Lab 4: Vibrations and beam-a-thon

Bio427 — Biomechanics

In this lab, we'll be investigating how stress and strain are distributed within loaded structures, primarily with reference to bending beams.

Goals

- Examine the time-dependent properties of biological materials
- Understand data acquisition using a computer and an Arduino
- Explore resonance and damping in natural systems
- Construct bio-inspired structures that minimize deflection under applied load.
- Construct bio-inspired structures that minimize material investment given a tolerable deflection.

Conceptual Basis

Many biomaterials are viscoelastic (as we have discussed in lecture), showing a time-dependence in their response to loads. There are a host of methods available for quantifying time-dependent properties including creep tests (constant force: isotonic loads), stress relaxation tests (constant length: isometric loads) as well as cyclic dynamic loads.

One fascinating application of dynamic loads is called a free vibration test (colloquially called a "kaboing-wonga-wonga" test) in which you impulsively load a structure and observe the oscillations that follow that load. Those oscillations have a particular frequency for that structure (called the natural frequency) and, in the presence of any visco-elasticity, they damp out in time. The natural frequency for a given structure will depend on the stiffness and density of the material from which it is made: stiffer materials increase the natural frequency, but denser materials decrease the natural frequency. For a cylindrical beam of uniform section, there is a simple relationship that relates the natural frequency to the geometry of that beam and the stiffness and density of the material of which that beam is composed:

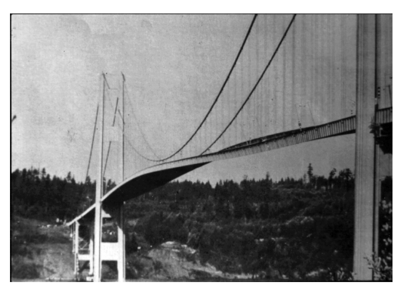

Figure 1: On November 7, 1940 at 11:00 am the Tacoma Narrows bridge was excited by wind vortices and establish large amplitude vibrations. This vibrating beam failed and fell into the sound. Though no lives were lost, it was a narrow escape

$$\omega_n = 3.5 \times \sqrt{\frac{EI}{\rho A L^4}} \tag{1}$$

where E is Young's modulus for the material from which the beam is made, A is the cross-sectional area of the beam, L is the beam length and ρ is the density of the beam. For nearly all biological materials, we take the density to be $1000\,\mathrm{kg\,m^{-3}}$, which is the approximate density of water. Thus, if you knew the length and radius of the beam and its density, you could observe its natural frequency and compute the stiffness of the material that constitutes the beam!

Measuring strain

In the lab exercises you have done so far, you have used measurements of length and time together with mathematical models to investigate biomechanical performance of humans in jumping and walking. In many cases, however, it is not possible to directly obtain the data we want, and in these cases we rely on transducers to convert signals from one type of energy to another.[1] If we wish to measure small changes in strain, we must convert this mechanical energy into an electrical signal using strain gauges. Strain gauges are sensors that change in resistance proportional to the strain they are undergo. These devices, as you will see if you look closely at one, are simply a very thin etched metal foil that essentially forms a very long, very thin wire that doubles back on itself many times – see Figure 2.

[1] A common example of a transducer is the microphone in your cell phone: this is an electroacoustic transducer which converts sound into an electrical signal. Your voice produces propagating changes in air pressure, which cause the motion of a coil within a magnetic field inside the microphone. This results in a potential difference across the conductor, which constitutes an electrical signal which may be transmitted.

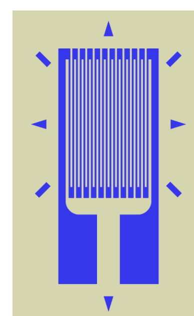

Figure 2: A typical foil strain gauge (Source: Wikimedia Commons)

A strain gauge is mounted onto a surface, and as the surface is deformed, the strain gauge is stretched very slightly. You may recall from physics that the resistance of a wire is given by

$$R = \rho \frac{l}{A} \tag{2}$$

where R is the resistance of the wire in ohms (Ω), ρ is the electrical resistivity of the wire material in ohm-meters ($\Omega\,\mathrm{m}$)[2], l is the length of the wire, and A is the cross sectional area of the wire.

[2] Note that ρ here is not the density of the material, it's the resistivity. They somewhat annoyingly share the same symbol.

So, as the strain gauge is stretched, its l increases, thereby increasing its resistance. This change in resistance can be measured using a Wheatstone bridge, which is a standard device used to measure unknown resistance which may be familiar from physics.

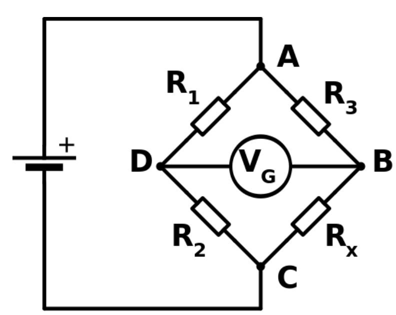

Figure 3: Schematic for a wheatstone bridge. R_1, R_2 and R_3 are all known resistors, but R_X is unknown. In a single-gauge strain gauge application, R_{1-3} are precision resistors, and R_X is a strain gauge.

Data acquisition

Once we have a tranducer capable of converting the time-varying strain signal to an electrical signal, we next need to take that electrical signal and record it for later analysis. In this lab course, we will use small, low-cost microcontroller boards called Arduinos to measure data from our sensors and relay this information to our laptops.

