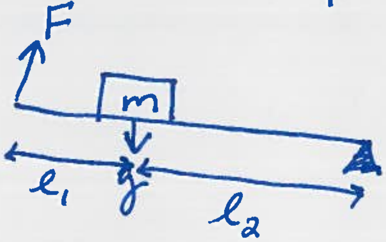
1a To compute the strain, we first need to find the stress in the tendon by finding the force produced by the muscle pulling on the tendon:



$$F(l_1+l_2) = mg\,l_2$$

$$\Rightarrow F = mg\frac{l_2}{l_1+l_2}$$

$$\Rightarrow F = (100\text{kg})(10\text{m/s}^2)\left(\frac{15}{20}\right) = 750\text{N}$$

Now find stress in the tendon:

$\sigma_t = \frac{F}{A}$ where F is the force produced by the muscle (above) and A is the cross-sectional area of the tendon.

$$\Rightarrow \sigma_t = \frac{750\text{N}}{0.0001\text{m}^2} \Rightarrow \boxed{\sigma_t = 7.5\text{MPa}}$$

Now that we have the stress, we can find the strain:

$$\sigma_t = E\varepsilon_t \Rightarrow \varepsilon_t = \frac{\sigma_t}{E} = \frac{7.5\text{MPa}}{600\text{MPa}} \Rightarrow \boxed{\varepsilon = 1.25\%}$$

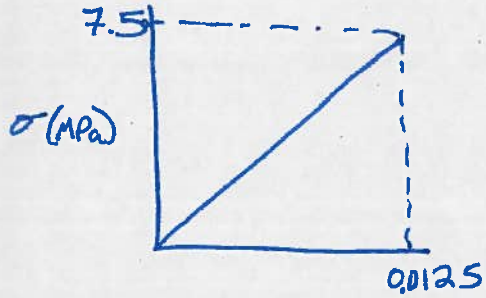
To find the length change:

$$(1+\varepsilon)\cdot\text{initial length} = \text{new length}$$

$$\Rightarrow 1.0125 \cdot 10\text{ cm} = 10.125\text{cm}$$

$$\Delta L = L_i - L_n \Rightarrow \boxed{\Delta L = 0.125\text{cm}}$$

1b Strain energy per unit volume is the area under the stress-strain curve:



$$U = \frac{1}{2}\sigma\varepsilon = \frac{1}{2}(7.5\,\text{MPa})\cdot(0.0125)$$

$$\Rightarrow U = 0.047\,\text{MPa} \quad \text{or} \quad 46{,}875\,\text{Pa}$$

*Note on units: $\text{Pa} = \frac{N}{m^2}$ and $J = Nm$, hence $\text{Pa} = \frac{J}{m^3}$ (or energy per unit volume).

Total strain energy = $U \cdot$ Volume

$$\Rightarrow U_{tot} = 0.047\,\frac{MJ}{m^3} \cdot \underbrace{L_t \cdot A_t}_{\text{Volume}}$$

$$= 0.047\,\frac{MJ}{m^3} \cdot 0.1\,m \cdot 0.0001\,m^2$$

$$\Rightarrow \boxed{U_{tot} = 0.47\,J}$$

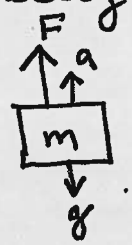
1c First we need to know the maximum force that can be applied to the tendon:

$$\sigma_{max} = \frac{F_{max}}{A} \Rightarrow 82\,MPa = \frac{F_{max}}{0.0001 m^2}$$

$$\Rightarrow F_{max} = 8200 N.$$

A simple way to consider this problem is to set F_{max} equal to the force that accelerates the body through the air:

$$F_{max} = m_{body} \cdot (a_{max} - g)$$

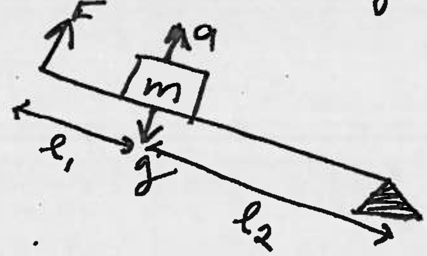


And then solve for a_{max}.

