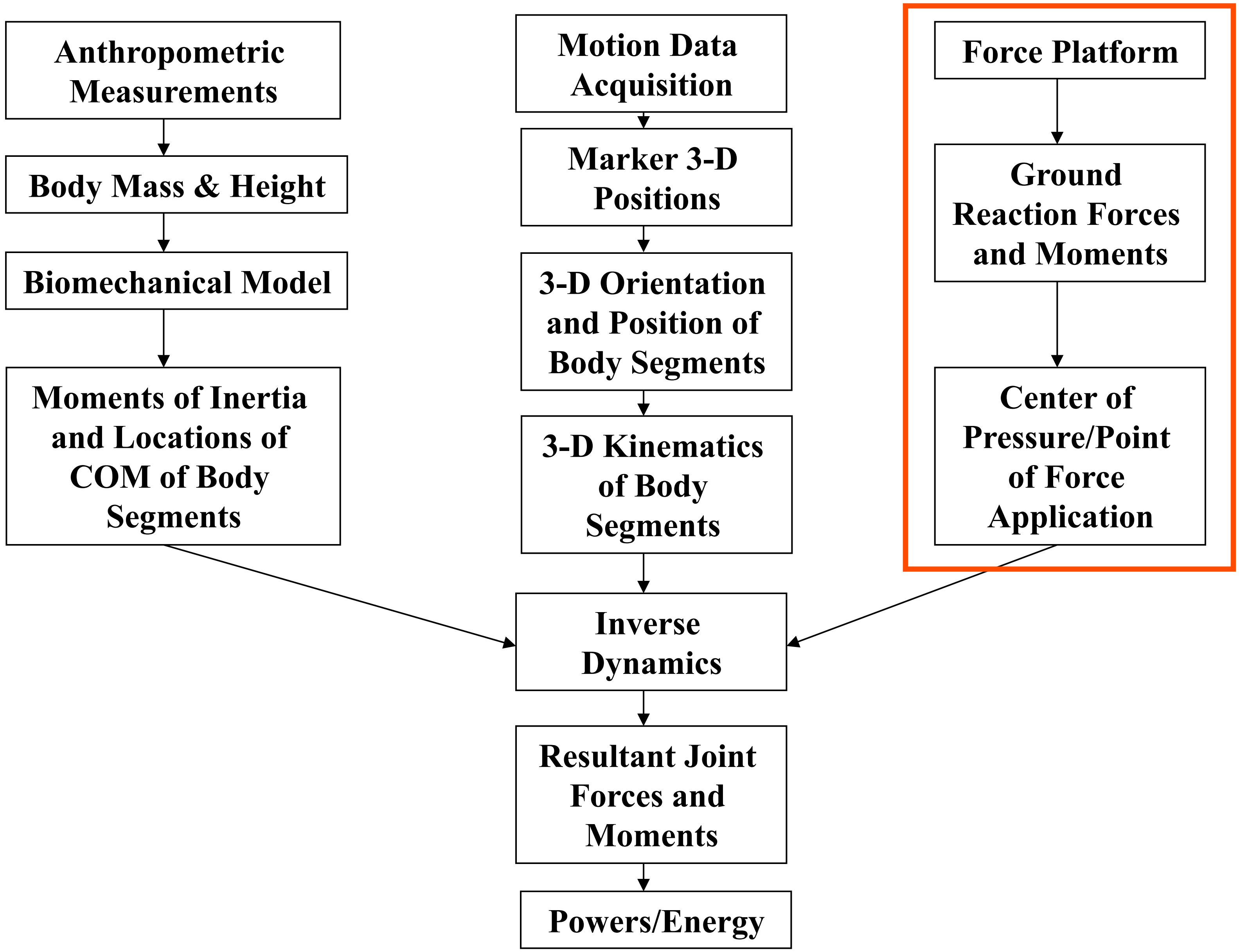
Three-Dimensional Biomechanical Analysis of Human Movement

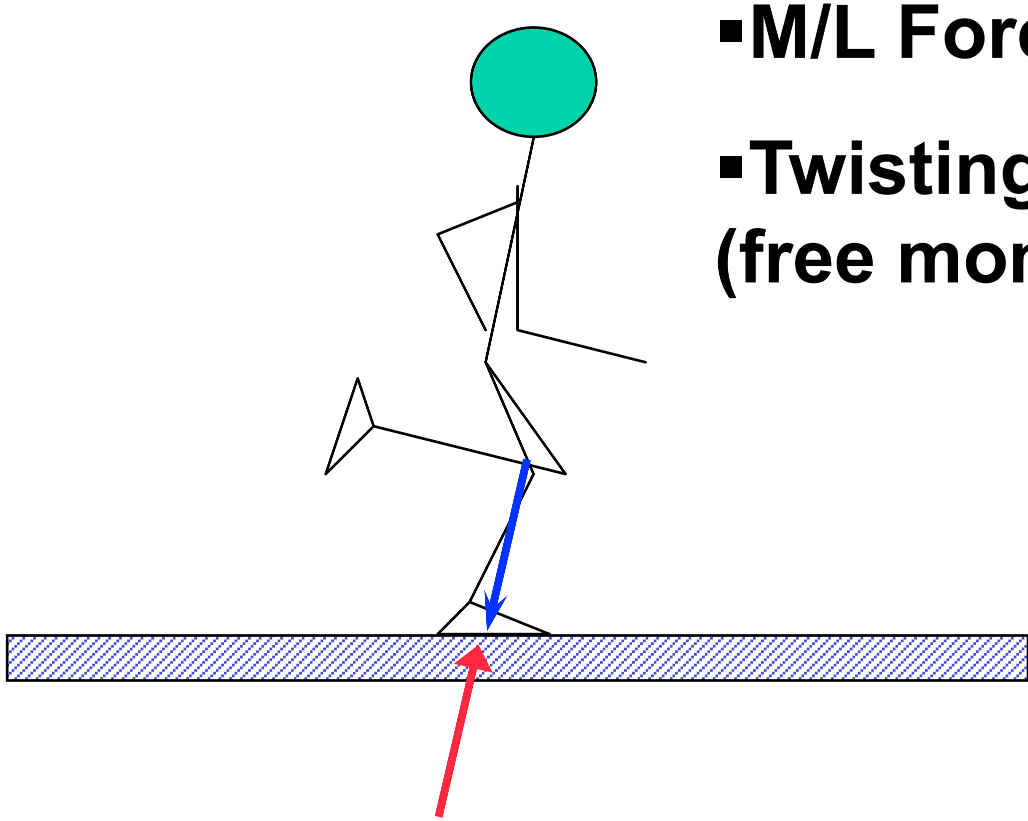


Ground Reaction Forces

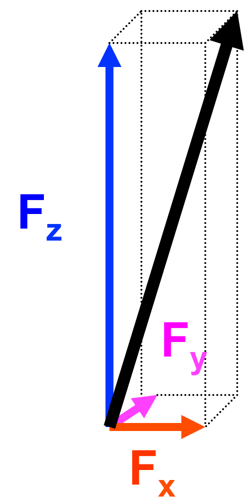
- Vertical Force
- A/P Force
- M/L Force

} Horizontal Shear

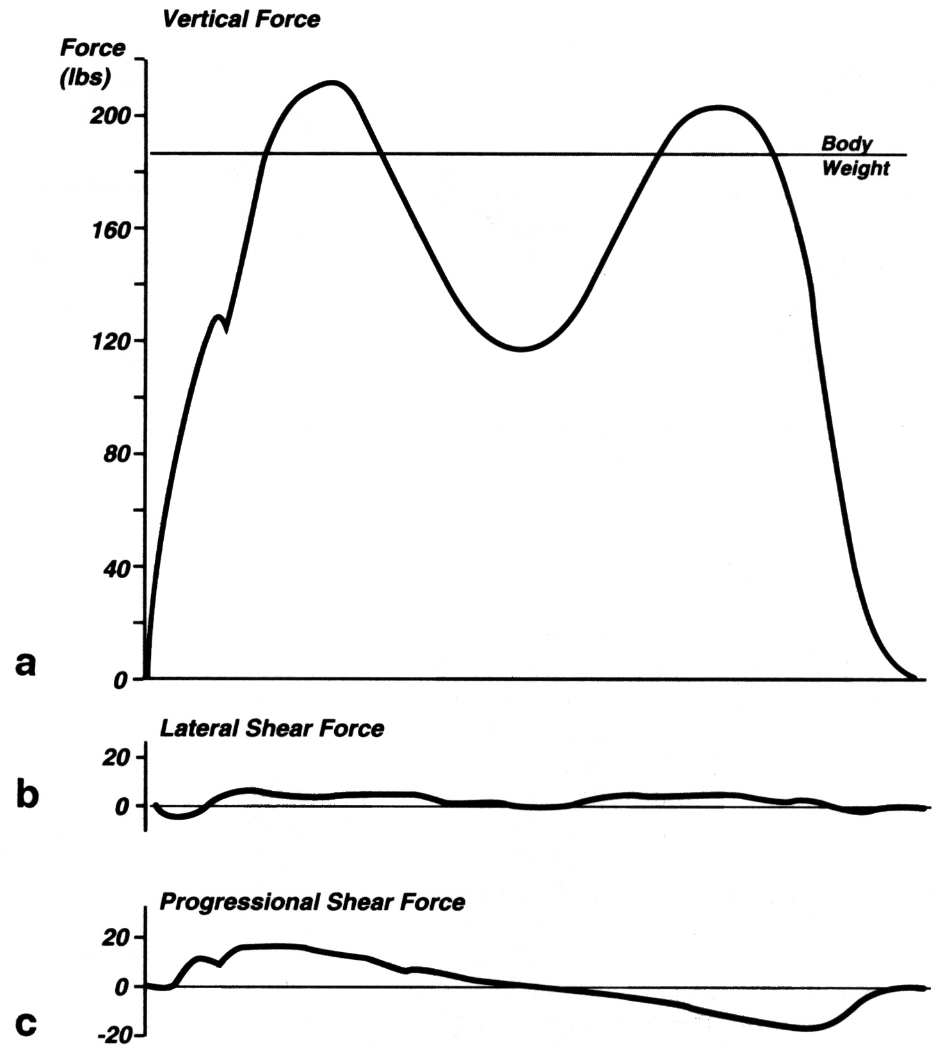
- Twisting Torque (free moment)



$F = F_x + F_y + F_z$



Ground Reaction Forces



Force Platforms

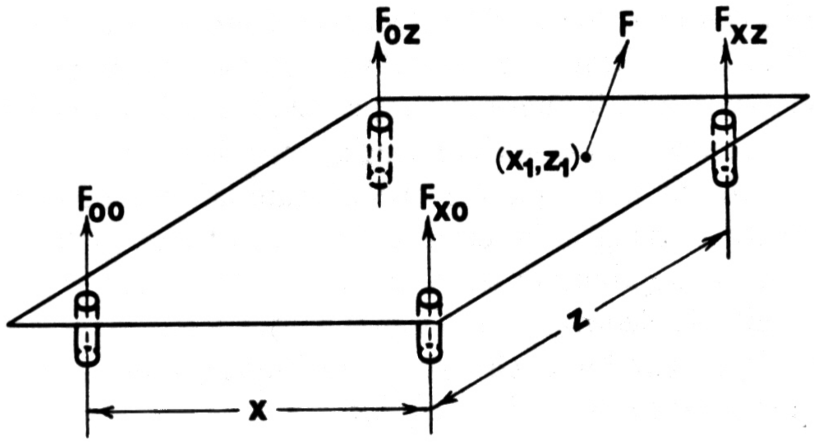
Piezoelectric type

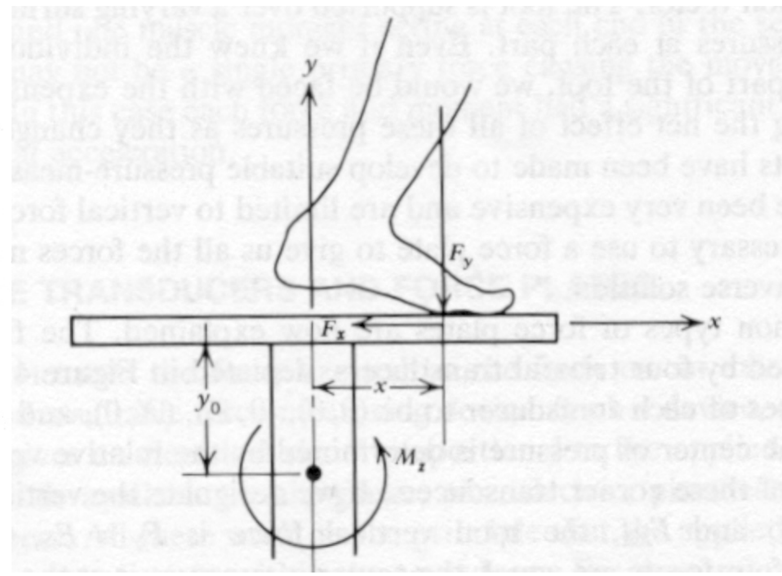
A piezoelectric material, quartz crystal, will generate an electric charge when subject to mechanical strain. Quartz crystals are cut into disks that respond to mechanical strain in a single direction.

