Homework 4, Bio 427, Autumn 2016
Graded by: Jorge

1. Jumping to conclusions
   a. You needed to write the spring constant (k) as a function of the tendon stiffness (E_{tendon}), the cross-sectional area of the tendon (A), and the original tendon length (L_o).
      Hooke's law tells us: $F = kx$

      Given:
      $A = 0.5\ \text{cm}^2 = 5 * 10^{-5}\ \text{m}^2$
      $E_{tendon} = 600\ \text{MPa} = 6 * 10^8\ \text{Pa}$
      $L_o = 20\ \text{cm} = 0.2\ \text{m}$

      We know:
      $$\sigma_{tendon} = \frac{F}{A} \quad \text{and} \quad \sigma_{tendon} = E_{tendon}\varepsilon$$
      $$\frac{F}{A} = E_{tendon}\varepsilon$$
      This implies:
      $$F = \varepsilon A E_{tendon} = E_{tendon} A \frac{\Delta L}{L_o} = \frac{E_{tendon} A}{L_o} \Delta L$$

      $$\boxed{k = \frac{E_{tendon} A}{L_o} = \frac{(6 \cdot 10^8\ \text{Pa})(5 \cdot 10^{-5}\text{m}^2)}{(0.2\ \text{m})} = 1.5 \cdot 10^5\ \frac{\text{N}}{\text{m}}}$$

   b. Given:
      Mass of the individual = m = 100 kg

      $$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{1.5 \cdot 10^5\ \frac{\text{N}}{\text{m}}}{100\ \text{kg}}} = 6.16\ \text{Hz}}$$

   c. I'll let you **jump** to conclusions on this answer. You would stand on the tips of your toes and hop. Record the hop response in a quantifiable manner (i.e. videography). Think about your free vibration test in lab.

2. Muscle force, power, and efficiency

a.

$$F = \frac{bT_o - av}{v + b}$$

Since T_o is the initial tension, it must have dimensions of force, $\boxed{T_o = \mathrm{N} = \mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}^2}}$

Breaking down the top of Hill's equation, $\frac{bT_o}{v+b}$ and $\frac{av}{v+b}$ must both have dimensions of force.

Let us take, $\frac{bT_o}{v+b}$ first. If we write as, $T_o \frac{b}{v+b}$ it becomes clear that $\frac{b}{v+b}$ must be dimensionless for the whole equation to have dimensions of force. Thus b must have the same dimensions as v,

$$\boxed{b = \frac{\mathrm{m}}{\mathrm{s}}}$$

Lastly, take $\frac{av}{v+b}$, if we write it as $a\frac{v}{v+b}$, we can see that $\frac{v}{v+b}$ will be dimensionless, and thus,

$$\boxed{a = \mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}^2}}$$

b. If we are given that

$$\frac{b}{v_{max}} \cong \frac{1}{4} \qquad \text{and} \qquad \frac{a}{T_o} \cong \frac{1}{4}$$

We can rewrite

$$F = \frac{bT_o - av}{v + b}$$

as

$$F = \frac{\frac{v_{max}}{4}T_o - \frac{T_o}{4}v}{v + \frac{v_{max}}{4}} = \frac{v_{max}T_o - T_o v}{4v + v_{max}} = T_o\frac{v_{max} - v}{4v + v_{max}}$$