$$\Rightarrow \boxed{a_{max} = 72\,m/s}$$ (or 82 m/s, if you do not account for g)

This is equivalent to a situation in which F_{max} directly pulls the body up into the air (like, say, a hook lifting the person vertically). Several students, however, noted that the force produced by the muscle acts through the lever system:

$$F_{max}(l_1 + l_2) = m_{body}(a_{max} - g) \cdot l_2$$

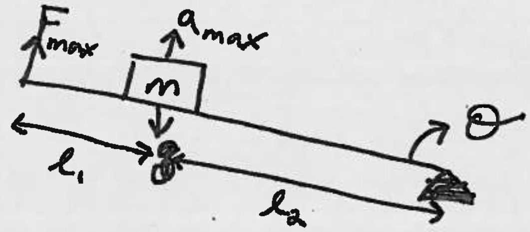


A few students noted that the system, however, is not static → once the person begins moving the torque balance is no longer in equilibrium, and you need to account for rotational inertial effects.

→

1c continued

Now the problem can be set up like this:



F_{max} is the maximum force produced by the muscle just as the foot leaves the ground, launching the body with an acceleration a_{max}. We need an equation of motion that accounts for angular motion such that the sum of the torques about the fulcrum equals the moment of inertia (I) times the angular acceleration (α):

$$I\alpha = \Sigma\tau$$

This is analagous to $ma = \Sigma F$.

The torques about the fulcrum are those produced by the force of the muscle and the accelerating body:

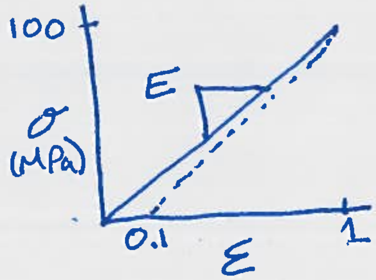
$$I\alpha = F_{max}(l_1 + l_2) + m(a_{max} - g)l_2$$

Note that $\alpha = \frac{a_{max}}{r}$, where $r = l_2$ in this case.

$$\Rightarrow I\frac{a_{max}}{l_2} = F_{max}(l_1 + l_2) + m(a_{max} - g)l_2$$

This equation can be solved for a_{max}, the maximum acceleration of your body that the muscle force can produce without harming the tendon.

2a Stiffness (E) is the slope of the linear region of the stress-strain curve:



$$\sigma = E\varepsilon \Rightarrow E = \frac{\sigma}{\varepsilon} = \frac{100\,MPa}{1}$$

$$\Rightarrow \boxed{E = 100\,MPa}$$

2b Maximum stress (strength) is the stress applied just before the material breaks. In this case, that's: $\boxed{\sigma_{max} = 100\,MPa}$.

Maximum strain (extensibility) is the strain when σ_{max} is applied: $\boxed{\varepsilon_{max} = 1}$.

2c The work of extension is the input strain energy per unit volume, which is the area under the stress-strain curve (the red line in the problem):

$$U = \frac{1}{2}\sigma\varepsilon = \frac{1}{2}(100\,MPa)\cdot 1 \Rightarrow \boxed{U = 50\,MPa \ (\text{or } \tfrac{MJ}{m^3})}$$

2d Resilience (R) $= \dfrac{\text{Energy returned}}{\text{Energy input}}$

The energy input is the answer to 2c, and the energy returned is the area under the blue curve:

$$R = \frac{45\,\frac{MJ}{m^3}}{50\,\frac{MJ}{m^3}} \Rightarrow \boxed{R = 0.9}$$