Strain Gauge type

Use strain gauge to measure stress in machined aluminum transducers (load cells). Deformation of the material causes a change in the resistance and thus a change in the voltage (Ohms Law: $V = I * R$).

Two Common Types of Force Plates

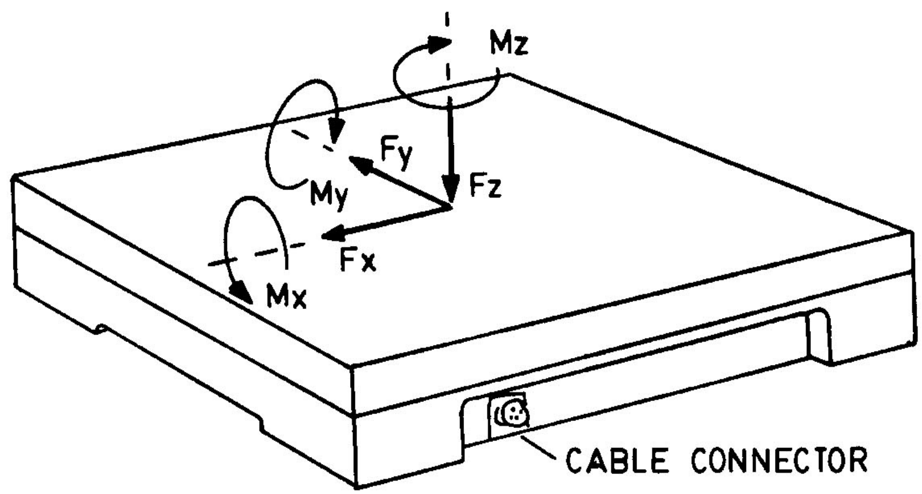




A flat plate supported by four triaxial transducers

A flat plate supported by one centrally instrumented pillar

AMTI (Strain Gauge) Force Platform



Output signals from the platform:

F_x: the anterior/posterior force

F_y: the medial/lateral force

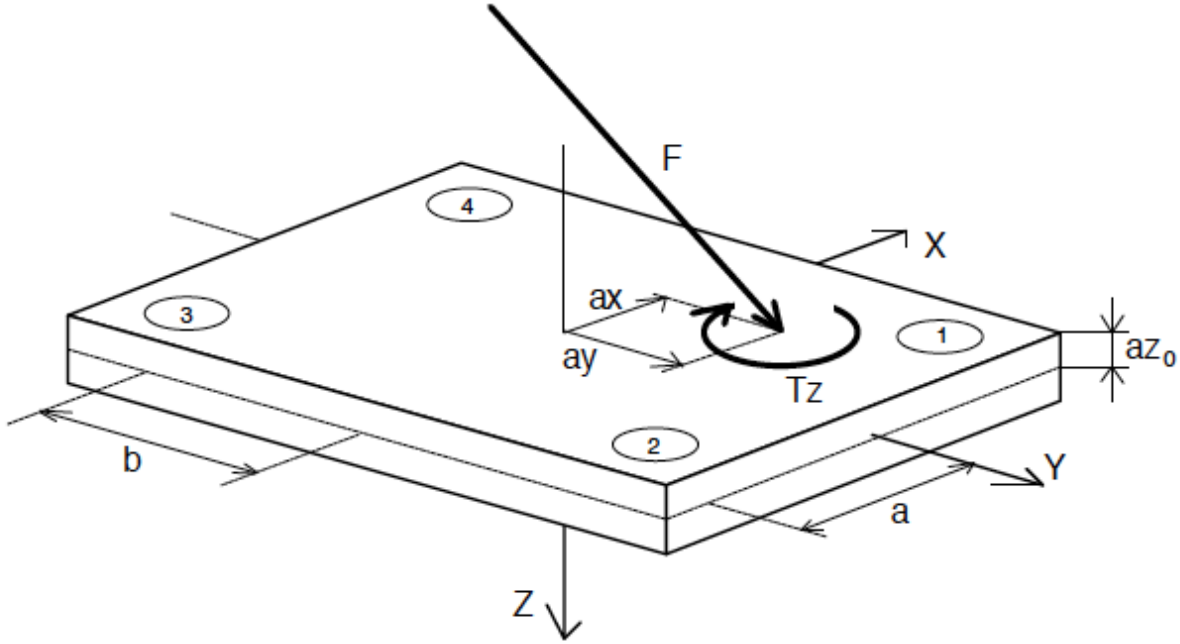
F_z: the vertical force

M_x: the moment about the anterior/posterior axis

M_y: the moment about the medial/lateral axis

M_z: the moment about the vertical axis

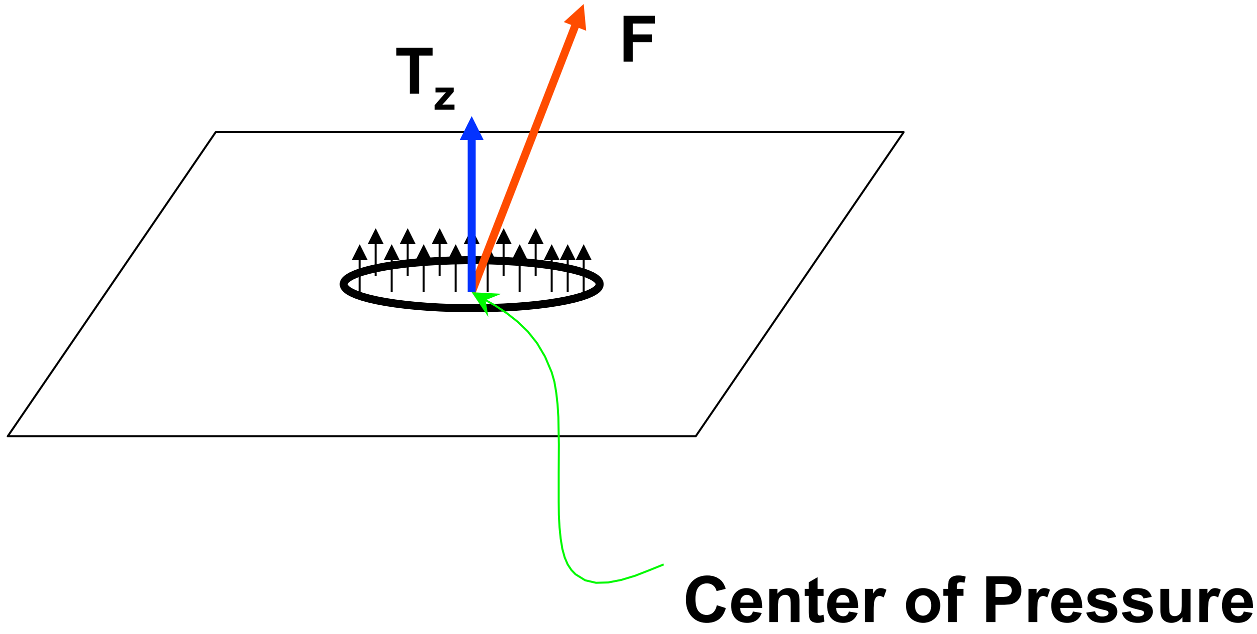
Kistler (Strain Gauge) Force Platform



Force plate output signals

Output signal	Channel	Description
fx12	1	Force in X-direction measured by sensor 1 + sensor 2
fx34	2	Force in X-direction measured by sensor 3 + sensor 4
fy14	3	Force in Y-direction measured by sensor 1 + sensor 4
fy23	4	Force in Y-direction measured by sensor 2 + sensor 3
fz1 ... fz4	5 ... 8	Force in Z direction measured by sensor 1 ... 4

Center of Pressure (COP)

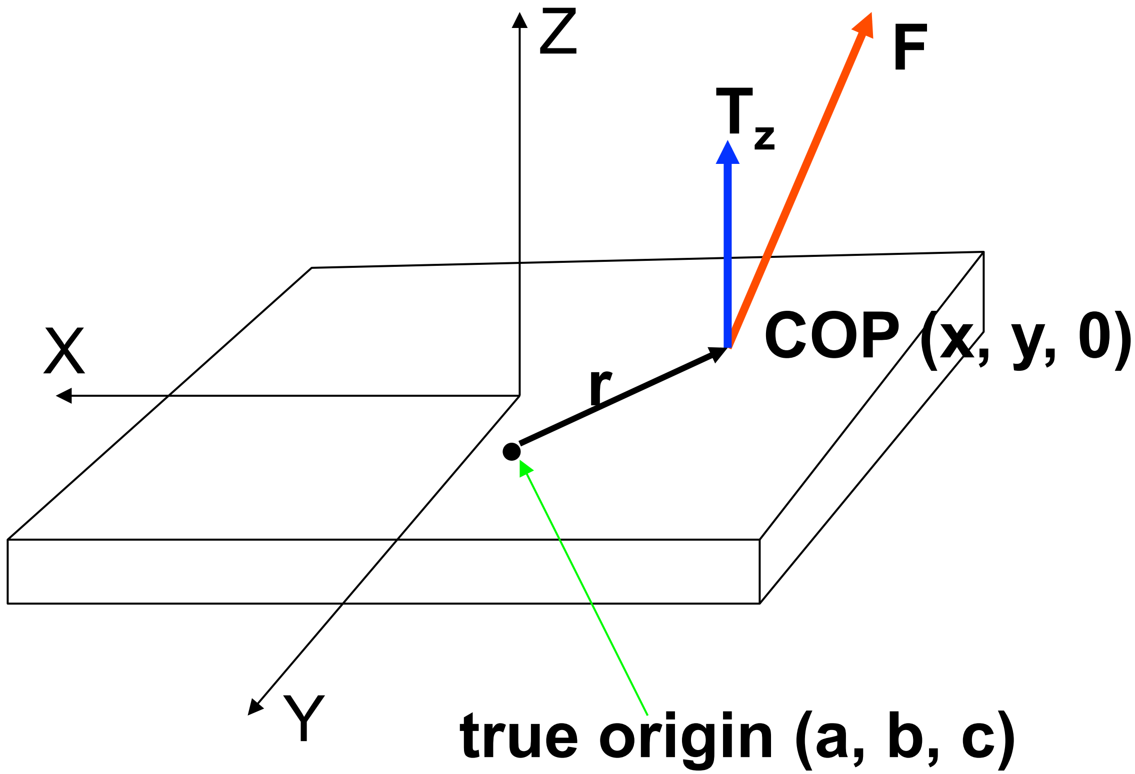


All the forces acting between the foot and the ground can be summed and yield a single reaction force vector (F) and a twisting torque vector (T_z about the vertical axis). Under normal condition there is no physical way to apply T_x and T_y.

The point of application of the ground reaction force on the plate is the center of pressure (COP).

