Biomechanics 427 Homework 4

1) Jumping to conclusions. (3 pts)

In this problem we would like to compute the natural frequency of your body supported by your ankle and foot (like in many previous ankle problems). As we mentioned in class, the natural frequency (f) is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

where k is the spring constant and m is the mass.

(a) Write an equation for the spring constant (k) as a function of the stiffness of tendon (E_{tendon}) and the area (A) and length (L) of the tendon and estimate it's value if E_{tendon} = 600 MPa, A = 0.5 cm^2 and L = 20 cm.

(b) If your mass is 100 kg, what is your natural frequency?

(c) Explain how you experimentally would test this hypothesis. Attempt to do so.

2) Muscle force, power, and efficiency (5 pts).

Muscle force (F) declines with increasing shortening velocity (v) according to Hill's equation: $F = \dfrac{bT_o - av}{v + b}$

where v is the shortening velocity, T_o is the isometric (non-shortening) force, and the constants a and b have values that vary among muscle.

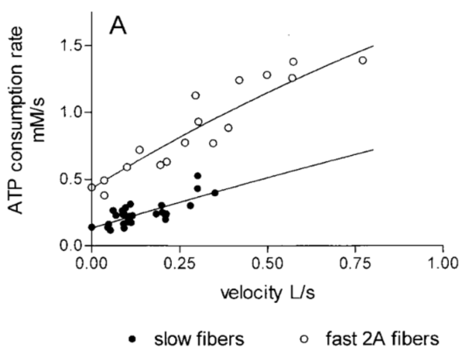
(a) For Hill's equation to be dimensionally correct, what must be the dimensions of the constants a, b, and T_o? (you can indicate either dimensions or specify units). (1 pt)

(b) Several authors (e.g. Y.C. Fung or T. McMahon) report that the non-dimensional ratios b/v_{max} and a/T_o are approximately equal to 1/4 for many muscles. Here, v_{max} is equal to the maximum shortening velocity for muscle (for zero force). Rewrite Hill's equation to express a dimensionless force (F/T_o) as a function of dimensionless shortening velocity (v/v_{max}). (1 pt)

(c) Like force, the mechanical power output of muscle also depends on shortening velocity. The mechanical power output is the rate at which mechanical work is done. Using your new form of Hill's equation, write an equation that expresses the mechanical power output of muscle in terms of shortening velocity (v), maximum shortening velocity v_{max} and isometric tension T_o. (1 pt)

(d) Graph both the force and power output as a function of velocity for a muscle whose maximum isometric tension is 1 N and the maximum shortening velocity is 1 m/s. (1 pt)

(e) Force and power are just two measures of muscle performance. Efficiency is yet another such measure and is defined as the ratio of the mechanical power output to the rate at which muscle consumes energy. The energy consumption rate of muscle (its metabolic rate) increases *linearly* as shortening velocity increases. At isometric conditions, muscle consumes energy (measured in ATP utilization) at a rate of about 0.4 mM ATP/s. At maximum shortening velocity, it consumes energy at a rate of about 1.5 mM/s. Using Power divided by ATP consumption rate, plot the efficiency of contraction as a function of shortening velocity (v) with v ranging from 0 to v_{max}. (1 pt)



The figure to the left (He et al., 2000, ATP Consumption and Efficiency of Human Single Muscle Fibers with Different Myosin Isoform Composition, *Biophysical Journal*, 79:945-961) plots the rate of energy consumption by muscle as a function of its shortening velocity. The maximum shortening velocity (v_{max}) occurs at about 1 L/s (that is 1 muscle length per second).

3. Discussion points: choose either (a) or (b) (1 pt)

(a) The ATP consumption (APTase) rate varies from muscle type to muscle type. For example, fast fibers in the figure above use more energy than slow ones and the slope of the ATP consumption rate versus shortening velocity is lower for slow fibers. Would velocity for the best efficiency occur at faster or slower speeds for the fast fiber?

(b) Under what conditions would operating a maximum power output be better than maximum efficiency?

4. Another paper (1 pt)

Like a prior paper you read previously in this class, the authors raised an idea (hypothesis) towards the end of the paper that may not have been directly tested by the experiments and data presented. The paper by George et al., (2013) asserts that a temperature gradient leads to a functional gradient and the potential of energy storage in cross-bridges. In particular, they state that

> "In addition to some attachment at the extrema of the length cycle. At these intermediate temperatures, cross-bridges that remain bound at the very end of lengthening or shortening can store energy in their axial or radial extension respectively. This stored energy may return energy into the lattice when the crossbridges detach at the start of the subsequent phase. In doing so, the deformed cross-bridges could assist antagonistic muscles. Prior studies have shown that elastic energy storage is indeed crucial for meeting the high inertial power costs of flight (3, 4). If even a portion of these crossbridges facilitate elastic energy saving via a temperature gradient, they would contribute to the overall energy savings I locomotion. Because temperature gradients are an inevitable consequence of internal energy generation and heat dissipation in both vertebrates and invertebrates, this mechanism of energy storage could be a general phenomenon in locomotor systems (11, 12)."

What experimental approach could you design to test the hypothesis that radial energy storage in cross-bridges contributes to locomotion efficiency? (1 pt)