empirical	Parametric
Describes Behavior	Describes Mechanism

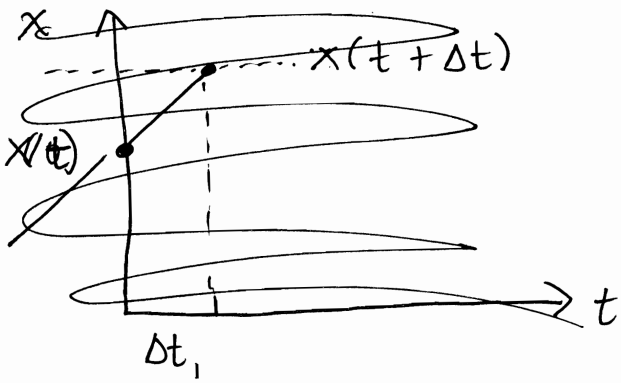
☆ Process of Modeling:

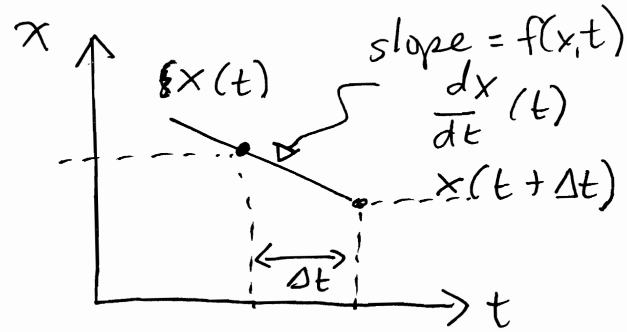


Recall ODE's:

$$\frac{dx}{dt} = f(x,t) + \text{Initial conditions} \rightarrow x(t) = ?$$

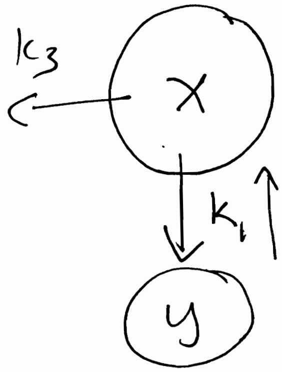
Analytical solutions - you learned in ODE class.
Numerical Solutions - you do this week in labs.





Compartmental Models

Compartment = ~~mass~~ amt. / mass / # of objects of material
(1 variable per compartment)
mass variables (mg) or conc. (mg/ml)



Transfer rate is proportional to amt. of material in old compart.
Constants of prop. are parameters

Conservation of mass translates directly to ODE's: ⇒ all terms = mass/time or #/time

If # or mass:

$$\frac{dx}{dt} = -k_1 x + k_2 y - k_3 x$$

k in sec^{-1}

$$\frac{dy}{dt} = +k_1 x - k_2 y$$

If conc.: X is mass/volume; V_x is volume; K is volume/sec

$$\frac{dX}{dt} \cdot V_x = -K_1 X + K_2 Y - K_3 X$$

$$\frac{dY}{dt} \cdot V_y = K_1 X - K_2 Y$$

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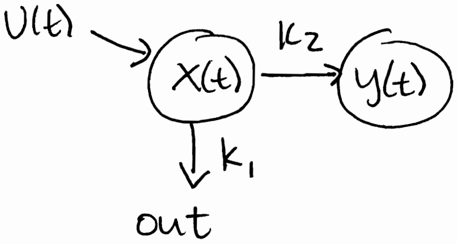
(4)

These are most useful for Chemistry & Biochemistry, or Organism modeling

Example:

- You are exposed to toxin, with dose D entering blood stream: $x(t)$ = mass in blood.
- Toxin is stored in fat at rate constant k_2. $y(t)$ = in fat.
- Toxin is cleared by kidneys (urine) at rate c. k_1.

Diagram: System:



Math:

$$\frac{dx}{dt} = -(k_1+k_2)\,x(t)$$

$$\frac{dy}{dt} = +k_2\,x(t)$$

Experiment:

inject dose D in blood at $t=0$.