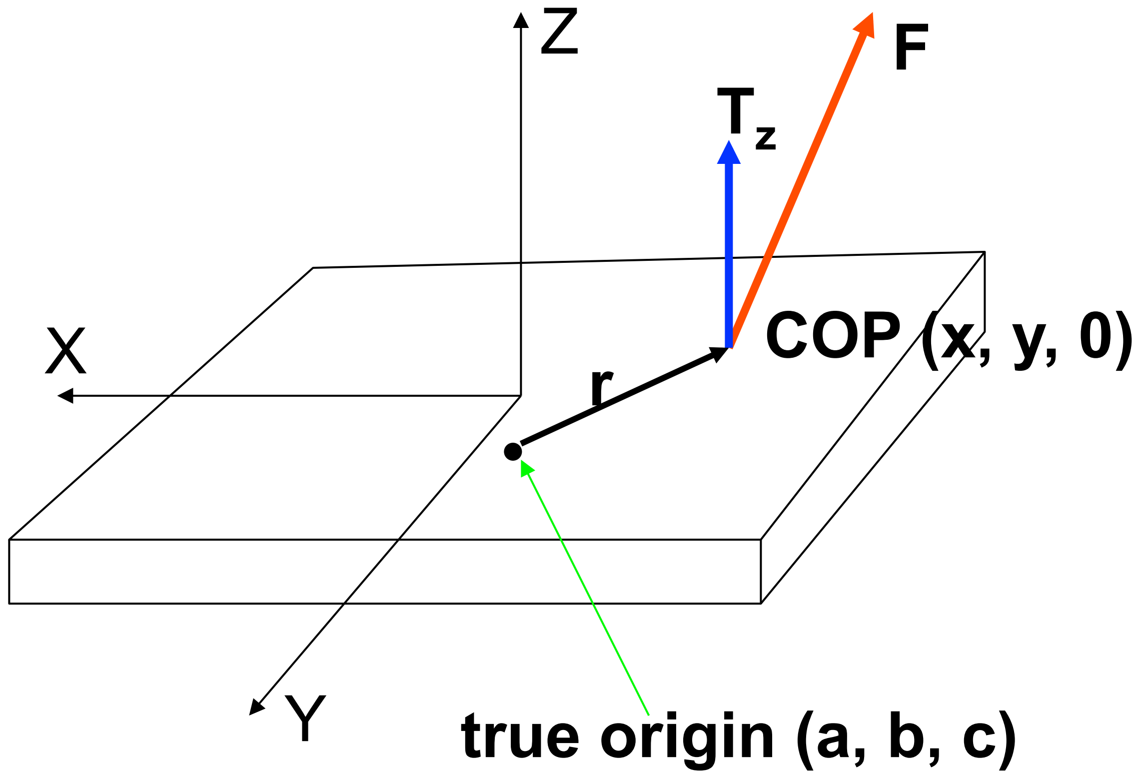
Computation of the COP

Generally, the true origin of the plate is not at the geometric center of the plate surface. The manufacturer usually provides the offset data.



The moment measured from the plate is equal to the moment caused by **F** about the true origin plus T_z.