Thus,

$$\boxed{\frac{F}{T_o} = \frac{v_{max} - v}{4v + v_{max}}}$$

c. We wanted power ($\mathcal{P}$) in terms of v, v_{max}, and T_o

$$\mathcal{P} = \frac{E}{t} = Fv$$

$$F = T_o \frac{v_{max} - v}{4v + v_{max}}$$

$$\boxed{\mathcal{P} = vT_o \frac{v_{max} - v}{4v + v_{max}}}$$

d. Force as a function of velocity

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Plot[1 (1 - v)/(4 v + 1), {v, 0, 1}, AxesLabel → {v, F}]
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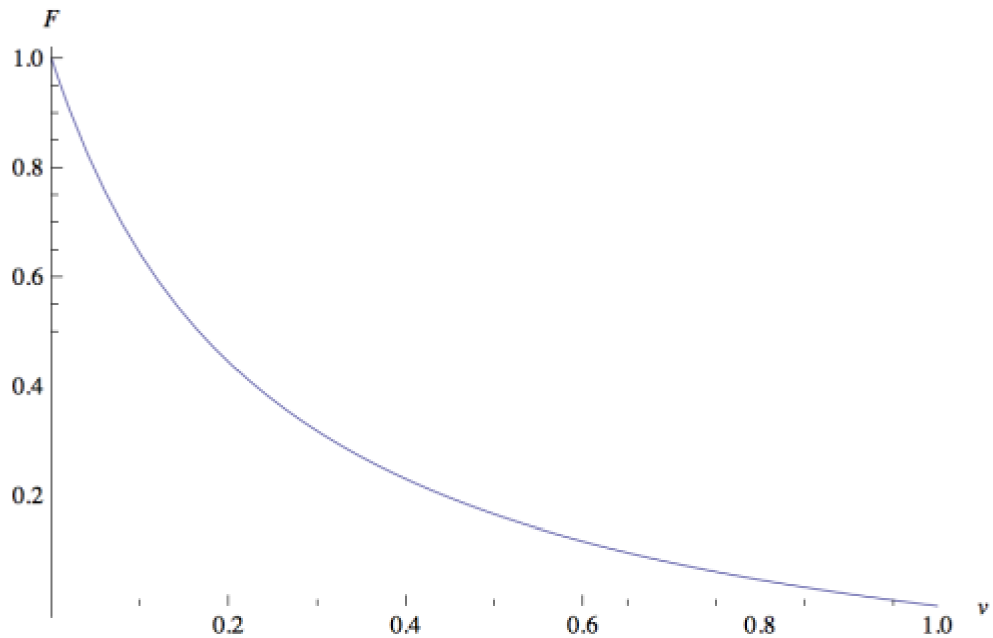
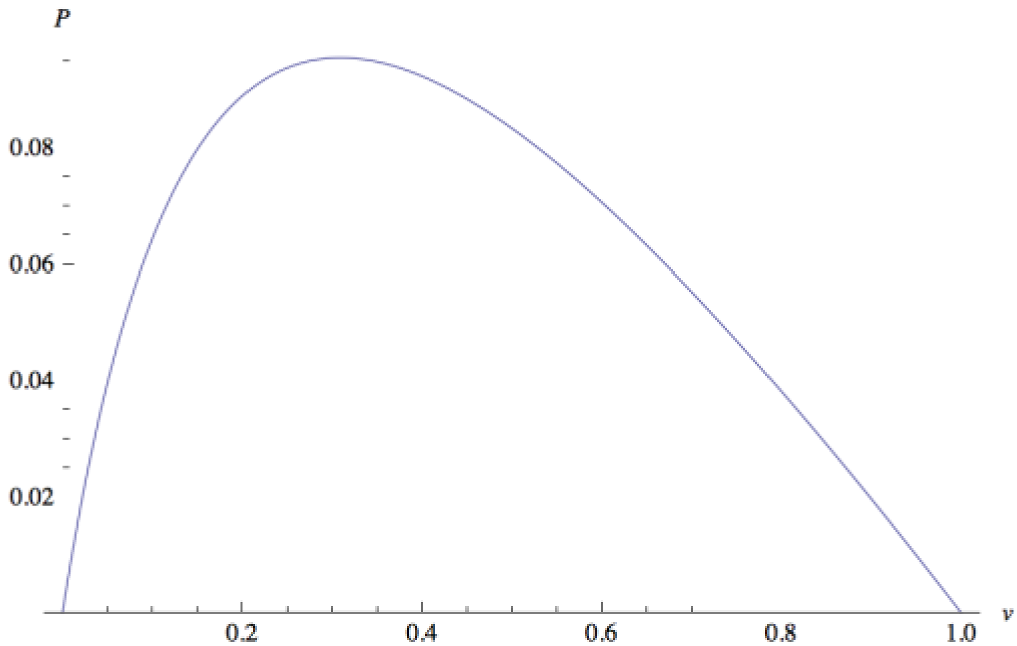