$U(t) = D\,\delta(t)$ ⟸ delta (dirac) function. impulse, integrates to 1, but infinitely tall & narrow.

But, easier to do as I.C. in this case.

$x(0) = D, \quad y(0) = 0.$

Solve Analytically

$$\frac{dx}{x} = -(k_1+k_2)\,dt$$

$$\ln x = -(k_1+k_2)t + C$$

$$x = C_1 \exp(-(k_1+k_2)t)$$

$$\frac{dy}{dt} = k_2 C_1 e^{-(k_1+k_2)t}$$

$$y = -\frac{k_2}{k_1+k_2} C_1 e^{-(k_1+k_2)t} + C_2$$

~~Math~~ Use I.C.'s:

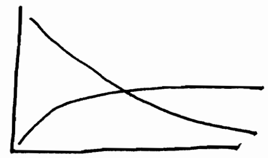
$$x(0) = D = C_1 e^0 = C_1$$

$$x = De^{-(k_1+k_2)t}$$

$$y(0) = 0 \Rightarrow C_2 = \frac{k_2}{k_1+k_2} De^0 = \frac{k_2}{k_1+k_2} D$$

$$y = \frac{k_2}{k_1+k_2} D \left[1 - e^{-(k_1+k_2)t}\right]$$

Solve numerically:
Need to pick a certain set of parameters
k_1, k_2, D.



* How do we know if this is correct?
Can we test Analytic or Numerical for likely mistakes?

(1) If you have both, plot vs each other for same params.

(2) Extreme times: $t=0$ should match I.C.

$x(0) = De^{-0} = D$ ✓ $y(0) = \frac{k_2}{k_1+k_2} D[1-e^0] = 0$ ✓

$t \to \infty$: what is expected?

x has outflows, but no inflows, so $\to 0$.

$x(\infty) = De^{-\infty} = 0$ ✓

y has inflows, no outflows, so > 0.

$y(\infty) = \frac{k_2}{k_1+k_2} D(1-e^{-\infty}) = \frac{k_2}{k_1+k_2} D > 0$ ✓

(3) Extreme Parameter Values vs common sense

$D \uparrow \Rightarrow y(\infty)\uparrow$ - makes sense here.

$k_1 >> k_2$: $x(t) \sim De^{-k_1 t}$

$y(t) \sim D\frac{k_2}{k_1}[1-e^{-k_1 t}]$

$y(t)$ gets very small as $\frac{k_2}{k_1} << 1$.

makes sense; ~~mos~~ little goes to fat. ✓

$k_2 >> k_1$: $x(t) \sim De^{-k_2 t}$

$y(t) \sim D(1-e^{-k_2 t}) \Rightarrow$ all goes into fat. ✓

(4) Physiological: $x > 0, y > 0 \ \forall t$. ✓

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Higher-Order ODE's

Can we recast higher order ODE?

To solve numerically, you will see in labs today, you need:

$$\frac{dx_1}{dt} = f_1(x_1, x_2, \ldots x_n)$$

$$\frac{dx_2}{dt} = f_2(x_1, x_2, \ldots x_n)$$

etc. – all 1^{st} order ODE's.

What if you have a 2^{nd} order ODE?

Example: van der Pol oscillator, used to model heart beat or action potentials of neurons

$$\frac{d^2x}{dt^2} = \mu(\overset{x^2-1}{\cancel{1-x^2}}) \frac{dx}{dt} + x = \cancel{0}$$

define $z_1 = x$

$z_2 = dx/dt$.

then $\frac{dz_1}{dt} = \frac{dx}{dt} = z_2$ is first ODE

$\frac{dz_2}{dt} = \frac{d^2x}{dt^2}$

So $\frac{dz_2}{dt} = \mu(\overset{z_1^2-1}{\cancel{\quad}}) z_2 + z_1$ is 2^{nd} ODE.

$$\begin{bmatrix} dz_1 \\ dz_2 \end{bmatrix} = \begin{bmatrix} z_1 \\ \mu(z_1^2-1)z_2 - z_1 \end{bmatrix}$$

This Example is Explained in MATLAB tutorials & in Wikipedia very nicely.