Arduinos are extraordinarily versatile devices with myriad applications, as you will see as this lab course progresses. In this particular lab, we have configured the Arduino boards to read in a single analog voltage on one of its analog input channels, convert this analog signal to a digital representation of that signal[3], and then relay a message containing this digital representation to the laptop via USB serial. We can then configure the laptop to listen for these messages and store these as data.

[3] This is roughly analogous to recording sound on a computer through a microphone and sound card.

Free vibration

For this part of the lab we will be measuring the natural frequency of vibration for a biomaterial (a stick), and use these data to estimate the stiffness of bamboo.

Recall from Eqn 1 that the natural frequency of a vibrating cylindrical beam is determined partly by its stiffness, as well as its density and shape. Therefore, if we can measure the natural frequency of vibration for a structure where we know the dimensions of the structure and the density of the material from which it is composed, we can estimate the stiffness of the material.

We will measure the natural vibration frequency of these beams by applying a foil strain gauge (see Section) directly to the surface of the beam, loading that beam and then releasing that load quickly (the kaboing-wonga-wonga test). As the beam vibrates, the strain gauge will be cyclically stretched and compressed, and we will be able to see this variation in strain as a function of time.

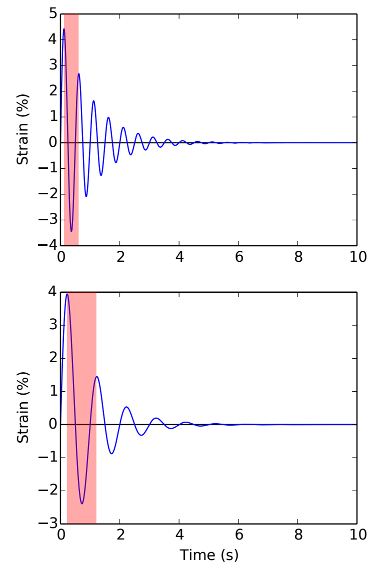

Figure 4: Kaboing-wonga-wonga tests for two otherwise identical cylindrical beams of differing length. A single period of oscillation is highlighted for each beam.

Begin by ensuring that you have an Arduino board connected via USB to your laptop, and that the Arduino is supplying power to the Industrologic SGAU strain gauge amplifier. The strain gauge will form the fourth arm of a Wheatstone bridge, where the other three arms are precision 350 Ω resistors.

Start `yodaq` again, and you should see an output voltage from the strain gauge amplifier that varies as strain is applied to the gauge. Apply a force to the end of the beam, and release this force to allow the beam to vibrate freely. You should see the output voltage from the strain gauge oscillate accordingly. Hit the "Stop" button after several such oscillatory periods, and measure the time between successive peaks of the oscillation by clicking on the plot area. This is the period T of the natural vibration, and from this value you can determine the natural frequency:

$$\omega_n = \frac{2\pi}{T}$$

Use this measured value along with Eqn 1 to estimate the Young's modulus of the beam.

Now, select a beam of a different size, measure its radius and length and calculate its area and second moment of area. Assuming the Young's modulus is the same for both beams, use Eqn 1 to estimate the natural frequency of the second beam. Verify your estimate empirically (measure the frequency).

Cantilever Beam Thunderdome

In this competitive exercise, you are challenged to construct cantilever beams out of paper that are resistant to deflection caused by an external load on its free end. You can and should feel free to use any of the plant stems from the greenhouse as inspiration.

Round 1! Make a stiff beam

In Round 1, your objective is to make your beam as flexurally stiff as possible.

Rules

1. You may use a single sheet of paper, scissors, and tape to construct your beam. Tape can only be used in small quantities to join edges of paper – excessive use of tape as a structural element may lead to disqualification!
2. Your beam will be clamped to the end of a yard stick and cantilevered 15 cm beyond the edge.
3. Whichever beam supports the largest load before failure (or a deflection greater than 3 cm) is the winner!

Round 2! Make a light, reasonably stiff beam

In Round 2, your objective is to make your beam as light as possible, given an acceptable amount of deflection.

Rules

1. Rules 1 and 2 from Round 1 apply. The TA will select the load requirement.
2. The beam may deflect by up to 3 cm vertically under the applied load.
3. Whichever beam is lightest, while undergoing a deflection within the accepted tolerance, is the winner!

Lab 4: Vibrations and beam-a-thon

Lab Section: ______________________

Your Name: ______________________ Partner's Name: ______________________

Free vibration data

		Beam A	Beam B	
1	Sample length (m)	____________	____________	
2	Sample outer radius (m)	____________	____________	
3	Sample inner radius (m)	____________	____________	
4	Sample second moment of area (m^4)	____________	____________	
5	Sample area (m^2)	____________	____________	
6	Measured natural frequency (Hz)	____________	____________	*For Beam B, do after 7b.*
7a	Young's modulus (Pa)	____________	____________	*Assume same as Beam A.*
7b	Predicted natural freq (Hz)		____________	

Questions

1. Let's consider the natural frequency of a cylindrical beam ($A = \pi r^2$, $I = \frac{\pi}{4} r^4$). How would natural frequency change if we (a) doubled the length of a beam? (b) doubled the radius of the beam? (c) doubled the Young's modulus?

2. Briefly describe your strategy for the Cantilever Beam Thunderdome competition in rounds 1 and 2? Sketch your beam cross sections. Compare your beam design to a classmate's. Explain the differences and why your beam performed better/worse.