Computation of the COP



$$M = r \times F + T_z$$

Computation of the COP

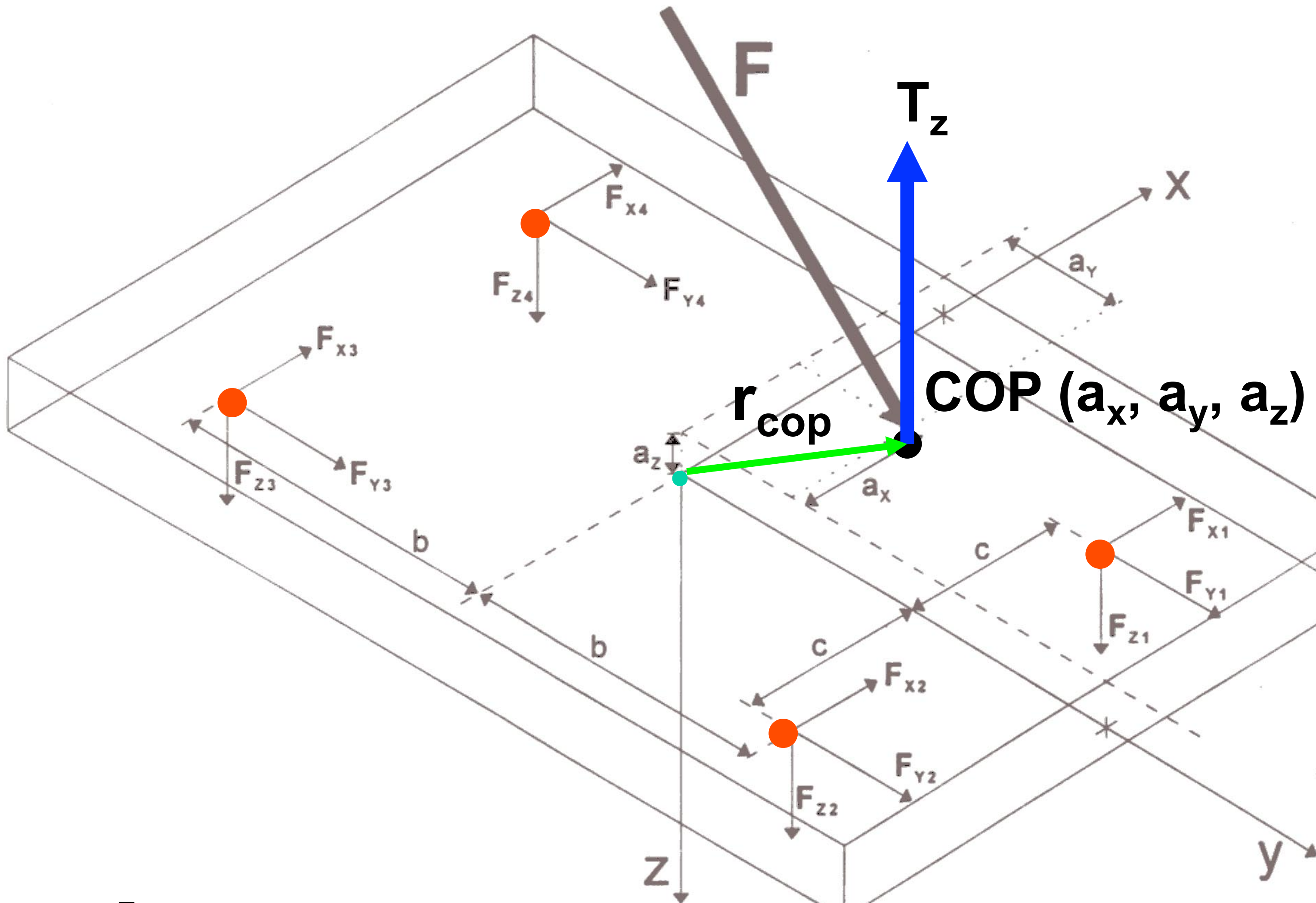
$$M = r \times F + T_z$$

$r = (x-a, y-b, -c)$ known: a, b, c; unknown: x, y

$F = (F_x, F_y, F_z)$ force values from plate outputs

$T_z = (0, 0, T_z)$ unknown: T_z

$M = (M_x, M_y, M_z)$ torque values from plate outputs



$$r_{cop} = [a_x, a_y, a_z]$$

$$M_{GRF} = r_{cop} \times F + T_z$$

Computation of the COP

$M_x = (y\text{-}b)\ F_z + c\ F_y$

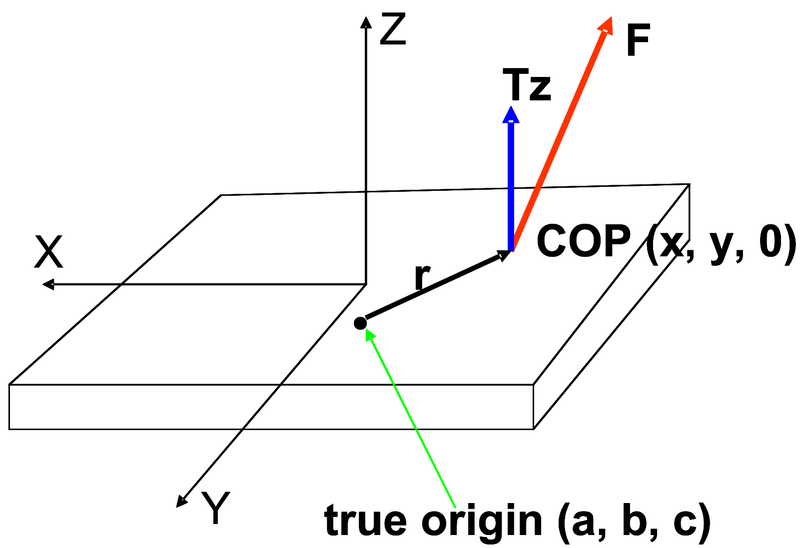
$M_y = -c\ F_x - (x\text{-}a)\ F_z$

$M_z = (x\text{-}a)\ F_y - (y\text{-}b)\ F_x + T_z$

$x = -(M_y + cF_x)/F_z + a$

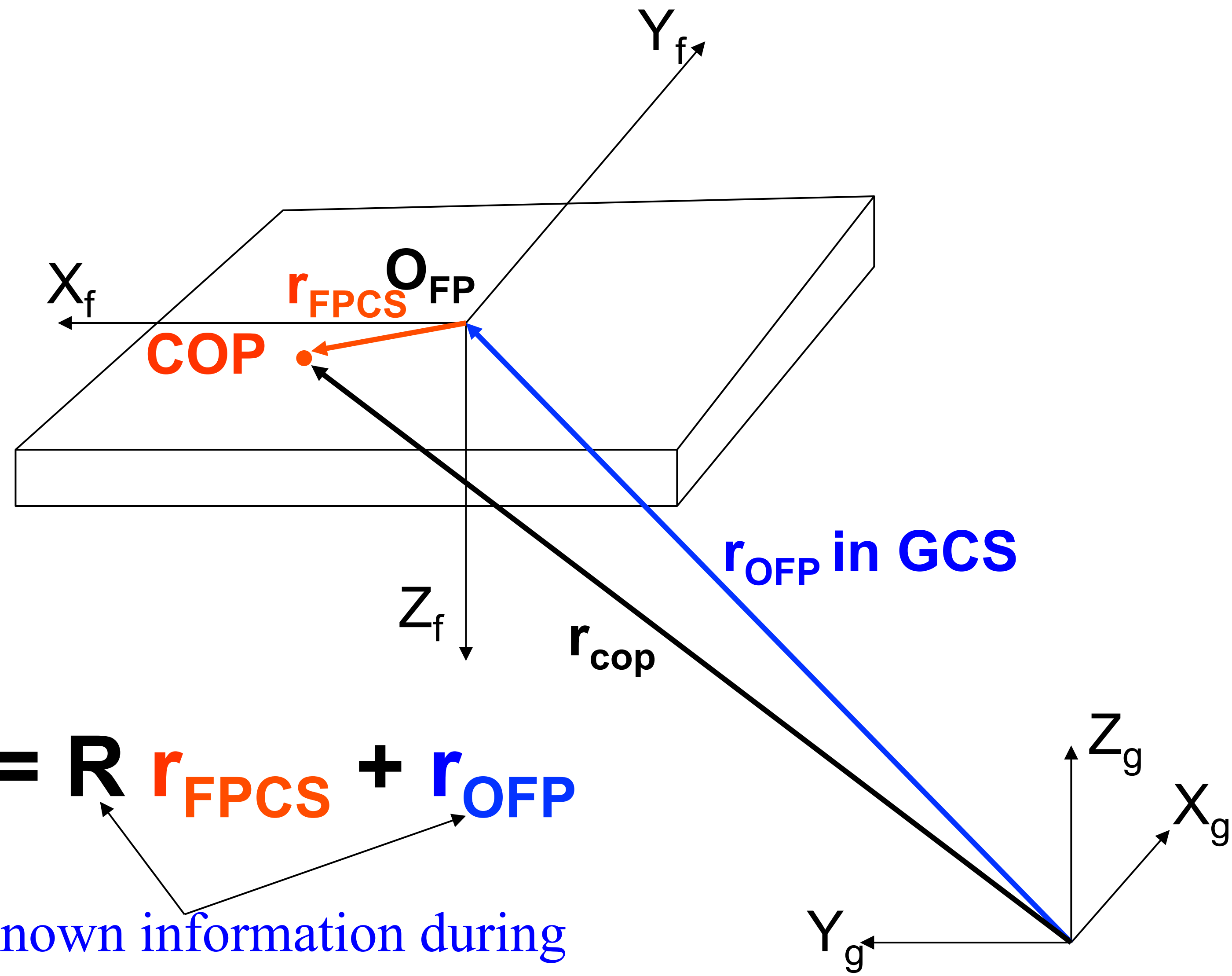
$y = (M_x - cF_y)/F_z + b$

$T_z = M_z - (x\text{-}a)F_y + (y\text{-}b)F_x$



$M = r \times F + Tz$

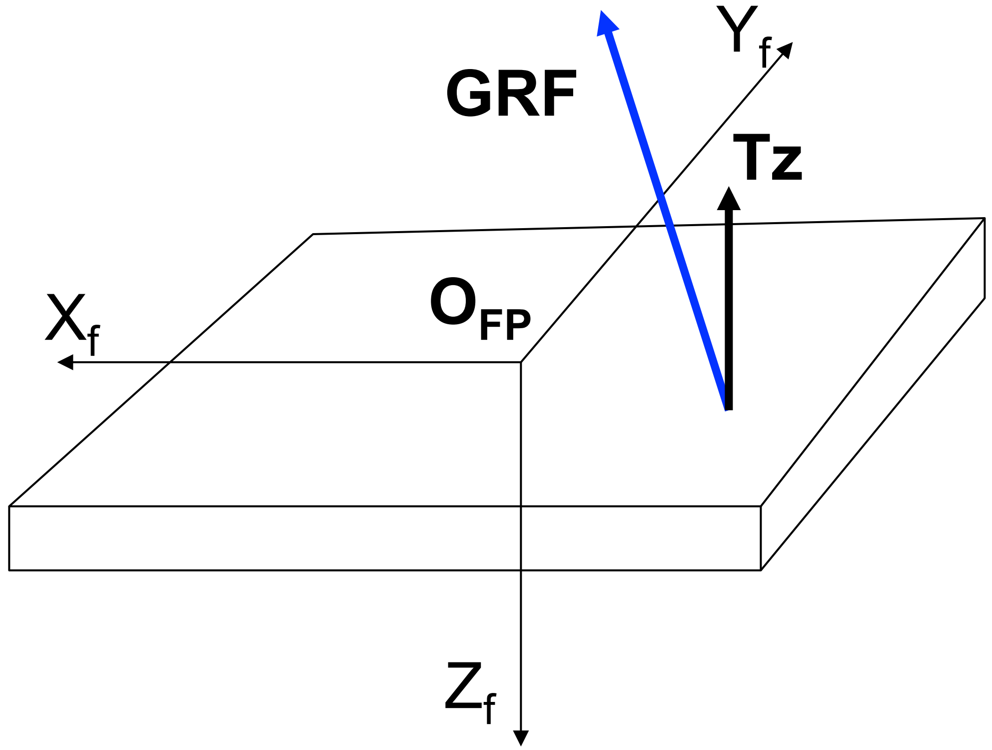
Force Plate Coordinate System



$$r_{cop} = R\, r_{FPCS} + r_{OFP}$$

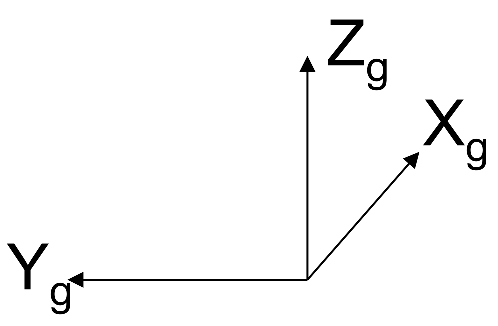
known information during laboratory setup (calibration)

GRF in Global Coordinate System



$$GRF_{GCS} = R\ GRF_{FPCS}$$

$$Tz_{GCS} = R\ Tz_{FPCS}$$



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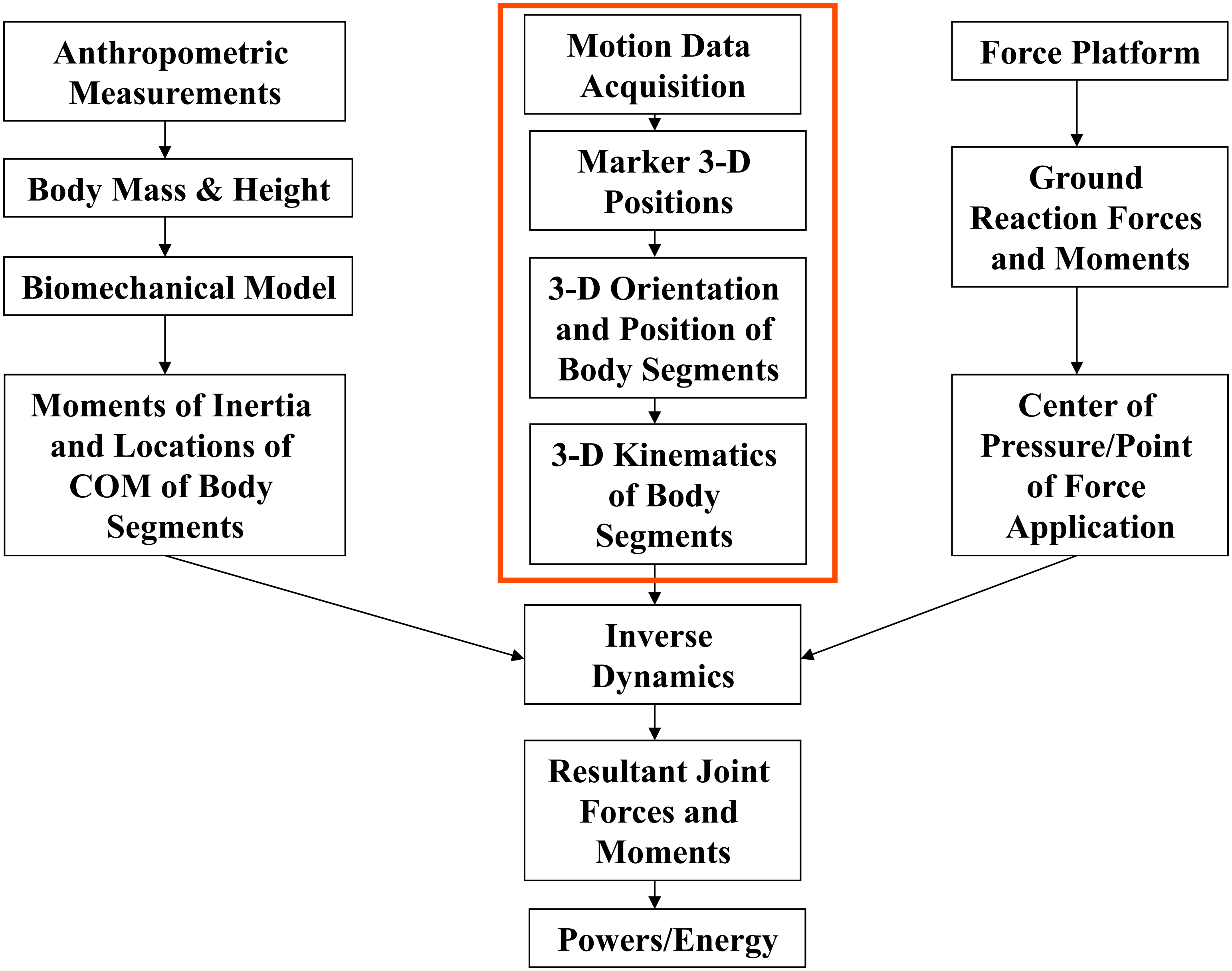


Determining Body Segment and Joint Kinematics

Three-step procedure

- Three-dimensional marker positions
- Body segment (limb) positions and orientations (assuming rigid body)
- Relative orientation and movement of limb segments (joint kinematics)

Step #1 … Marker Position

- 3-D reconstruction from several 2-D images
  - Each point seen by at least 2 cameras

- Vicon system displays reconstructed points
  (saves you a ton of time)

- Now, for Step #2

Step #2 … Segment positions and orientations

- Defining segment coordinate systems
  - Position described by the segment origin
  - Orientation provides the "absolute" angles

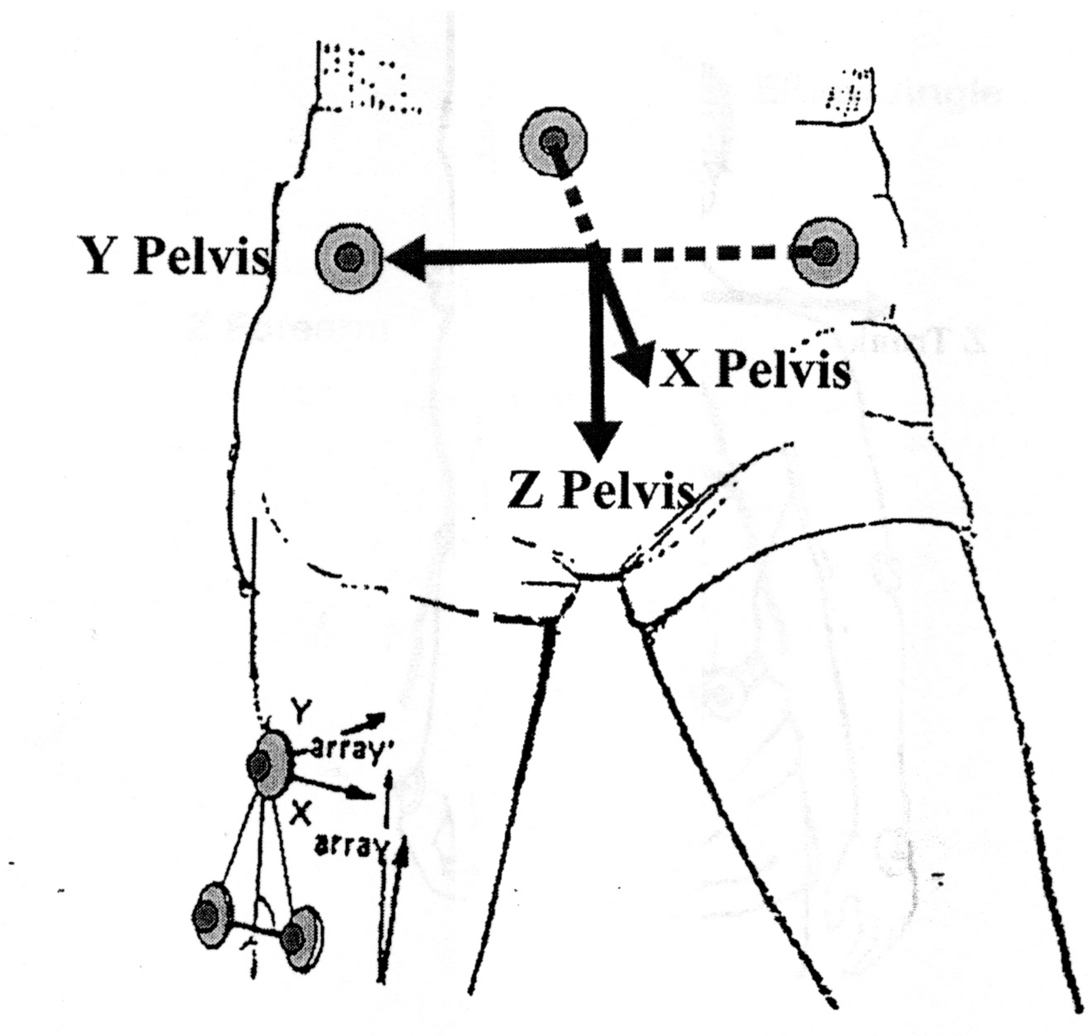
Segment definitions
(the need for marker sets)

- Absolutely necessary for kinematic variables to be measured/calculated

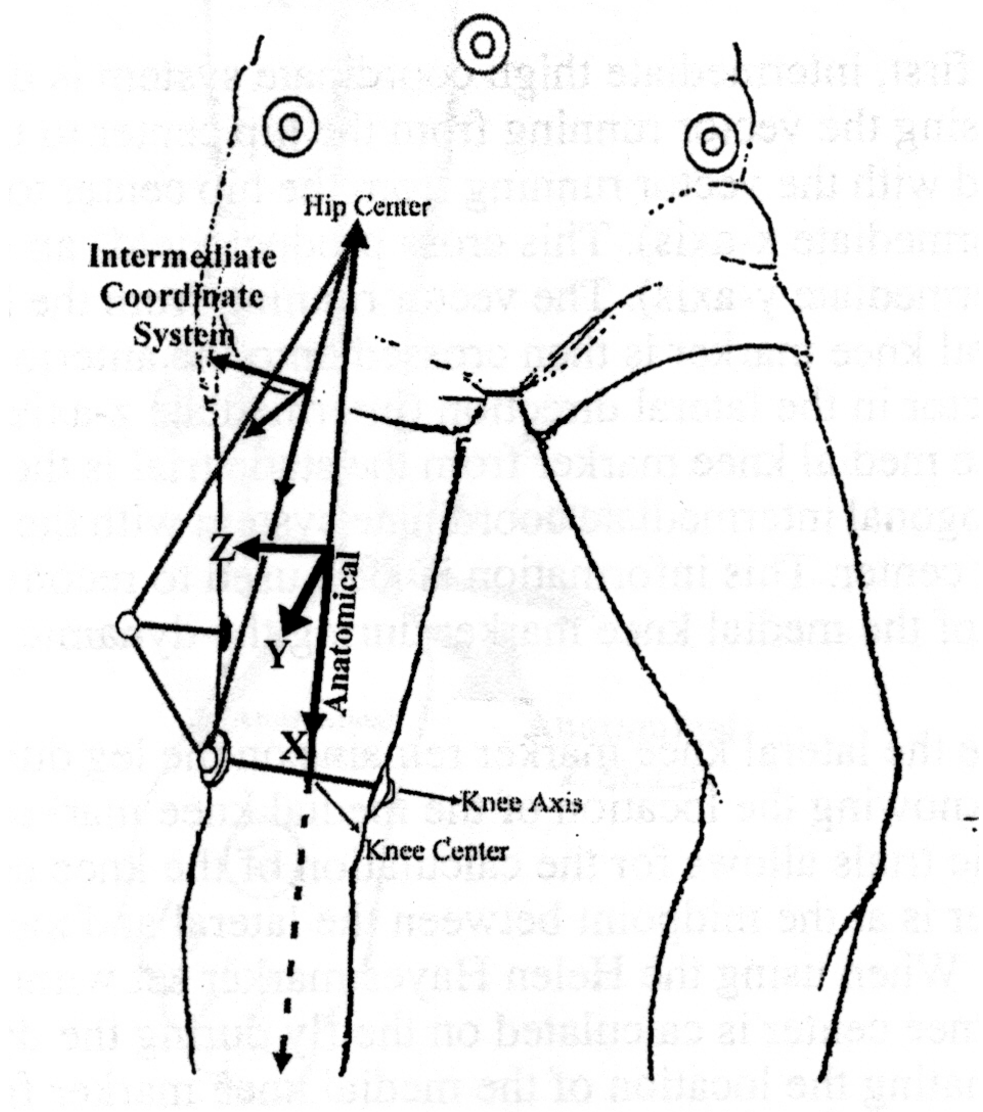
Key definitions in 2D and 3D kinematics:

