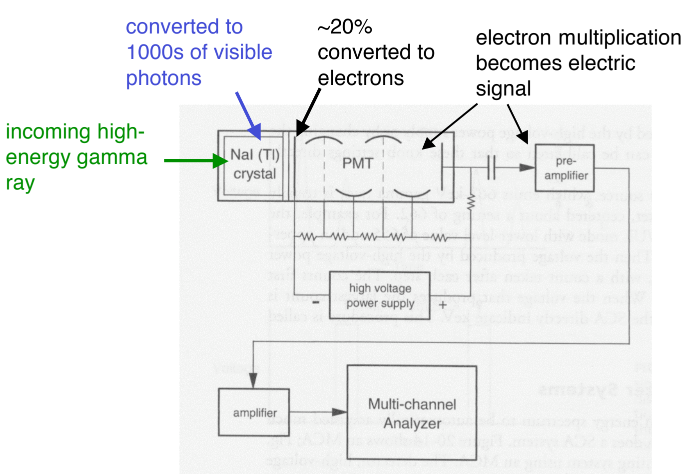
Semiconductor Detectors
vs.
Scintillator+PMT Detectors

- Semiconductors are emerging technology —- Scint.PMT systems relatively unchanged in 50 years. NaI(Tl) excellent for single-photon, new scintillation materials being developed for PET (lutetium-based, GSO, $LaBr_3$, $LaCl_3$).
- Direct detection in semicond. permits **superior energy resolution** —- Scint.PMT is indirect (2-step process) with marginal quantum efficiency, which limits energy resolution.
- Purity/growth of semicond. is still a challenge (i.e. expensive) —- Scint.PMT is well established stable technology.
- Semicond. can be finely pixilated for spatial resolution (μm theoretically) —- Scint.PMT pixel size cannot be made arbitrarily small (~ 0.5mm – 1.0mm lower limit thus far). Note spatial resolution / sensitivity trade-offs!
- Semicond. cannot be made arbitrarily thick for photon absorption efficiency (sensitivity) —- Scintillation crystals can be arbitrarily thick. Note spatial resolution / sensitivity / energy resolution trade-offs!
- Semicond. require more sensitive electronics generally, and less amiable to signal multiplexing, resulting in potential need of many, many more processing channels.

Sample Spectroscopy System Hardware

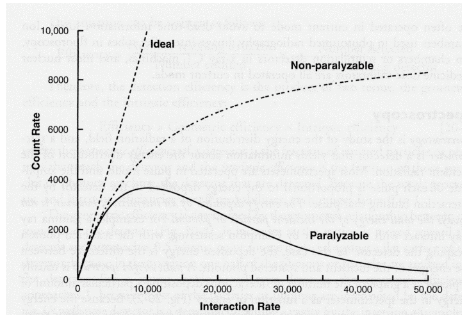


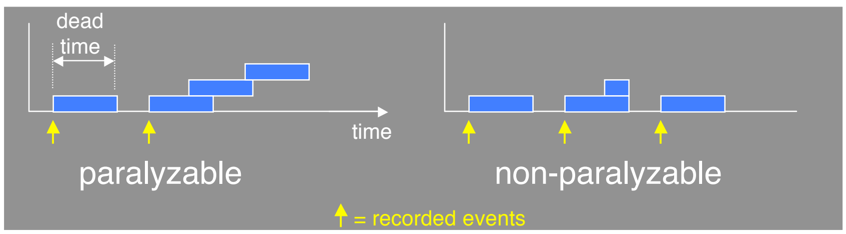
(statistical uncertainties!)

larger current or voltage
|
more electrons
|
more scintillation photons
|
higher gamma energy

From: The Essential Physics of Medical Imaging (Bushberg, et al)

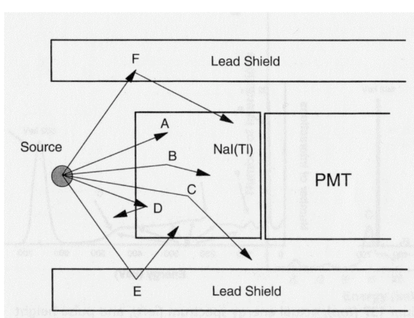
Interaction Rate and Dead-time





From: The Essential Physics of Medical Imaging (Bushberg, et al)

