

1. Work of Ascent on Moving Ladder (1pt)

Answers could discuss the following (but explained in more detail for full credit):

- Work is being done to prevent loss of potential energy.
- Even if your center of mass remains in the same place, your muscles are doing work to move your arms and legs.
- The work you do reduces the amount of work that needs to be done by the motor of the ladder.

2a. What is the total air time? (1pt)

Time = Distance/Velocity

Total distance traveled in air = $(1 - \beta)L$

So total air time = $\frac{(1-\beta)L}{v}$

2b. Show how we got h . (1 pt)

A common mistake was to use the given equation for PE to get h by simply canceling mg from both sides. This is incorrect, because $PE = mgh$ is the general formula for potential energy. To determine PE, it is required to find h . We give you h , but for Part B we are now asking you – how did we get h ? In general, “show” means to derive the given relationship using physical principles.

In this case, the key was to notice that h is the maximum height of the jumper in its ballistic phase. We showed you in lecture that $h = \frac{1}{2} * g * \left(\frac{airtime}{2}\right)^2$. It's fine to begin your answer using this formula. It comes from the kinematic equation for y position:

$$h = y_{top} = y_{initial} + v_{y,initial} * t_{top} - \frac{1}{2}gt_{top}^2$$

At the apex of the jump, $v_y = 0 = v_{y,initial} - gt_{top}$, so we can find $v_{y,initial} = gt_{top}$ (Note the equation for v_y is simply the derivative of the position equation.)

Now that we know $v_{y,initial}$, we can find h : $h = 0 + gt_{top}^2 - \frac{1}{2}gt_{top}^2 = \frac{1}{2}gt_{top}^2$.

But $t_{top} = \frac{t_{airtime}}{2}$ so $h = \frac{1}{2}g\left(\frac{airtime}{2}\right)^2$.

The airtime is the time you found in 2a. Plug this into the equation for h .

2c. Power per stride? (1pt)

Average power per stride = Energy of stride/Time of stride:

$$\left[(1 - \beta)^2 * \frac{mg^2L^2}{8V^2} + \frac{1}{2}mV^2 \right] * \frac{V}{L} = (1 - \beta)^2 * \frac{mg^2L}{8V} + \frac{mV^3}{2L}$$

2d. As velocity goes to zero? (1pt)

This is the limit $V \rightarrow 0$ of the answer to 2c, which is ∞ . Or think of it this way: as V gets really small, what happens to power assuming everything else – like stride length – stays the same? The first term, with V in the denominator, will get really big. The second term, with V in the numerator, will get really small. But a really big number plus a really small number is still a really big number.

3a. Dimensionless? (1pt)

Credit was given whether this was shown in units or in dimensions.

$$Power = \frac{Work}{Time} = \frac{Fd}{t} = \frac{\frac{ML}{T^2} * L}{T} = \frac{ML^2}{T^3}$$

Which is the same as:

$$mgV = M * \frac{L}{T^2} * \frac{L}{T} = \frac{ML^2}{T^3}$$

3b. Cost? (1pt)

This is simply the answer for 2c divided by mgV :

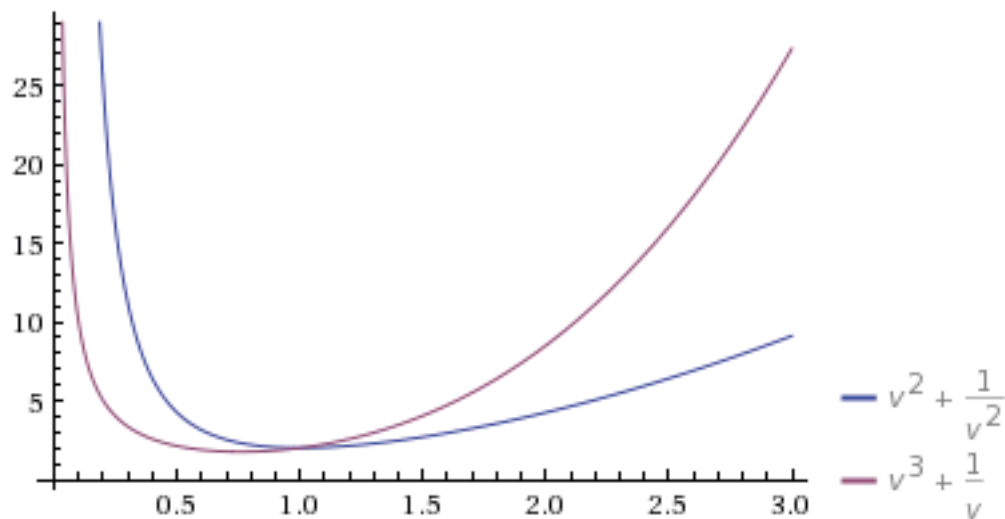
$$(1 - \beta)^2 * \frac{mg^2L}{8V} + \frac{mV^3}{2L} * \frac{1}{mgV} = (1 - \beta)^2 * \frac{gL}{8V^2} + \frac{V^2}{2Lg}$$

3c. Plot Power and Cost (1pt)

If you aren't able to intuit what the plots should look like based on the equations (which is hard!) a great way to begin is to simply plug in some values, plot points, and see how the two graphs diverge.

Or, use Mathematica (or Wolfram Alpha online):

Plot:



For the above plot, I assumed $\beta = 0$ and everything else 1, so the equations are only dependent on V . (You could have assumed any values for the other variables so long as they were the same between the two equations.) Blue is Cost, red is Power.

Note that Cost and Power approach infinity at different rates. This means that cost and power are minimized at different velocities.

4. How to test the hypothesis? (2pts)

Key to full credit is to fully explain what the results from your experiment would imply about the hypothesis.

Here are some good answers from that got full credit (lots of other possibilities though!)

“A possible experiment to test this hypothesis is to mutate *Issus* nymphs so they don’t have the gears. Using high speed cameras, one could measure the difference in the leg movements of the *Issus* nymphs and see if they are in sync with each other. If they are, the gears are not the structure that causes the synchrony.”

“In order to test this hypothesis I would make 2 groups of *Issus*: one with normal gears and a group without gears (or gears removed/broken). I would try to match the 2 groups on every other aspect as much as possible. I would then use the same protocol in the paper to measure the synchronization of the *Issus* jumping in each group with a high speed camera. If the hypothesis is correct I would expect to see the *Issus* group with gears to display significantly smaller synchronization times compared to the group without gears.”