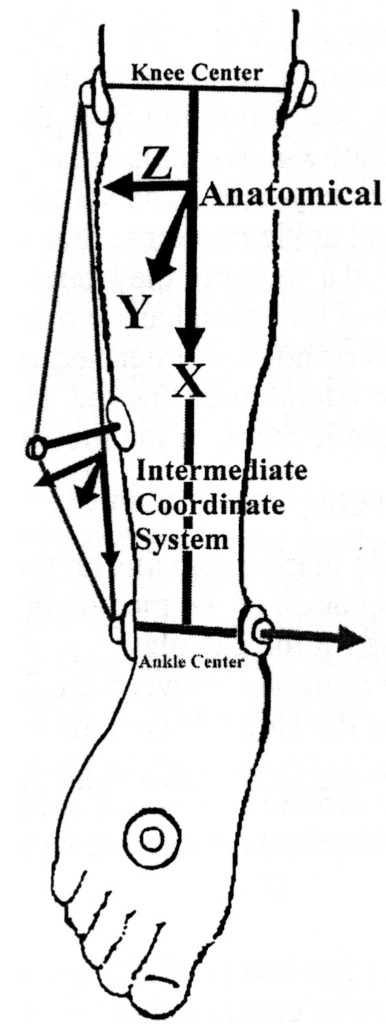
- Segment endpoints for creation of links
- Segment dimensions (body segment parameters)
- Orientation of segments, for angular data

Pelvis marker set (general)

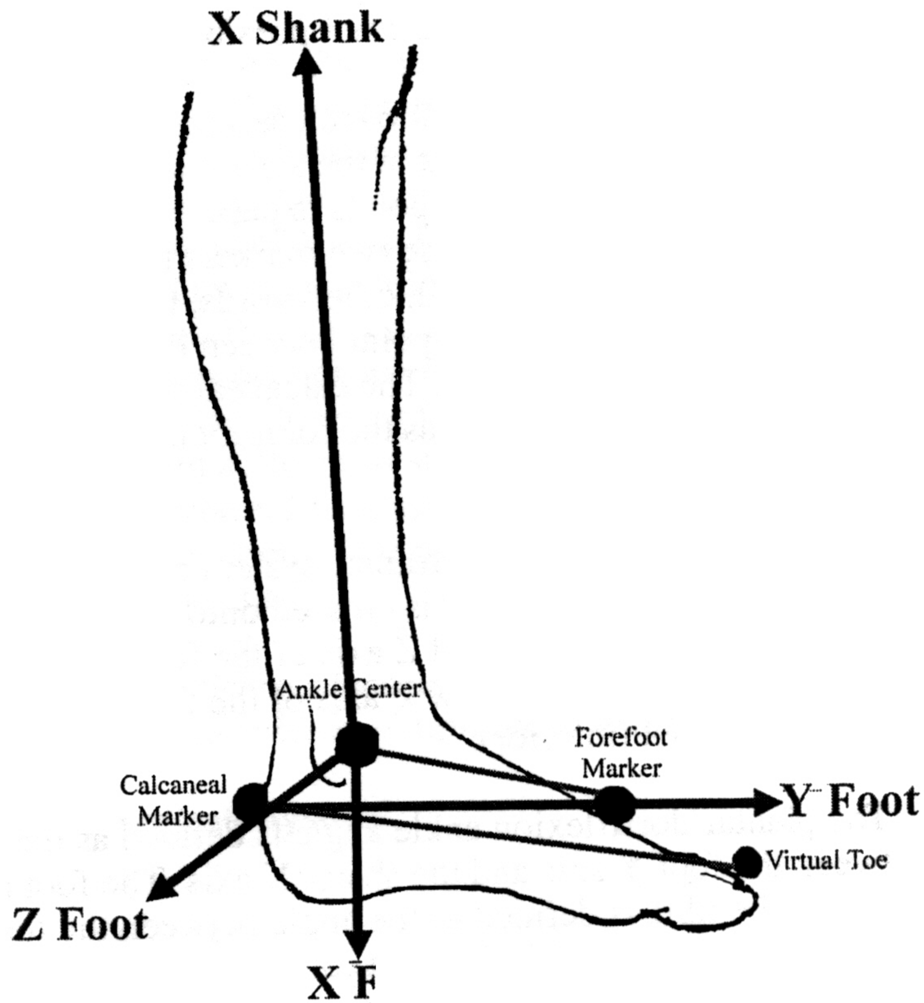


Helen Hayes marker set





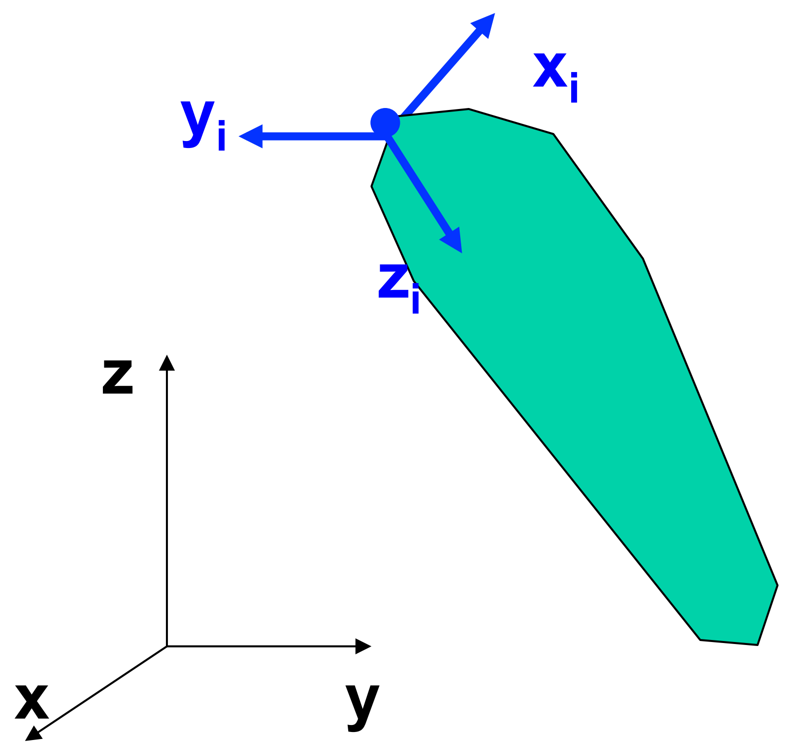
Foot marker set (general)



Step #3 … Relative position and orientations between segments

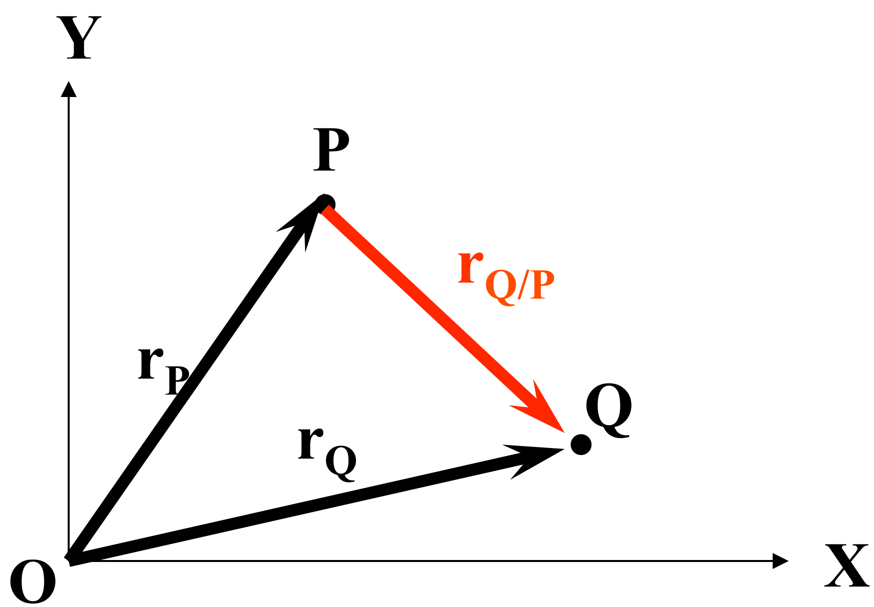
Relationship between the Local c.s. (LCS) and the Global c.s. (GCS)

- Linear
- Rotational



Linear Kinematics of a Rigid Body

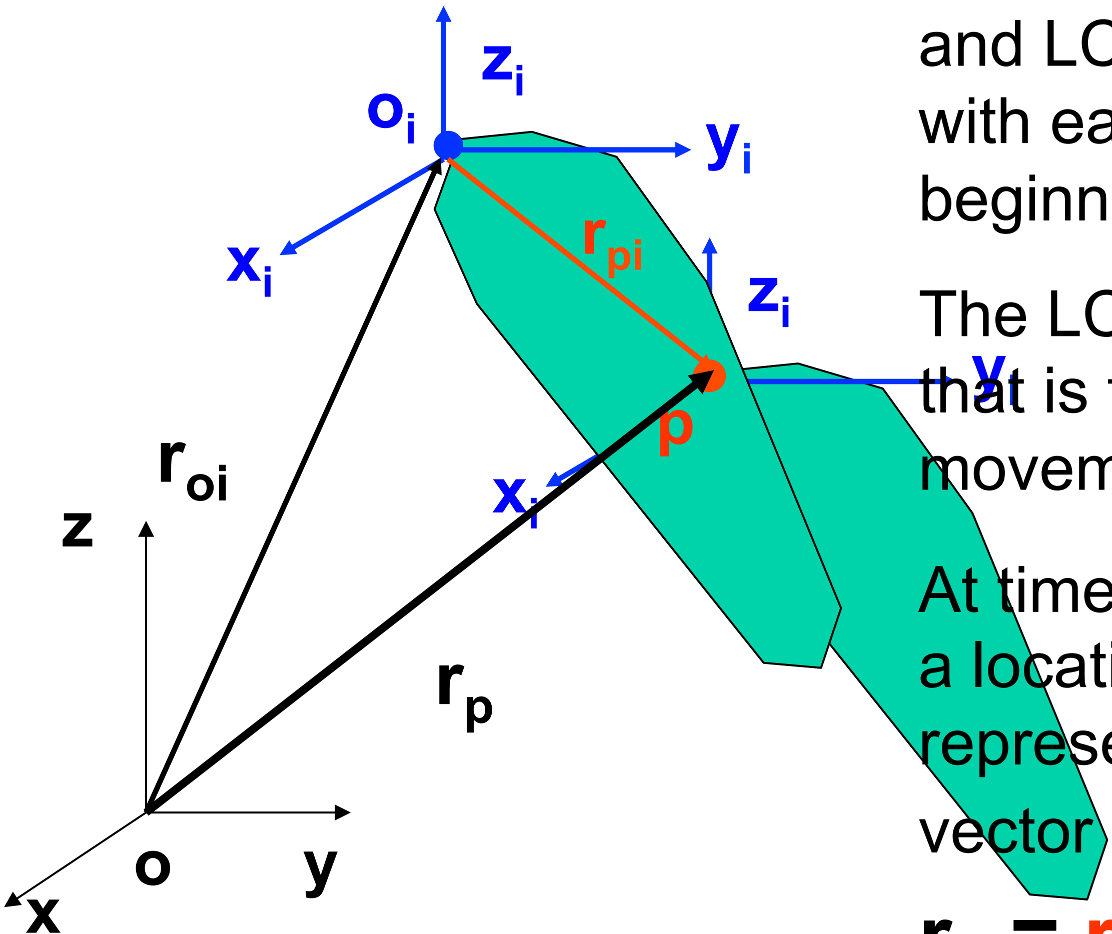
Position Vector: A vector starting from the origin of a coordinate system to a point in the space is defined as the position vector of that point.