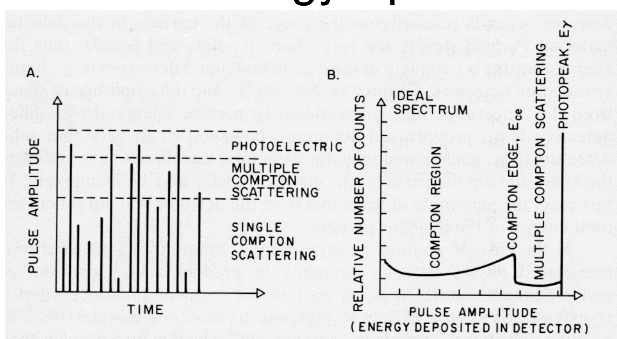
Interactions of Photons with a Spectrometer



A. Photoelectric
B. Compton + Photoelectric
C. Compton
D. Photoelectric with characteristic x-ray escape
E. Compton scattered photon from lead shield
F. Characteristic x-ray from lead shield

Sample Spectroscopy System
Output

Ideal Energy Spectrum



counting mode

Energy Resolution

Realistic Energy Spectrum

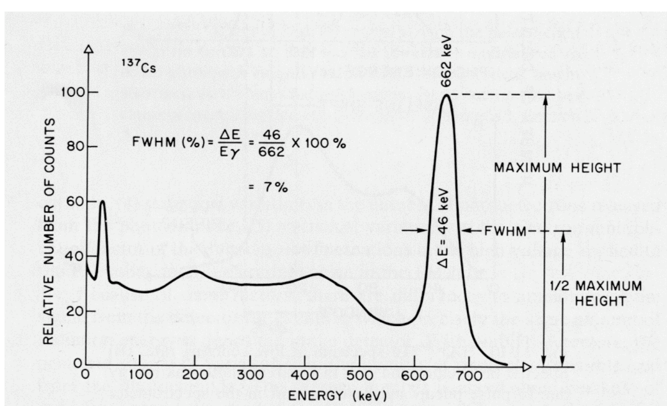
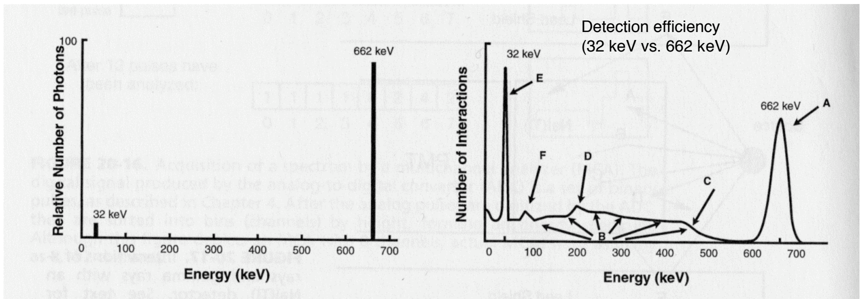


Fig. 11-11. Calculation of FWHM energy resolution of a NaI(Tl) detector for ^{137}Cs 662 keV γ rays.

From: Physics in Nuclear Medicine (Sorenson and Phelps)

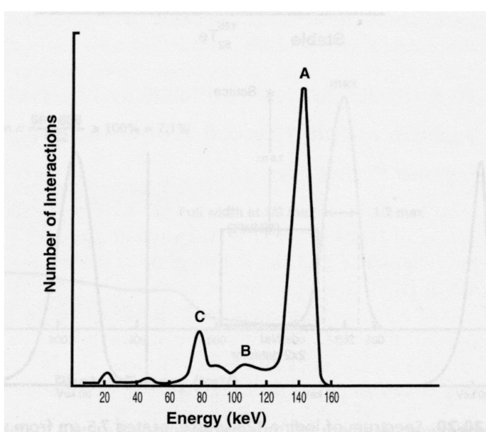
Sample Spectrum (Cs-137)



A. Photopeak
B. Compton continuum
C. Compton edge
D. Backscatter peak
E. Barium x-ray photopeak
F. Lead x-rays

From: The Essential Physics of Medical Imaging (Bushberg, et al)

Sample Spectrum (Tc-99m)



A. Photopeak
B. Photoelectric with iodine K-shell x-ray escape
C. Absorption of lead x-rays from shield

Note absence of Compton continuum

Why?

