Biology 427 Biomechanics
Problem Set 7: The last of 2011

Some basic relationships:

Assume induced grad is negligible for all that follows.

Lift = L = $0.5\ \rho\ S\ C_L\ U^2$

Drag = D = $0.5\ \rho\ S\ C_D\ U^2$

For gliders:

$L = m\ g\ \cos(\theta)$

$D = m\ g\ \sin(\theta)$

θ = glide angle

Data (ignore effects of angle of attack).

For vultures

mass = 1 kg

span of wings = 2.0 m

Area of wings = 1.0 m^2

Wing drag coefficient = 0.3

Wing lift coefficient = 1.0

Body drag coefficient = 0.3

Body projected area = 0.2 m^2

Density of air = 1 kg/m^3

Viscosity of air = $15\ 10^{-5}$ kg/ m s

1. The descent of (a spherical) man

The drag coefficient is a function of Reynolds number and, for a sphere, is approximated by the following equation:

$$C_D = \frac{24}{Re} + \frac{6}{1+\sqrt{Re}} + 0.4$$

with

$$Re = \frac{\rho U L}{\mu}$$

Creatures falling in air experience drag forces that resist their downward plunge. We are interested in plotting their velocity as they begin their fall. We know that, at equilibrium, their drag is equal to their weight (the downward gravitational pull). Until equilibrium is reached, however, they accelerate from rest – asymptotically reaching their terminal velocity. In this problem, we will derive the equation for this motion and then use *Mathematica* to solve and plot it.

(a) Write an equation that sets the difference between drag and weight equal to the downward force acting on the animal (mass times acceleration). Assume that the drag coefficient is as above.

(b) To solve an equation like the one above requires a numerical approximation, since the equation is quite non-linear. *Mathematica* provides a way to do so – and it is in a file called "FallingSphere.nb".

The line "NDSolve[….] " solves the differential equation that should look a lot like what you had developed in part (a) above. Explore how size affects the time it takes to reach terminal velocity. (This is just playing with radius.)

(c) Select are particular size and check to be sure the terminal velocity you predict with the Mathematic file matches one you would predict from the simple equation developed in lecture where weight = drag.

2. A soar topic, but uplifting nonetheless.
A sailplane pilot followed a vulture traveling across the Kalahari desert. With no noticeable flapping, the bird was able to travel many kilometers by clever use of thermals spaced throughout the desert. In this problem, we will use simple aerodynamics to compute the average distance between thermals that would permit long distance migration of vultures.

(a) Compute the glide angle of the vulture (be sure to include both wing and body drag in your analysis).

(b) You are now armed with the glide angle. If vultures rise vertically in a thermal to a height of 100m, how far apart must these thermals be for long distance migration? (i.e. What is the spacing of thermals that would allow vultures to travel without flapping wings?)

(c) If thermals elevate vultures at an average vertical velocity of 1 m/s, how long will it take a vulture to travel 1 km with thermals spaced every 200 m?

(d) If vultures rise in a thermal by flying in a circular spiral with a radius of 50 m, what is the average speed of the vulture relative to the ground?

(e) Use "ParametricPlot3D" to plot the trajectory of the vulture.