Displacement Vector: The vector difference of two position vectors is defined as the displacement vector from the first point (P) to the second point (Q).

$$r_{Q/P} = r_Q - r_P$$

Linear Transformation



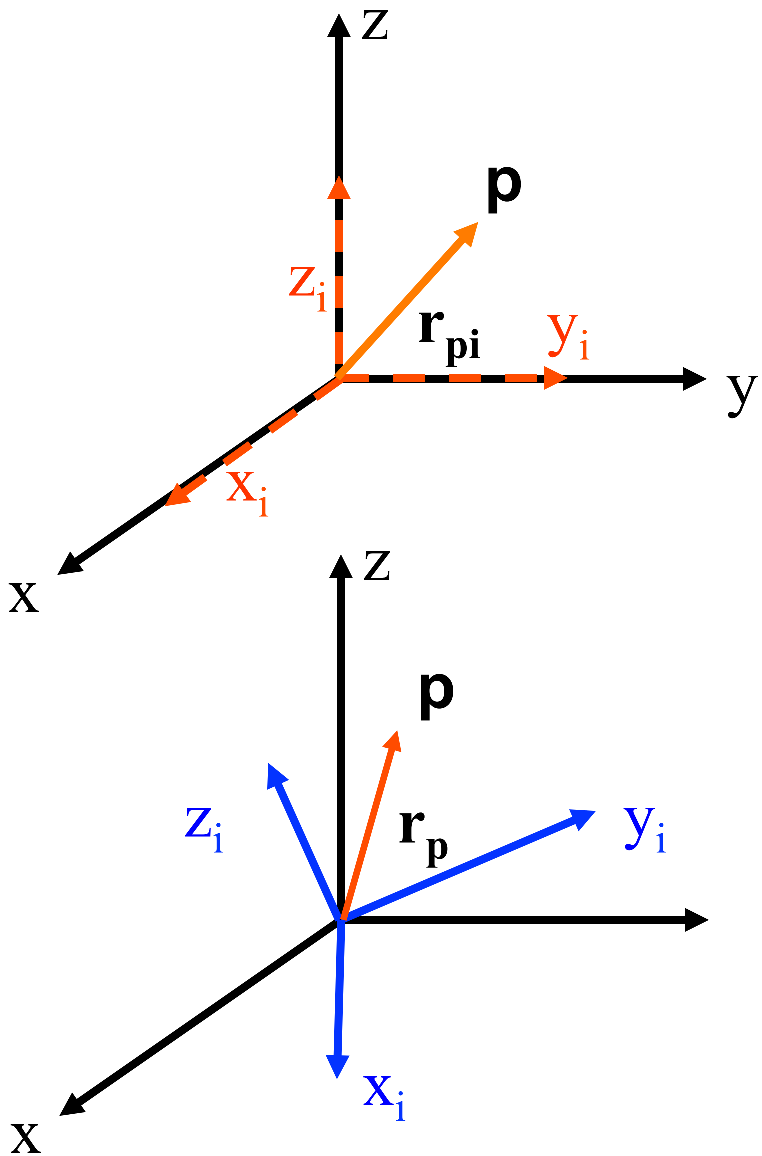
Assume that GCS (x,y,z) and LCS (x_i,y_i,z_i) coincide with each other in the beginning $(t = 0)$.

The LCS is only translating, that is there is no rotational movement.

At time t, the LCS moves to a location which is represented by a position vector of $\mathbf{r}_{oi}$.

$$\mathbf{r}_p = \mathbf{r}_{pi} + \mathbf{r}_{oi}$$

Rotational Transformation



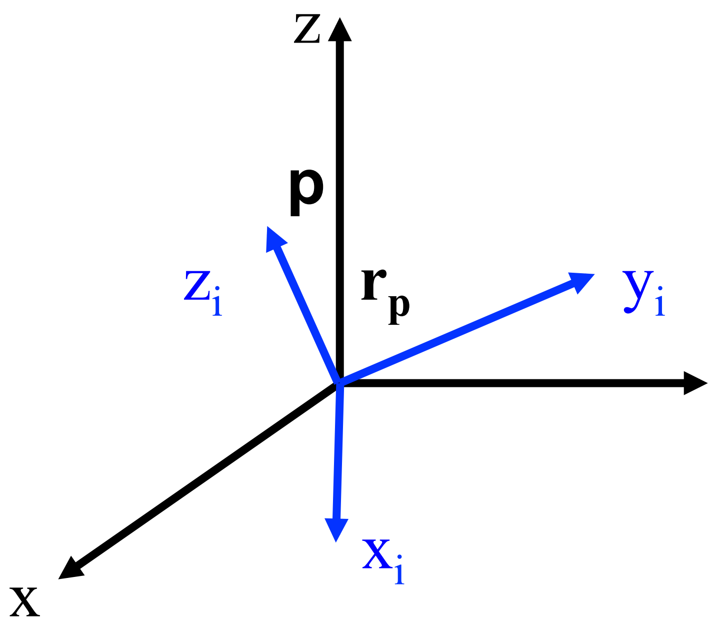
Assume that GCS (x,y,z) and LCS (x_i,y_i,z_i) coincide with each other in the beginning (t = 0).

At time t, the LCS rotates with respect to the GCS and reaches a final orientation.

$$\mathbf{r_p} = \mathbf{R}\,\mathbf{r_{pi}}$$

R: rotation matrix from LCS to GCS

Rotational Matrix



$$\mathbf{r}_p = \mathbf{R}\,\mathbf{r}_{pi}$$

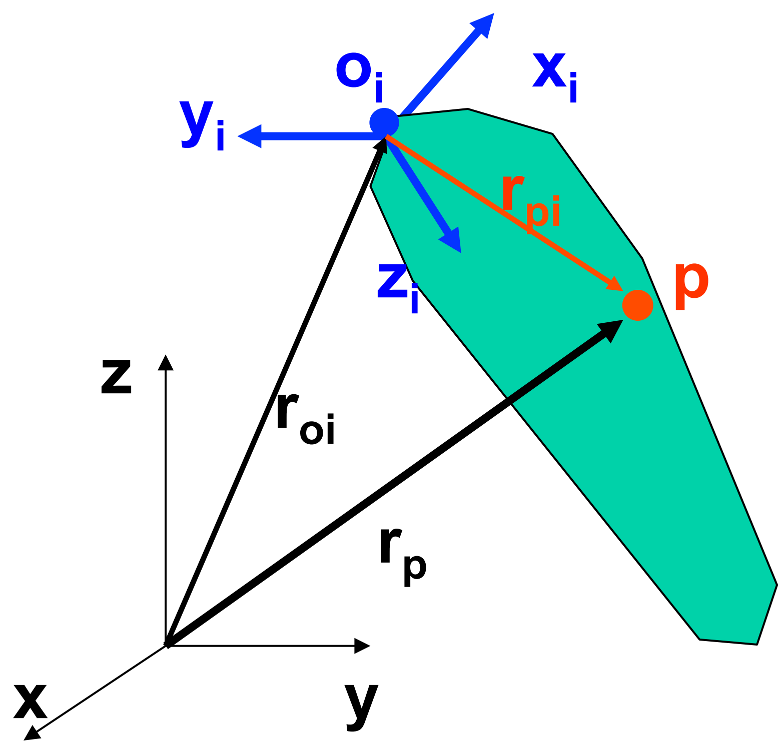
If directions of the x_i, y_i, and z_i of the LCS axes can be expressed by unit vectors v_1, v_2, and v_3, respectively, in the GCS, the rotation matrix from the LCS to GCS is defined as **R**

$$R = \begin{bmatrix} v_1 \bullet i & v_2 \bullet i & v_3 \bullet i \\ v_1 \bullet j & v_2 \bullet j & v_3 \bullet j \\ v_1 \bullet k & v_2 \bullet k & v_3 \bullet k \end{bmatrix} = \begin{bmatrix} v_{1x} & v_{2x} & v_{3x} \\ v_{1y} & v_{2y} & v_{3y} \\ v_{1z} & v_{2z} & v_{3z} \end{bmatrix}$$

Relationship between the LCS and Fixed (Global) coordinate system (GCS)

Linear transformation+Rotational transformation

$$r_p = R\, r_{pi} + r_{oi}$$



Relationship between the LCS and Fixed (Global) coordinate system (GCS)

4×4 Transformation Matrix

$$r_p = R\, r_{pi} + r_{oi}$$

$$\begin{bmatrix} r_{px} \\ r_{py} \\ r_{pz} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} r_{pxi} \\ r_{pyi} \\ r_{pzi} \end{bmatrix} + \begin{bmatrix} r_{oix} \\ r_{oiy} \\ r_{oiz} \end{bmatrix}$$

$$\begin{bmatrix} r_{px} \\ r_{py} \\ r_{pz} \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & r_{oix} \\ a_{21} & a_{22} & a_{23} & r_{oiy} \\ a_{31} & a_{32} & a_{33} & r_{oiz} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{pxi} \\ r_{pyi} \\ r_{pzi} \\ 1 \end{bmatrix}$$

Determining Joint Kinematics

If the orientation of two local coordinate systems (two adjacent body segments) are known, then the relative orientation between these two segments can be determined.

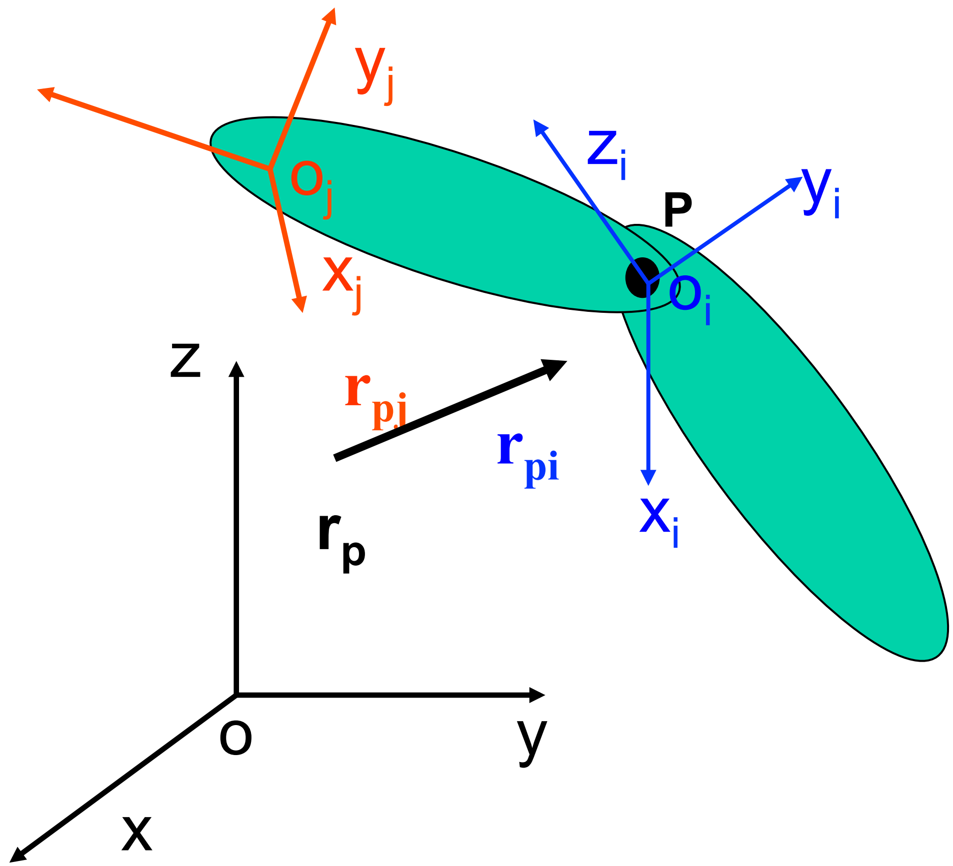
$$\mathbf{r_p} = \mathbf{R_i}\, \mathbf{r_{pi}} \qquad (1)$$

$$\mathbf{r_p} = \mathbf{R_j}\, \mathbf{r_{pj}} \qquad (2)$$

(1)=(2)

$$\mathbf{R_i}\, \mathbf{r_{pi}} = \mathbf{R_j}\, \mathbf{r_{pj}}$$

$$\mathbf{r_{pi}} = \mathbf{R_i}^{-1}\, \mathbf{R_j}\, \mathbf{r_{pj}}$$



Determining Joint Angles

3-D joint angles are concerned about the relative orientation between any two adjacent body segments, therefore, only the rotation matrix is needed for computation.