From: The Essential Physics of Medical Imaging (Bushberg, et al)

Effects of Pulse Pileup

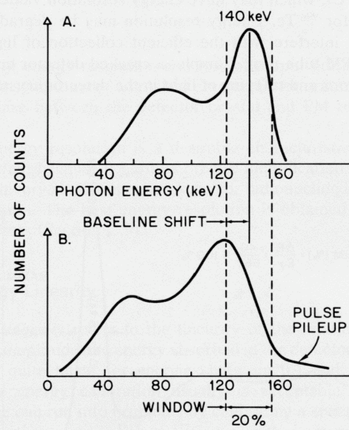


Fig. 11-10. (A) ^{99m}Tc spectrum at low counting rate. (B) Spectral broadening and shift in apparent photopeak energy due to pulse pileup and baseline shift in the spectrometer amplifier at high counting rate.

From: Physics in Nuclear Medicine (Sorenson and Phelps)

Calibrations

- Energy calibration (imaging systems/spectroscopy)
  - Adjust energy windows around a known photopeak
  - Often done with long-lives isotopes for convenience
    Cs-137: $E\gamma$= 662 keV (close to PET 511 keV), $T_{1/2}$=30yr
    Co-57: $E\gamma$= 122 keV (close to Tc99m 140 keV), $T_{1/2}$=272d

- Dose calibration (dose calibrator)
  - Measure activity of know reference samples (e.g., Cs-137 and Co-57)
  - Linearity measured by repeated measurements of a decaying source (e.g., Tc-99m)

Overview of today's lecture

- Emission vs. Transmission Imaging
- Nature of nuclear radiation
  - Isotopes used in nuclear medicine
- Detection methods
- **Counting statistics**
- Imaging systems
  - Planar gamma scintigraphy

Random Processes in Nuclear Medicine

Radiation decay and detection are random processes described by probabilistic statistical distribution functions; *P(x, t,…)*.

The means and variances of these probability distributions are part of the data analysis process in nuclear medicine.

Examples governed by quantitative law:

Radioactive decay in time;

$P(t)dt = exp(-t/\tau)dt$ = probability atom will decay between the times t and $t+dt$. [$\tau = T_{1/2}$ / ln(2), $T_{1/2}$ = half-life of isotope]

Gamma ray absorption in medium;

$P(x)dx = exp(-\mu x)dx$ = probability γ will be absorbed between x and $x+dx$. μ = attenuation coefficient of material at specified energy.

Examples governed by empirical law:

Scintillation photons created upon absorption of a gamma ray;

There is a mean number created per unit of absorbed energy, and an associated variance, for each scintillation material. Have to work with law of averages —-> probabilities.

Scintillation photon creates a photo-electron at a PMT photocathode;

Photocathodes have an intrinsic quantum efficiency (QE) – the chance that any given photon creates a photo-electron is QE%. After observing 1000s of photons, QE% will have generated photo-electrons.