$\boxed{3}$ To stop the train, the spider silk must be able to absorb the kinetic energy of the train:

$$KE_{train} = U_{absorbed} \leftarrow U \text{ is energy per unit volume, } J/m^3,$$

so we will multiply by volume to get both sides in terms of energy:

$$KE_{train} = U_{silk} \cdot Vol_{silk}$$

$$\Rightarrow \frac{1}{2} m_t V_t^2 = \frac{\sigma_{silk\_max}^2}{2E} \cdot \underbrace{\pi r^2 L}_{\text{Volume}} \cdot \underbrace{20}_{\text{\# of threads}}$$

We want to solve for r, the radius of 1 thread (because 2r = diameter of thread):

* Note that $V_t = 80\text{mph} = 35.8\ m/s$.

$$\Rightarrow r = \sqrt{\frac{\frac{1}{\cancel{2}} m_t (V_t)^2 (\cancel{2} \cdot E)}{20 \cdot \sigma_{mx}^2 \cdot L \cdot \pi}} = \sqrt{\frac{(3000\,kg)(35.8\,m/s)^2 (5\times10^9\,Pa)}{20 \cdot (350\times10^6\,Pa)^2 (30\,m)\pi}}$$

$$\Rightarrow r = 9.12\times10^{-3}\ m$$

$$\Rightarrow \boxed{d = 1.82\times10^{-2}\ m} \text{ or } 1.82\ cm$$

$\boxed{4}$ The key to this problem is to use the formula for stress in a beam:

$$\sigma = \frac{My}{I}$$

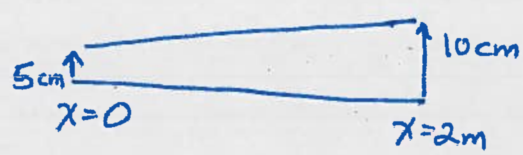
and turn this into $\sigma(x)$, stress as a function of x.

* $\boxed{M(x) = F \cdot x}$ (for a point force at the end of a beam)

* $y(x) = r$ → the distance to the outer fiber of the tree branch from the neutral axis.

r varies as a function of x:

$$r(x) = 0.05 + 0.025x$$



so $\boxed{y(x) = 0.05 + 0.025x}$

* $I(x)$ is the second moment of area for a circular cross-section: $I = \frac{\pi}{4} r^4$, But since r varies with x:

$$I(x) = \frac{\pi}{4}(0.025x + 0.5)^4$$

$$\Rightarrow \sigma(x) = \frac{Fx(0.025x + 0.5)}{\frac{\pi}{4}(0.025x + 0.5)^4}$$

Plotting this (see attached graph) will show you where the maximum stress occurs along the length of the branch. This is where it is most likely to break.

$\boxed{5}$ Lots of possible answers!

Length From End (L)	Branch Radius (y)	Moment (M)	Second Moment of Area (I)	Stress (Sigma)		
0	0.05	0	4.90874E-06	0		
0.1	0.0525	100	5.9666E-06	879897.7525		
0.2	0.055	200	7.18688E-06	1530565.944		
0.3	0.0575	300	8.58541E-06	2009221.604		
0.4	0.06	400	1.01788E-05	2357851.009		
0.5	0.0625	500	1.19842E-05	2607594.588		
0.6	0.065	600	1.40198E-05	2781770.512		
0.7	0.0675	700	1.63044E-05	2897989.717		
0.8	0.07	800	1.88574E-05	2969654.915		
0.9	0.0725	900	2.16991E-05	3007035.868		
1	0.075	1000	2.48505E-05	3018049.291	BREAK HERE	
1.1	0.0775	1100	2.83333E-05	3008830.316		
1.2	0.08	1200	3.21699E-05	2984155.183		
1.3	0.0825	1300	3.63836E-05	2947756.633		
1.4	0.085	1400	4.09983E-05	2902561.144		
1.5	0.0875	1500	4.60386E-05	2850868.718		
1.6	0.09	1600	5.153E-05	2794490.084		
1.7	0.0925	1700	5.74985E-05	2734852.081		
1.8	0.095	1800	6.39712E-05	2673079.085		
1.9	0.0975	1900	7.09755E-05	2610056.283		
2	0.1	2000	7.85398E-05	2546479.089