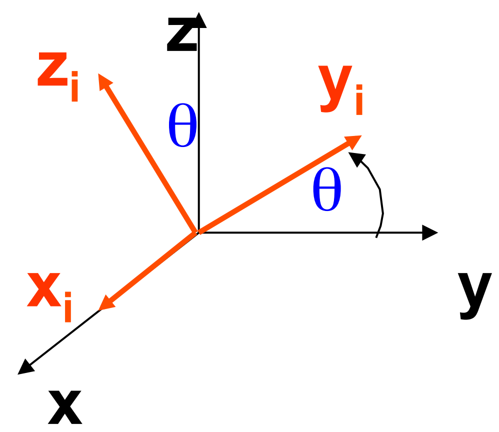
$$r_{pi} = R_i^{-1} R_j r_{pj}$$

$$= R_{i/j} r_{pj}$$

Basic Rotational Matrices

Rotation about the X-axis

$$r_p = \mathbf{R}\, r_{pi}$$

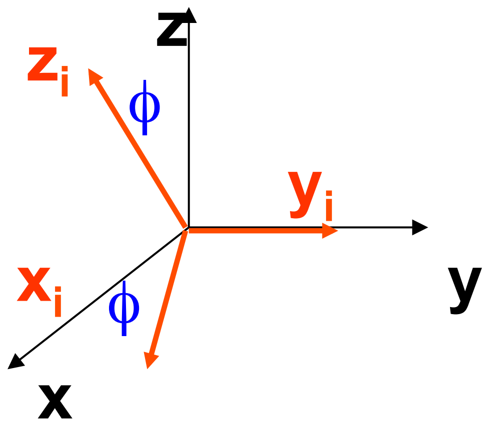


$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

Basic Rotational Matrices

Rotation about the Y-axis

$$r_p = R\, r_{pi}$$

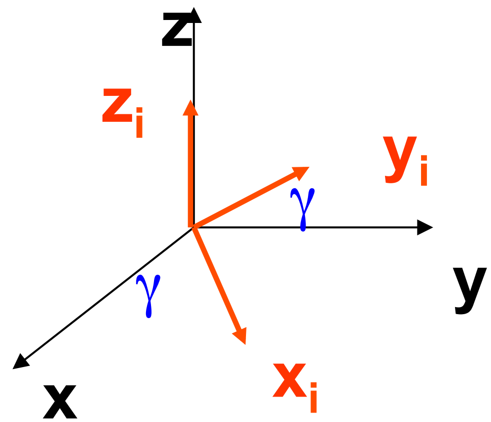


$$R = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}$$

Basic Rotational Matrices

Rotation about the Z-axis

$$r_p = \mathbf{R}\, r_{pi}$$



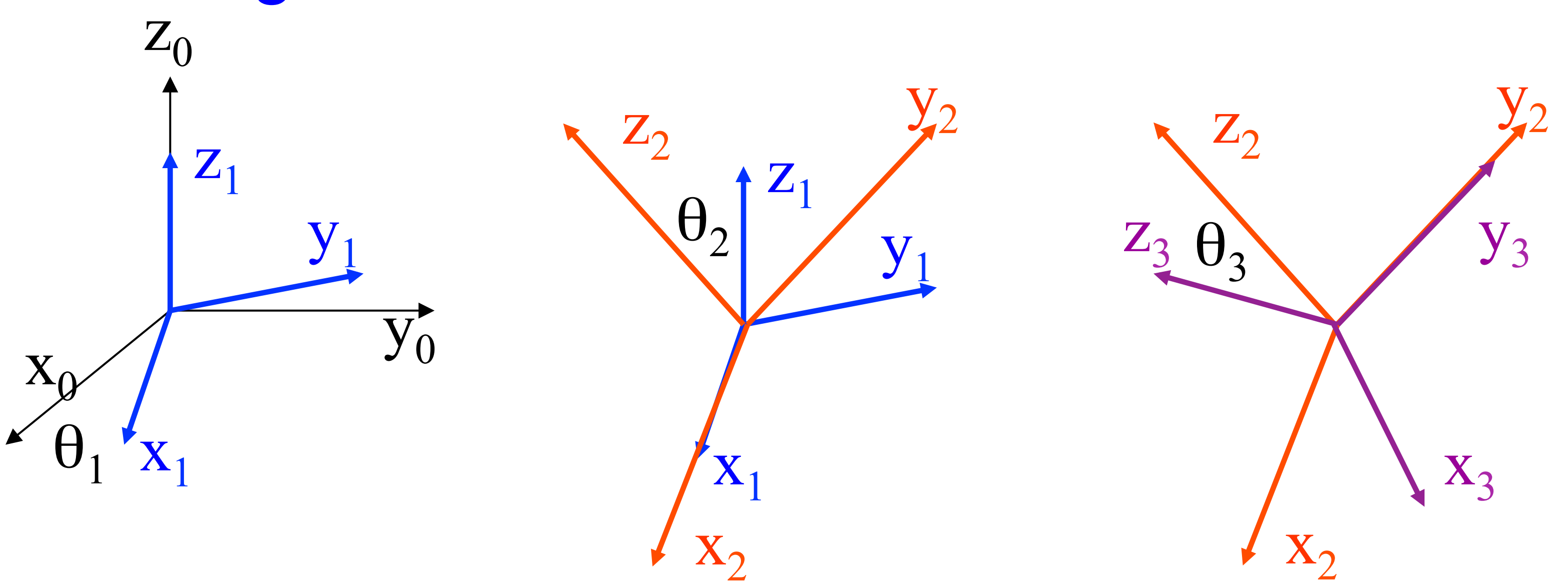
$$R = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Cardan / Euler Angles

Cardan/Euler angles are defined as a set of three finite rotations assumes to take place in sequence to achieve the final orientation (x_3,y_3,z_3) from a reference frame (x_0,y_0,z_0).

Cardan angles: all three axes are different

Euler angles: the 1st and last axes are the same



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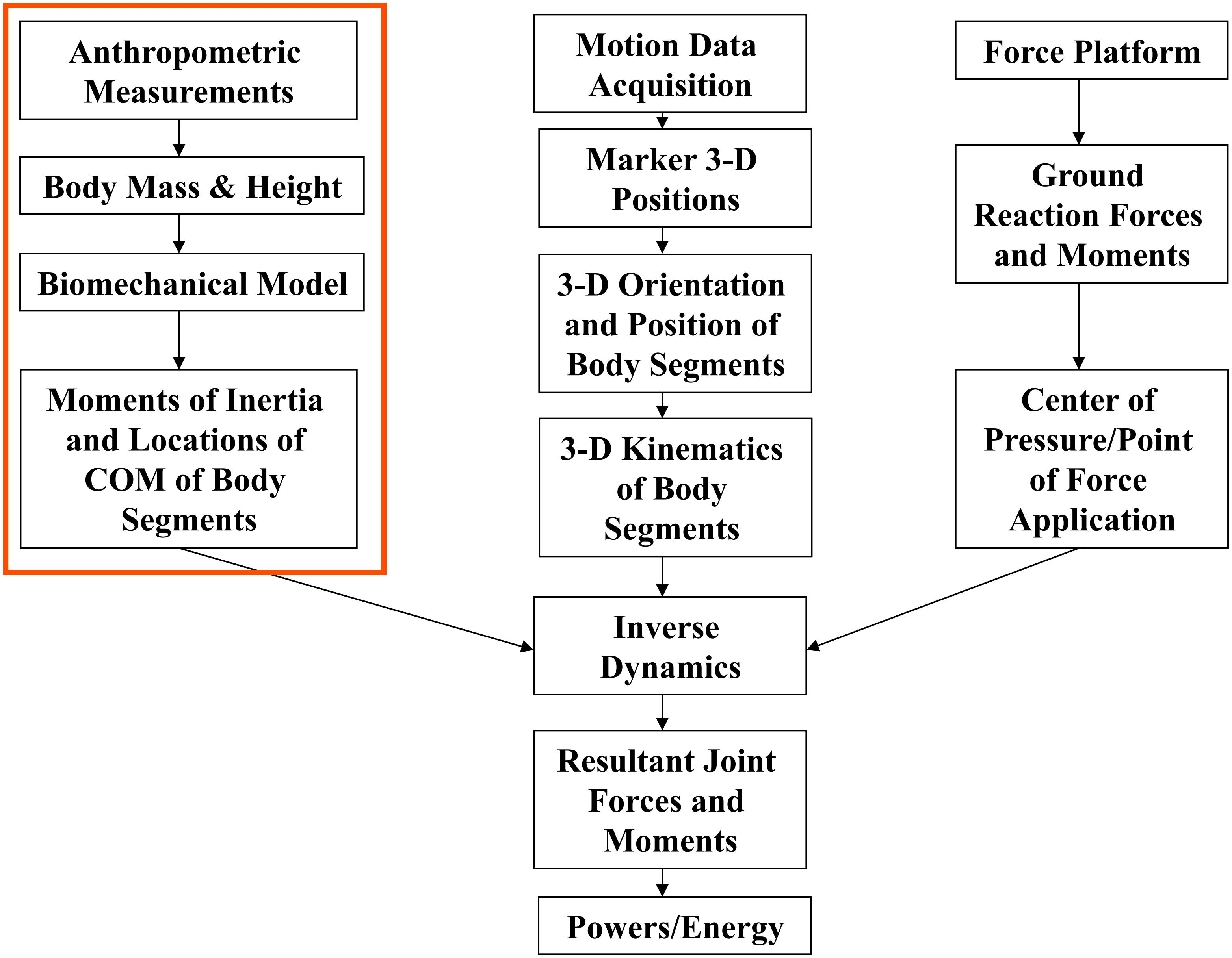
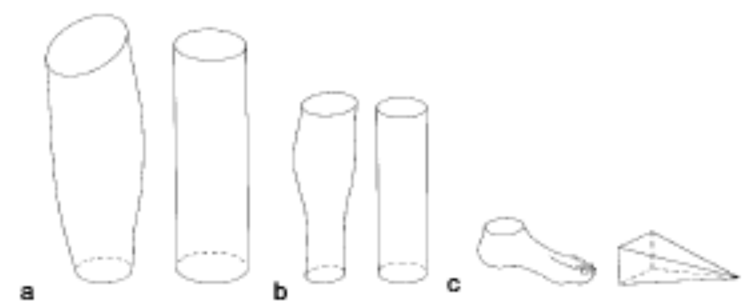


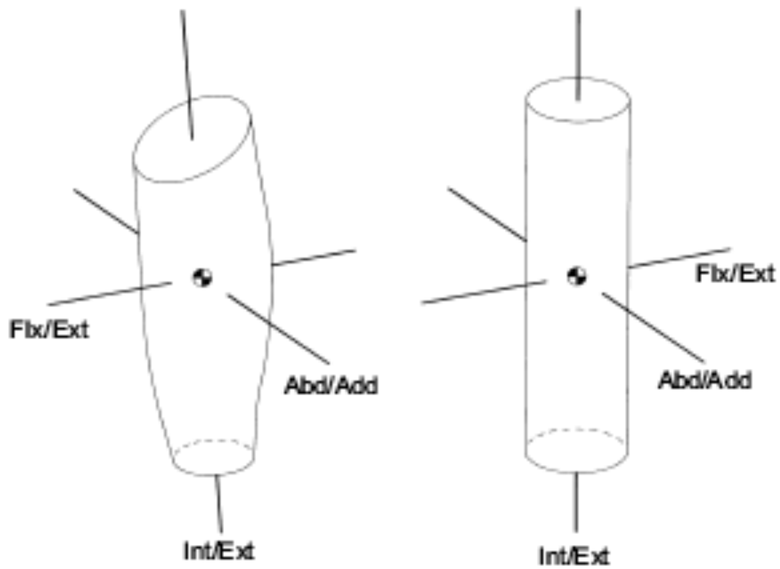
Inertial parameters



Mass of calf = (0.0226)(Total body mass)
+ (31.33)(Calf length)(Calf circumference)^2
+ (0.016)