Power as a function of velocity

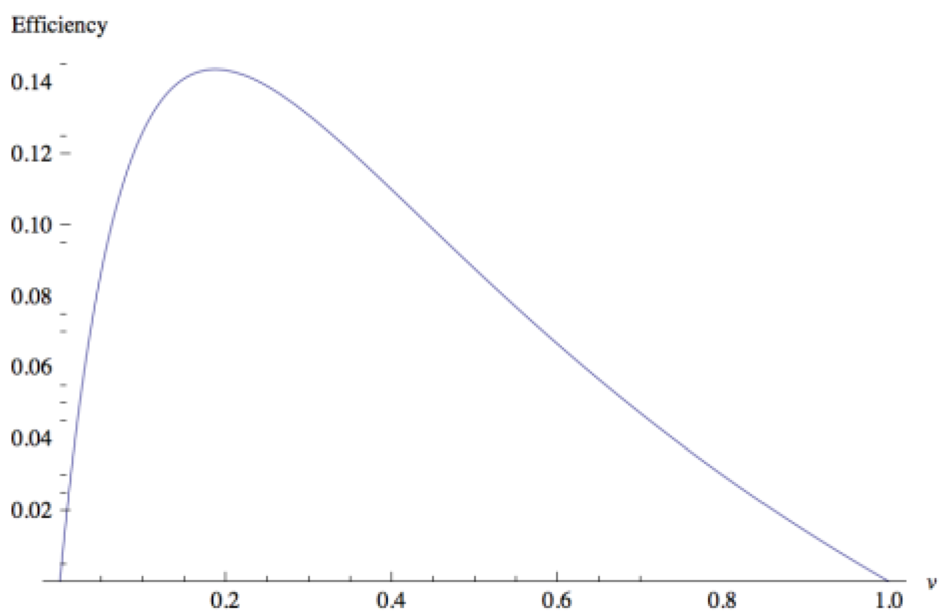
$$\text{Plot}\left[v\ 1\ \frac{1-v}{4\,v+1},\ \{v,\ 0,\ 1\},\ \text{AxesLabel}\to\{v,\ P\}\right]$$



e. Finding the relationship between velocity and ATP utilization. The equation is as follows:

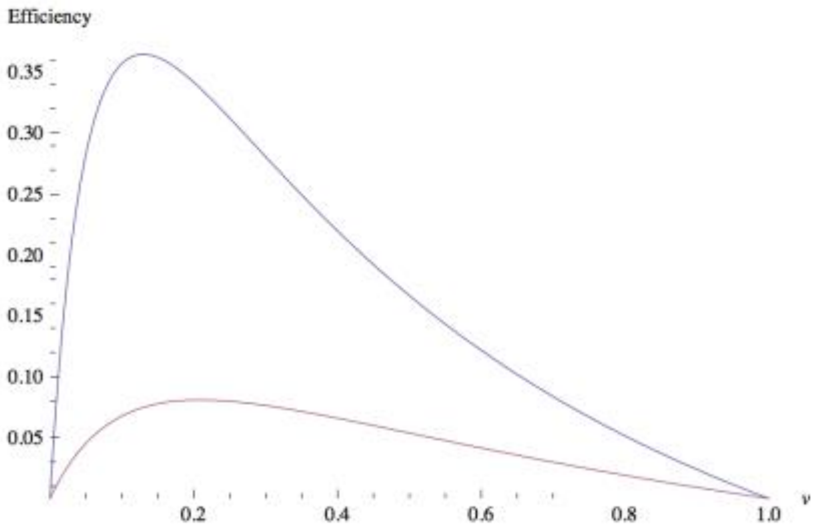
$$ATP\ consumption\ rate = 0.4 + 1.1v$$

$$\text{Plot}\left[\frac{v\ 1\ \frac{1-v}{4\,v+1}}{.4+1.1\,v},\ \{v,\ 0,\ 1\},\ \text{AxesLabel}\to\{v,\ \text{Efficiency}\}\right]$$



3. Discussion points:
    a. Let’s see… Plot two cases with an offset in ATP use but the same slope of ATP use:

    $$\text{Plot}\left[\left\{\frac{v\,1\,\frac{1-v}{4v+1}}{0.1+0.8v},\ \frac{v\,1\,\frac{1-v}{4v+1}}{0.8+1.5v}\right\},\ \{v,\ 0,\ 1\},\ \text{AxesLabel}\rightarrow\{v,\ \text{Efficiency}\}\right]$$

    The slow fibers are more efficient at all speeds below v_{max}. <u>Changing the ATP use rate offset does NOT change the speed of maximal efficiency</u>.

    b. When it is a matter of great and acute importance (i.e. outrunning a predator).

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4. There will be a variety of answers to this.