Characterizing Random Phenomena (and Errors)

Measures of Central Tendency:

•Mode – Most Frequent Measurements (not necessarily unique)

•Median – Central Value dividing data set into 2 equal parts (unique term)

•Mean (Arithmetic Mean) $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

Measures of Dispersion:

•Range – Difference of largest and smallest values

•Variance – Measures dispersion around mean: $\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$

•Standard Deviation: $\sigma = \sqrt{\sigma^2}$

Statistical Models for Random Trials

- **Binomial Distribution**
  - Random independent processes with two possible outcomes
- **Poisson Distribution**
  - Simplification of binomial distribution with certain constraints
- **Gaussian or Normal Distribution**
  - Further simplification if average number of successes is large (e.g., >20)

Characterizing Random Errors With a Distribution

Binomial Distribution - Independent trials with two possible outcomes

Binomial Density Function: $$P_{\text{binomial}}(r)=\frac{n!}{r!(n-r)!}p^r(1-p)^{n-r}$$

Probability of r successes in n tries when p is probability of success in single trial

$$\bar{x} = pn \qquad \sigma = \sqrt{pn(1-p)}$$

Example: What is the probability of rolling a 1 on a six sided die exactly 10 times when the die is rolled for a total of 24 times.
r = 10, n = 24, p = 1/6, $P_{\text{binom}}(r=10)$ = 0.0025 ~ 1 in 400

Poisson Distribution - Limiting form of binomial distribution as p ⇒ 0 and n ⇒ ∞

As in nuclear decay. Have many, many nuclei, probability of decay and observation of decay very, very small

$$P_{\text{Poisson}}(r)=\frac{\mu^r\exp(-\mu)}{r!}$$

Only one parameter: μ = Mean = Variance ---> $\bar{x} = \sigma^2$

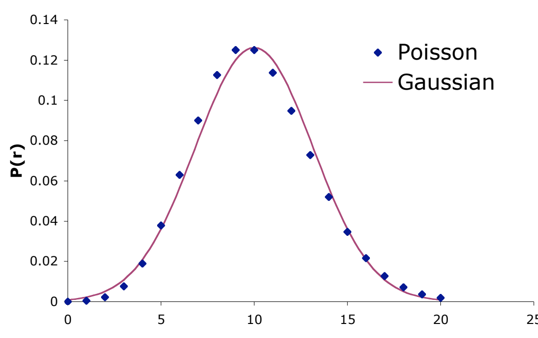
Example: A radioactive source is found to have a count rate of 5 counts/second. What is probability of observing no counts in a period of 2 seconds?

$$P_{\text{Poisson}}(r=0)=\frac{(\mu=10)^{r=0}\exp(-(\mu=10))}{(r=0)!}=4.54*10^{-5}$$

Gaussian (Normal) Distribution

- Symmetric about the mean
- Useful in counting statistics because distributions are **approximately** normal when n>20
- Variance and mean not necessarily equal

$$P_{\text{Gaussian}}(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$



Variance / Error in Counting Photons

Poisson process: mean = variance

—-> Number measured, *N*, is best estimate of mean number for that phenomenon (e.g. *N* emitted gamma rays per unit time, *N* scintillation photons per absorbed gamma ray, …)

—-> variance = mean = *N*

—-> standard deviation: $\sigma = \sqrt{\text{variance}} = \sqrt{N}$

Relative error, *e*, in counting experiments:
(signal-to-noise ratio SNR N/σ = $\sqrt{N}$)

$$e = \frac{\sigma}{\bar{x}} = \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}}$$

Relative error *decreases* as number of events *increases*

Emphasizes the importance of detecting as many gamma rays as possible, and the sensitivity (absorption efficiency) of nuclear medicine cameras

- This applies to *individual image pixels* in nuclear medicine (also applies to x-ray imaging, but number of photons is not limited there)
- Also applies to *energy resolution* in radiation detection systems

Simple Propagation of Error

- Quantities of interest are often determined from several measurements prone to random error.
- If the quantities are *independent*, then add independent contributions to error in quadrature as follows:

The simplest examples are addition, subtraction, and multiplication by a constant.

If the quantities *a* and *b* are measured with known error δ_a and δ_b, then the error in the quantities *x, y, z* when

$x = a + b$

$y = a - b$

$z = k*a$, k = constant (no error)

are:

$$\delta_x = \delta_y = \sqrt{\delta_a^2 + \delta_b^2}$$

$$\delta_z = k*\delta_a$$

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The Planar Gamma Camera