Mass of right calf = (0.0226)(64.90)
+ (31.33)(0.430)(0.365)^2+0.016
= 3.28 kg

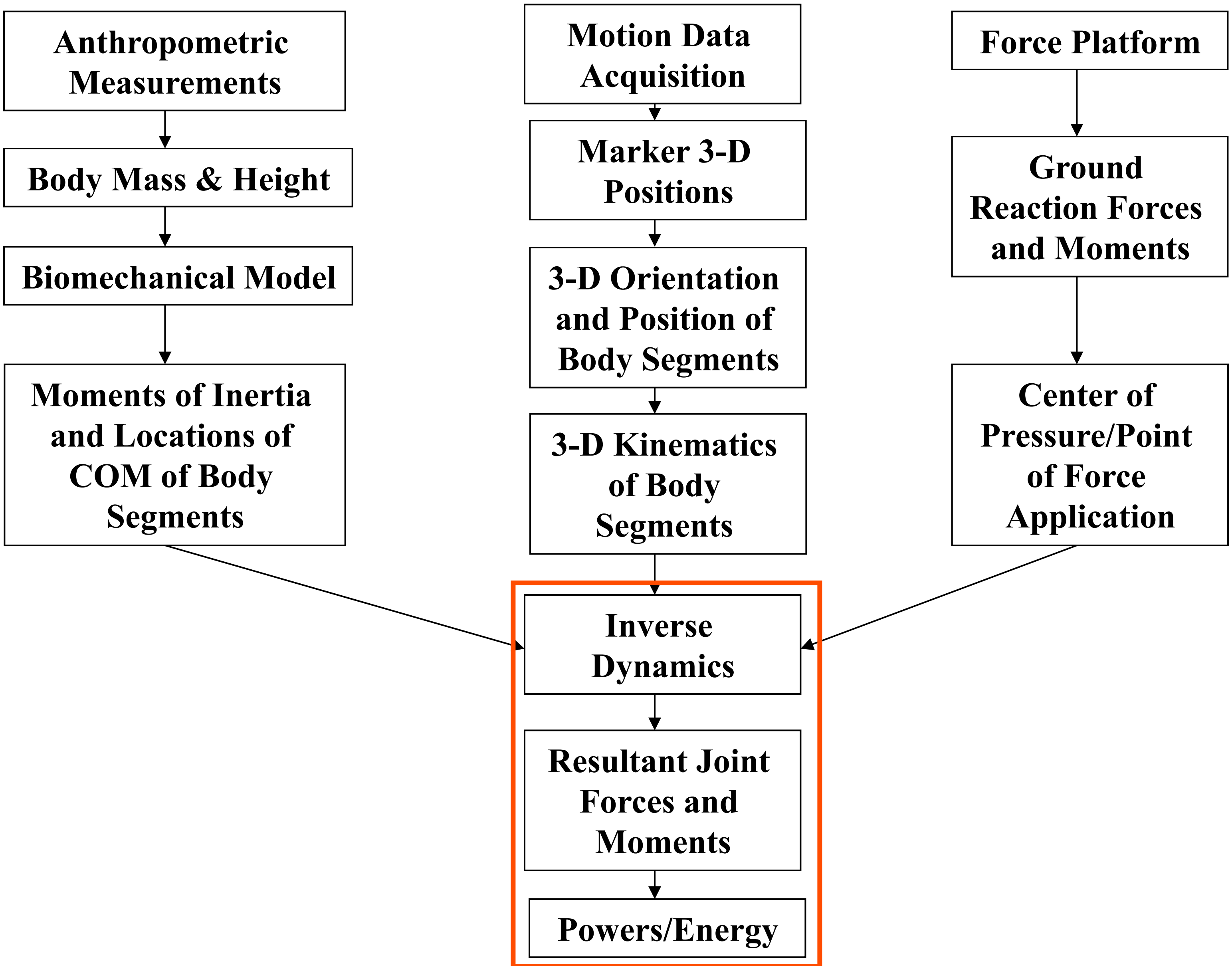
Inertial parameters



Moment of inertia of thigh about the flexion/extension axis=
(0.00762)(Total body mass) x
[(Thigh length)^2 + 0.076 (Midthigh circumference)^2] + 0.0115

Moment of inertia of right thigh about the flexion/extension axis =
$(0.00762)(64.90) \times [(0.460)^2 + 0.076(0.450)^2] + 0.0115 = 0.1238\ kg \cdot m^2$

Kinetic Analysis of Human Movement



Assumptions of the "Link-Segment" Model

- Each segment has a point mass located at its individual COM
- Location of the segmental COM remains fixed (w.r.t. segment endpoints) during the movement
- Joints are considered as hinge or ball & socket joints (max. 3 DOF each)
- Segment length and Mass moment of inertia about the COM are constant during movement

Forces Acting on the Link-Segment

- **Gravitational Force**

  acting at the COM of the body segment

- **Ground Reaction or External Contact Forces**

  acting at the COP or contact point

- **Net Muscle and/or Ligament Forces**

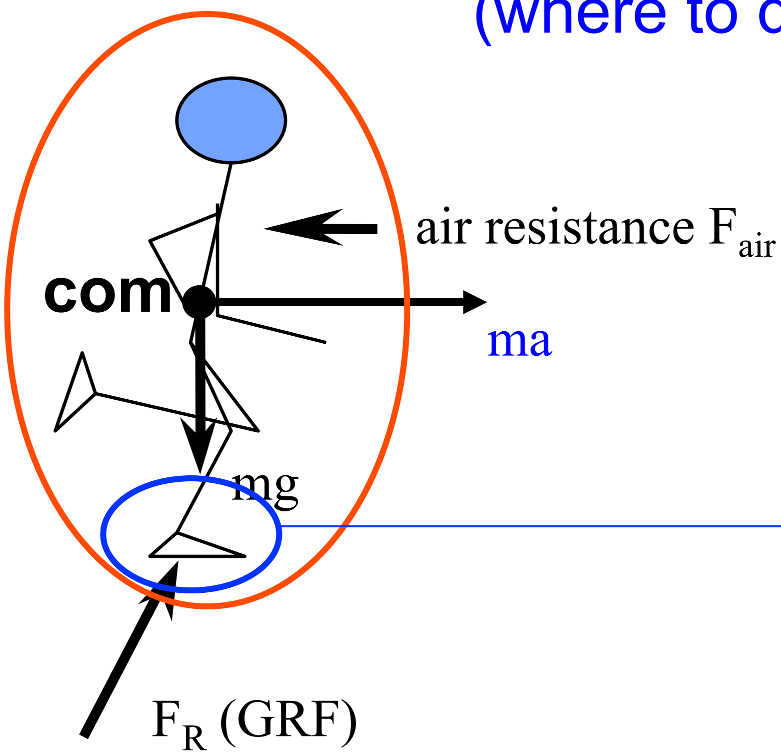
  acting at the joint

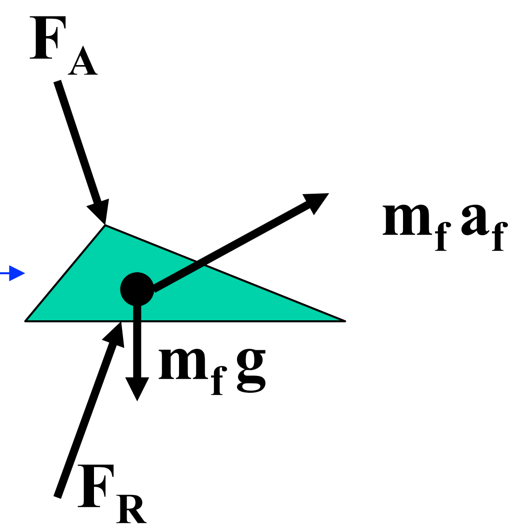
Free-Body Diagram

- **A free-body diagram is constructed to help identify the forces and moments acting on individual parts of a system and to ensure the correct use of the equations of motion**
- **The parts constituting a system are isolated from their surroundings and the effects of the surroundings are replaced by proper forces and moments**
- **In a free-body diagram, all known and unknown forces can be shown**

Free-Body Diagram

(where to draw the line …)





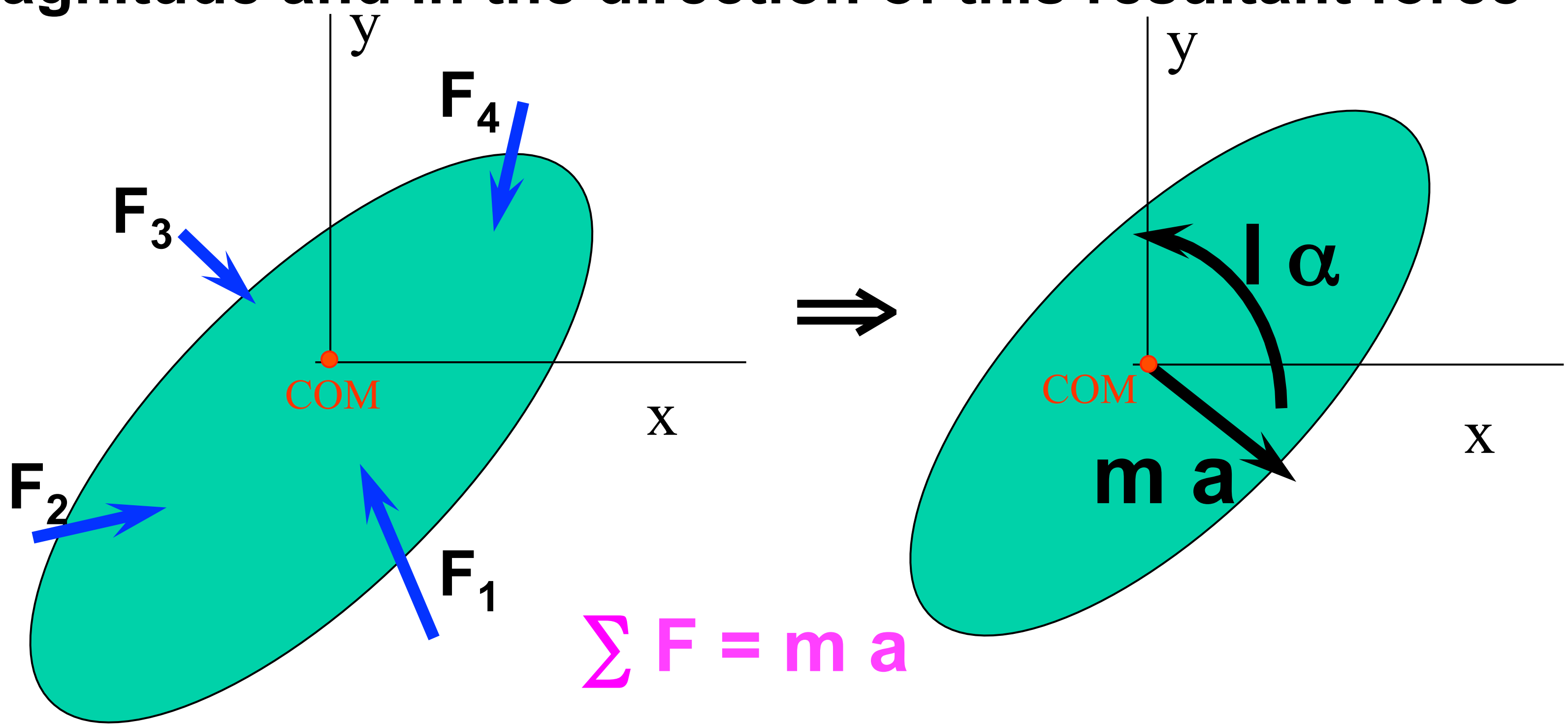
$\sum F = F_R + mg + F_{air} = ma$

$\sum F = F_R + m_f g + F_A = m_f a_f$

Equations of Motion of a Rigid Body

If the resultant force acting on a body is not zero,

→ the body's acceleration will be proportional to the magnitude and in the direction of this resultant force



$$\sum F = m\,a$$

$$\sum M = I\,\alpha$$